# ARL Formulas of MA Control Chart with Zero-Inflated Binomial Model when Underlying Distribution is Ratio of Two Poisson Means

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***Abstract*—The aim of this research is to derive an explicit formulas of Average Run Length (ARL) for the Moving Average (MA) control chart with Zero-Inflated Binomial model when the underlying distribution is ratio of two Poisson means. The ARL is used in term of comparison the performance in detecting a change on control chart. The derived formulas of ARL is carried out and showed to be beneficial for quality controller.**

***Index Terms*— Moving Average control chart, Zero-Inflated Binomial, Average Run Length**

## I. Introduction

Control chart is one of the important tools of Statistical Process Control (SPC). SPC are widely used for detecting changes in a processes, measuring, controlling, and improving quality of the process in many areas of interest including medicine, finance and economics, engineering, and others. Shewhart control chart was invented by Dr. Walter Shewhart in the 1924 as tool to determine if a manufacturing process mean is constant. It is well-known fact that Shewhart control chart do not perform well for small shift in the process mean (see, e.g., Pawar [1]). To resolve this defective point, several types of control chart were developed for detecting small shift in the process mean.

In 1954 the Cumulative Sum (CUSUM) control chart was first presented by Page [2]. Next, Exponentially Weighted Moving Average (EWMA) control chart was presented by Roberts [3] in 1959. Both were effective in detecting small shifts in the process mean and better than Shewhart control chart. Recently, Khoo[4] first presented Moving Average (MA) control chart for monitoring the non-conforming or defective fraction in discrete process.

The productions of high quality have been produced by industrial entrepreneurs. The occurrence of large number of zero failures on the products has commonly presented. In particular health engineering, it is called rare health events (see in [5],[6]). Applying to traditional np-chart based on a binomial distribution, a large number of observed zero data for an attribute quality characteristic can be fit the binomial distribution [5], In many cases, binomial distribution may be more flexible and natural to use in place of Poisson distribution with the control charts for ratio of two Poisson means [7], because binomial model is simple and flexible alternative to the Poisson model including case of over-dispersion too[8]. Therefore, developed model can be based on Zero Inflated Binomial (ZIB) distribution. In this paper, analytical ARL formulas of MA control charts when observations are from a zero-inflated binomial model when underlying distribution is ratio of two Poisson means is derived.

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## II. Zero-Inflated Binomial model when the underlying distribution is ratio of two Poisson means

Let observations $X_1, X_2, ..., X_n$ be i.i.d. random variables with binomial distribution underlying ratio of two Poisson means, where $X_i$ number of nonconforming is items in sample *i* of *m* samples of size *n*. A simple way to model Zero-Inflated is to include a proportion $\theta$ of extra-zeros follow from a Binomial distribution. Let *Y* and *Z* be two independent Poisson variates with parameter $\alpha$ and $\beta$ respectively. Then, the condition distribution of *Y* given *Y* + *Z* follows a Binomial distribution with parameters *n* and $p = \frac{\alpha}{\alpha+\beta}$. The Zero-Inflated Binomial Underlying distribution is ratio of two Poisson means density function $X \sim ZIB_{poi}(n, \alpha, \beta, \theta)$ can be written as

$$f(x) = \begin{cases} \theta \binom{n}{x} \left(\frac{\alpha}{\alpha+\beta}\right)^x \left(1-\frac{\alpha}{\alpha+\beta}\right)^{n-x} & ; \ x = 1,2,...,n \\ (1-\theta) + \theta\left(1-\frac{\alpha}{\alpha+\beta}\right)^n & ; \ x = 0 \end{cases}$$

The mean and variance of the above distribution are as follows

$$E(X) = n\left(\frac{\alpha}{\alpha+\beta}\right)\theta$$

and $$Var(X) = \theta n(n-1)\left(\frac{\alpha}{\alpha+\beta}\right)^2 + n\left(\frac{\alpha}{\alpha+\beta}\right)\theta - n^2\left(\frac{\alpha}{\alpha+\beta}\right)^2\theta^2$$

## III. THE MOVING AVERAGE CONTROL CHART FOR $ZIB_{POI}$ OBSERVATIONS

A MA control chart is a type of memory control chart based on equal weighted moving average. Suppose individual observations, $X_1, X_2, ..., X_i, ...$ are collected, the moving average of width *w* at time *i* can be computed as

$$MA_i = \begin{cases} \dfrac{X_i + X_{i-1} + \ldots + X_{i-w+1}}{i} = \dfrac{\sum_{j=1}^{i} X_j}{i} & i < w \\ \dfrac{X_i + X_{i-1} + \ldots + X_{i-w+1}}{w} = \dfrac{\sum_{j=i-w+1}^{i} X_j}{w} & i \geq w \end{cases}$$

where *w* is the width of the MA control chart. The mean and variance of the moving average are as follows:

$$E(MA_i) = E(X_i) = n\left(\frac{\alpha}{\alpha+\beta}\right)\theta$$

and

$$Var(MA_i) = \begin{cases} \dfrac{\theta n(n-1)\left(\frac{\alpha}{\alpha+\beta}\right)^2 + n\left(\frac{\alpha}{\alpha+\beta}\right)\theta - n^2\left(\frac{\alpha}{\alpha+\beta}\right)^2\theta^2}{ni}, & i < w \\ \dfrac{\theta n(n-1)\left(\frac{\alpha}{\alpha+\beta}\right)^2 + n\left(\frac{\alpha}{\alpha+\beta}\right)\theta - n^2\left(\frac{\alpha}{\alpha+\beta}\right)^2\theta^2}{nw}, & i \geq w \end{cases}$$

For periods $i < w$, the control limits are given by

$$UCL / LCL = n\left(\frac{\alpha}{\alpha+\beta}\right)\theta \pm k\sqrt{\frac{\theta n(n-1)\left(\frac{\alpha}{\alpha+\beta}\right)^2 + n\left(\frac{\alpha}{\alpha+\beta}\right)\theta - n^2\left(\frac{\alpha}{\alpha+\beta}\right)^2\theta^2}{in}}$$

and for periods $i \geq w$

$$UCL / LCL = n\left(\frac{\alpha}{\alpha+\beta}\right)\theta \pm k\sqrt{\frac{\theta n(n-1)\left(\frac{\alpha}{\alpha+\beta}\right)^2 + n\left(\frac{\alpha}{\alpha+\beta}\right)\theta - n^2\left(\frac{\alpha}{\alpha+\beta}\right)^2\theta^2}{wn}}$$

where *k* is the width of control limit.

## IV. THE EXPLICIT FORMULA FOR EVALUATING ARL FOR MA CONTROL CHART

The ARL values of MA control chart for Zero-Inflated Binomial model underlying two Poisson Means can be derived as follows. Let ARL = n and $p_j = \dfrac{\alpha_j}{\alpha_j + \beta_j}$, then

$$\begin{aligned} \frac{1}{ARL} &= \left(\frac{1}{n}\right)P(o.o.c. signal\ at\ time\ i < w) \\ &+ \left[\frac{n-(w-1)}{n}\right]P(o.o.c. signal\ at\ time\ i \geq w) \\ &= \left(\frac{1}{n}\right)\left\{\sum_{i=1}^{w-1}\left[P\left(\frac{\sum_{j=1}^{i} np_j\theta_j}{i} > UCL_i\right) + P\left(\frac{\sum_{j=1}^{i} np_j\theta_j}{i} < LCL_i\right)\right]\right\} \\ &+ \left[\frac{n-(w-1)}{n}\right]\left[P\left(\frac{1}{w}\sum_{j=i-w+1}^{i} np_j\theta_j > UCL_w\right)\right. \\ &\left. + P\left(\frac{1}{w}\right)\sum_{j=i-w+1}^{i} np_j\theta_j < LCL_w\right] \end{aligned}$$

$$=\left(\frac{1}{n}\right)\left\{\sum_{i=1}^{w-1}\left[P\left(\frac{\sum_{j=1}^{i} n\left(\frac{\alpha_j}{\alpha_j+\beta_j}\right)\theta_j}{i} > UCL_i\right) + P\left(\frac{\sum_{j=1}^{i} n\left(\frac{\alpha_j}{\alpha_j+\beta_j}\right)\theta_j}{i} < LCL_i\right)\right]\right\}$$

$$+\left[\frac{n-(w-1)}{n}\right]\left[P\left(\frac{1}{w}\sum_{j=i-w+1}^{i} n\left(\frac{\alpha_j}{\alpha_j+\beta_j}\right)\theta_j > UCL_w\right)\right.$$

$$\left.+P\left(\frac{1}{w}\sum_{j=i-w+1}^{i} n\left(\frac{\alpha_j}{\alpha_j+\beta_j}\right)\theta_j < LCL_w\right)\right]$$

$$=\left(\frac{1}{n}\right)\left\{\sum_{i=1}^{w}\left[P\left(\frac{\sum_{j=1}^{i} n\left(\frac{\alpha_j}{\alpha_j+\beta_j}\right)\theta_j}{i}\right.\right.\right.$$

$$\left. > n\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)\theta_0 + 3\sqrt{\frac{\theta_0 n(n-1)\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)^2 + n\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)\theta_0 - n^2\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)^2\theta_0^2}{in}}\right)$$

$$\left.\left.+P\left(\frac{\sum_{j=1}^{i} n\left(\frac{\alpha_j}{\alpha_j+\beta_j}\right)\theta_j}{i} < n\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)\theta_0 - 3\sqrt{\frac{\theta_0 n(n-1)\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)^2 + n\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)\theta_0 - n^2\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)^2\theta_0^2}{in}}\right)\right]\right\}$$

$$+\left[\frac{n-(w-1)}{n}\right]\left[P\left(\frac{1}{w}\sum_{j=i-w+1}^{i} n\left(\frac{\alpha_j}{\alpha_j+\beta_j}\right)\theta_j\right.\right.$$

$$\left. > n\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)\theta_0 + 3\sqrt{\frac{\theta_0 n(n-1)\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)^2 + n\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)\theta_0 - n^2\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)^2\theta_0^2}{wn}}\right)$$

$$+P\left(\frac{1}{w}\sum_{j=i-w+1}^{i} n\left(\frac{\alpha_j}{\alpha_j+\beta_j}\right)\theta_j\right.$$

$$\left.\left. < n\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)\theta_0 - 3\sqrt{\frac{\theta_0 n(n-1)\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)^2 + n\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)\theta_0 - n^2\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)^2\theta_0^2}{wn}}\right)\right]$$

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Defining

$$Z_1 = \frac{\sum_{j=1}^{i} n\left(\frac{\alpha_j}{\alpha_j+\beta_j}\right)\theta_j - n\left(\frac{\alpha_1}{a_1+\beta_1}\right)\theta_1}{\sqrt{\frac{\left(\theta_1 n(n-1)\left(\frac{\alpha_1}{a_1+\beta_1}\right)^2 + n\left(\frac{\alpha_1}{a_1+\beta_1}\right)\theta_1 - n^2\left(\frac{\alpha_1}{a_1+\beta_1}\right)^2\theta_1^2\right)}{in}}}$$

and

$$Z_2 = \frac{\sum_{j=i-w+1}^{i} n\left(\frac{\alpha_j}{\alpha_j+\beta_j}\right)\theta_j - n\left(\frac{\alpha_1}{\alpha_1+\beta_1}\right)\theta_1}{\sqrt{\frac{\left(\theta_1 n(n-1)\left(\frac{\alpha_1}{\alpha_1+\beta_1}\right)^2 + n\left(\frac{\alpha_1}{\alpha_1+\beta_1}\right)\theta_1 - n^2\left(\frac{\alpha_1}{\alpha_1+\beta_1}\right)^2\theta_1^2\right)}{wn}}}$$

,then

$$\frac{1}{ARL} = \left(\frac{1}{n}\right)\left\{\sum_{i=1}^{w-1}\left[P\left(Z_1 > \frac{\left(n\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)\theta_0 + 3\sqrt{\frac{\theta_0 n(n-1)\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)^2 + n\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)\theta_0 - n^2\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)^2\theta_0^2}{in}}\right) - n\left(\frac{\alpha_1}{a_1+\beta_1}\right)\theta_1}{\sqrt{\frac{\left(\theta_1 n(n-1)\left(\frac{\alpha_1}{a_1+\beta_1}\right)^2 + n\left(\frac{\alpha_1}{a_1+\beta_1}\right)\theta_1 - n^2\left(\frac{\alpha_1}{a_1+\beta_1}\right)^2\theta_1^2\right)}{in}}}\right)\right.\right.$$

$$\left.\left. + P\left(Z_1 < \frac{\left(n\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)\theta_0 - 3\sqrt{\frac{\theta_0 n(n-1)\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)^2 + n\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)\theta_0 - n^2\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)^2\theta_0^2}{in}}\right) - n\left(\frac{\alpha_1}{a_1+\beta_1}\right)\theta_1}{\sqrt{\frac{\left(\theta_1 n(n-1)\left(\frac{\alpha_1}{a_1+\beta_1}\right)^2 + n\left(\frac{\alpha_1}{a_1+\beta_1}\right)\theta_1 - n^2\left(\frac{\alpha_1}{a_1+\beta_1}\right)^2\theta_1^2\right)}{in}}}\right)\right]\right\}$$

$$+\left[\frac{n-(w-1)}{n}\right]\left[P\left(Z_2 > \frac{\left(n\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)\theta_0 + 3\sqrt{\frac{\theta_0 n(n-1)\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)^2 + n\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)\theta_0 - n^2\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)^2\theta_0^2}{wn}}\right) - n\left(\frac{\alpha_1}{a_1+\beta_1}\right)\theta_1}{\sqrt{\frac{\left(\theta_1 n(n-1)\left(\frac{\alpha_1}{a_1+\beta_1}\right)^2 + n\left(\frac{\alpha_1}{a_1+\beta_1}\right)\theta_1 - n^2\left(\frac{\alpha_1}{a_1+\beta_1}\right)^2\theta_1^2\right)}{wn}}}\right)\right.$$

$$\left. + P\left(Z_2 < \frac{\left(n\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)\theta_0 - 3\sqrt{\frac{\theta_0 n(n-1)\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)^2 + n\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)\theta_0 - n^2\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)^2\theta_0^2}{wn}}\right) - n\left(\frac{\alpha_1}{a_1+\beta_1}\right)\theta}{\sqrt{\frac{\left(\theta_1 n(n-1)\left(\frac{\alpha_1}{a_1+\beta_1}\right)^2 + n\left(\frac{\alpha_1}{a_1+\beta_1}\right)\theta_1 - n^2\left(\frac{\alpha_1}{a_1+\beta_1}\right)^2\theta_1^2\right)}{wn}}}\right)\right] \quad (1)$$

Let

$$U=\left\{\sum_{i=1}^{w-1}\left[P\left(Z_1>\frac{\left(n\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)\theta_0+3\sqrt{\frac{\theta_0 n(n-1)\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)^2+n\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)\theta_0-n^2\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)^2\theta_0^2}{in}}\right)-n\left(\frac{\alpha_1}{a_1+\beta_1}\right)\theta_1}{\sqrt{\frac{\left(\theta_1 n(n-1)\left(\frac{\alpha_1}{a_1+\beta_1}\right)^2+n\left(\frac{\alpha_1}{a_1+\beta_1}\right)\theta_1-n^2\left(\frac{\alpha_1}{a_1+\beta_1}\right)^2\theta_1^2\right)}{in}}}\right)\right.\right.$$

$$\left.\left.+P\left(Z_1<\frac{\left(n\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)\theta_0-3\sqrt{\frac{\theta_0 n(n-1)\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)^2+n\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)\theta_0-n^2\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)^2\theta_0^2}{in}}\right)-n\left(\frac{\alpha_1}{a_1+\beta_1}\right)\theta_1}{\sqrt{\frac{\left(\theta_1 n(n-1)\left(\frac{\alpha_1}{a_1+\beta_1}\right)^2+n\left(\frac{\alpha_1}{a_1+\beta_1}\right)\theta_1-n^2\left(\frac{\alpha_1}{a_1+\beta_1}\right)^2\theta_1^2\right)}{in}}}\right)\right]\right\}$$

$$V=\sum_{i=1}^{w-1}\left[P\left(Z_2>\frac{\left(n\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)\theta_0+3\sqrt{\frac{\theta_0 n(n-1)\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)^2+n\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)\theta_0-n^2\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)^2\theta_0^2}{wn}}\right)-n\left(\frac{\alpha_1}{a_1+\beta_1}\right)\theta_1}{\sqrt{\frac{\left(\theta_1 n(n-1)\left(\frac{\alpha_1}{a_1+\beta_1}\right)^2+n\left(\frac{\alpha_1}{a_1+\beta_1}\right)\theta_1-n^2\left(\frac{\alpha_1}{a_1+\beta_1}\right)^2\theta_1^2\right)}{wn}}}\right)\right.$$

$$\left.+P\left(Z_2<\frac{\left(n\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)\theta_0-3\sqrt{\frac{\theta_0 n(n-1)\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)^2+n\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)\theta_0-n^2\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)^2\theta_0^2}{wn}}\right)-n\left(\frac{\alpha_1}{a_1+\beta_1}\right)\theta}{\sqrt{\frac{\left(\theta_1 n(n-1)\left(\frac{\alpha_1}{a_1+\beta_1}\right)^2+n\left(\frac{\alpha_1}{a_1+\beta_1}\right)\theta_1-n^2\left(\frac{\alpha_1}{a_1+\beta_1}\right)^2\theta_1^2\right)}{wn}}}\right)\right]$$

Substituting $U$ and $V$ into Eq. (1), we obtain:

$$\frac{1}{n} = \frac{1}{n}U + \left[\frac{n-(w-1)}{n}\right]V$$

$$n = (1-U)V^{-1} + (w-1)$$

Therefore

$$ARL = \left\{ 1 - \sum_{i=1}^{w-1} \left[ P\left( Z_1 > \frac{\left( n\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)\theta_0 + 3\sqrt{\frac{\theta_0 n(n-1)\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)^2 + n\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)\theta_0 - n^2\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)^2\theta_0^2}{in}} \right) - n\left(\frac{\alpha_1}{a_1+\beta_1}\right)\theta_1}{\sqrt{\frac{\left(\theta_1 n(n-1)\left(\frac{\alpha_1}{a_1+\beta_1}\right)^2 + n\left(\frac{\alpha_1}{a_1+\beta_1}\right)\theta_1 - n^2\left(\frac{\alpha_1}{a_1+\beta_1}\right)^2\theta_1^2\right)}{in}}} \right) \right.\right.$$

$$\left.\left. + P\left( Z_1 < \frac{\left( n\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)\theta_0 - 3\sqrt{\frac{\theta_0 n(n-1)\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)^2 + n\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)\theta_0 - n^2\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)^2\theta_0^2}{in}} \right) - n\left(\frac{\alpha_1}{a_1+\beta_1}\right)\theta_1}{\sqrt{\frac{\left(\theta_1 n(n-1)\left(\frac{\alpha_1}{a_1+\beta_1}\right)^2 + n\left(\frac{\alpha_1}{a_1+\beta_1}\right)\theta_1 - n^2\left(\frac{\alpha_1}{a_1+\beta_1}\right)^2\theta_1^2\right)}{in}}} \right) \right] \right\}$$

$$\times P\left( Z_2 < \frac{\left( n\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)\theta_0 - 3\sqrt{\frac{\theta_0 n(n-1)\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)^2 + n\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)\theta_0 - n^2\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)^2\theta_0^2}{in}} \right) - n\left(\frac{\alpha_1}{a_1+\beta_1}\right)\theta_1}{\sqrt{\frac{\left(\theta_1 n(n-1)\left(\frac{\alpha_1}{a_1+\beta_1}\right)^2 + n\left(\frac{\alpha_1}{a_1+\beta_1}\right)\theta_1 - n^2\left(\frac{\alpha_1}{a_1+\beta_1}\right)^2\theta_1^2\right)}{in}}} \right) \Bigg] \Bigg] \Bigg\}$$

$$\times \left[ P\left( Z_2 > \frac{\left( n\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)\theta_0 + 3\sqrt{\frac{\theta_0 n(n-1)\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)^2 + n\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)\theta_0 - n^2\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)^2\theta_0^2}{wn}} \right) - n\left(\frac{\alpha_1}{a_1+\beta_1}\right)\theta_1}{\sqrt{\frac{\left(\theta_1 n(n-1)\left(\frac{\alpha_1}{a_1+\beta_1}\right)^2 + n\left(\frac{\alpha_1}{a_1+\beta_1}\right)\theta_1 - n^2\left(\frac{\alpha_1}{a_1+\beta_1}\right)^2\theta_1^2\right)}{wn}}} \right) \right.$$

$$\left. + P\left( Z_2 < \frac{\left( n\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)\theta_0 - 3\sqrt{\frac{\theta_0 n(n-1)\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)^2 + n\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)\theta_0 - n^2\left(\frac{\alpha_0}{\alpha_0+\beta_0}\right)^2\theta_0^2}{wn}} \right) - n\left(\frac{\alpha_1}{a_1+\beta_1}\right)\theta}{\sqrt{\frac{\left(\theta_1 n(n-1)\left(\frac{\alpha_1}{a_1+\beta_1}\right)^2 + n\left(\frac{\alpha_1}{a_1+\beta_1}\right)\theta_1 - n^2\left(\frac{\alpha_1}{a_1+\beta_1}\right)^2\theta_1^2\right)}{wn}}} \right) \right]^{-1} \quad (2)$$

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## V. Numerical Results

In this section, the numerical results of average run length, in-control state ($ARL_0$) and out-of-control state ($ARL_1$), for Zero-Inflated Binomial model underlying two Poisson means MA control chart was calculated from Eq.(2) and are demonstrated. The parameter values for MA control chart were chosen by setting the desired $ARL_0$ = 370, $\theta = 0.05$ for probability of the observation is zero, the mean of Poisson $\alpha_0 = 3, \beta_0 = 10$ ,the simulation observation n = 500 and the shifts parameter δ = 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.1, 0.2, 0.3, 0.4, 0.5, 1.0, 2.0. The ARL vales calculated from explicit formula are shown in Table 1 and Table 2.

Table 1 shows the ARL values of Zero-Inflated Binomial model underlying two Poisson means. By varying $\alpha$ shifts, the results show that MA control chart with *w* = 20 has the best performance when shifts are small. For large shifts $2 \le w \le 5$ has the best performance.

Table 2 shows the ARL of Zero-Inflated Binomial model underlying two Poisson means. By varying $\beta$, the results show that the MA control chart with *w* = 20 has the best performance when shifts are small. For large shifts $2 \le w \le 15$ has the best performance.

Note that, calculation of ARL with the explicit formulas in Eq. (2) are simple and very fast with computational time of less than 1 second.

## VI. Conclusion

Explicit formulas is derived for ARL on a MA control chart for observations from a Zero-Inflated Binomial underlying two Poisson means. The novel formulas of ARL is easy to use and easy to calculate and program. The performance of the novel formulas for ARL is demonstrated. The results show that for given ARL = 370 of MA control chart has best performance when the shifts in parameter values from an in-control to an out-of-control state are small.

TABLE I

The ARL results for $ZIB_{poi}$ MA chart where $\theta = 0.05$ and $ARL_0$ =370, when $\alpha$ shifts.

| $\beta$ | $\alpha$ | *w* | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 2 | 3 | 4 | 5 | 10 | 15 | 20 |
| 10 | 3 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 |
| | 3.01 | 360.640 | 360.340 | 360.042 | 359.745 | 358.280 | 356.847 | **355.445** |
| | 3.02 | 350.075 | 348.933 | 347.801 | 346.678 | 341.212 | 335.981 | **330.976** |
| | 3.03 | 338.840 | 336.405 | 334.009 | 331.651 | 320.403 | 310.003 | **300.377** |
| | 3.04 | 327.073 | 322.998 | 319.026 | 315.154 | 297.183 | 281.279 | **267.152** |
| | 3.05 | 314.910 | 308.951 | 303.210 | 297.678 | 272.816 | 251.898 | **234.149** |
| | 3.06 | 302.483 | 294.494 | 286.903 | 279.684 | 248.376 | 223.436 | **203.261** |
| | 3.07 | 289.916 | 279.842 | 270.415 | 261.579 | 224.681 | 196.911 | **175.492** |
| | 3.08 | 277.321 | 265.184 | 254.012 | 243.701 | 202.294 | 172.862 | **151.194** |
| | 3.09 | 264.799 | 250.685 | 237.918 | 226.323 | 181.555 | 151.479 | **130.307** |
| | 3.1 | 252.440 | 236.483 | 222.308 | 209.646 | 162.623 | 132.722 | **112.555** |
| | 3.2 | 148.651 | 124.681 | 106.916 | 93.306 | 56.523 | 41.615 | **34.780** |
| | 3.3 | 86.239 | 66.213 | 53.306 | 44.448 | 25.116 | 19.991 | **19.064** |
| | 3.4 | 52.051 | 37.684 | 29.329 | 24.054 | 14.557 | **13.533** | 14.636 |
| | 3.5 | 33.158 | 23.203 | 17.877 | 14.759 | **10.344** | 11.021 | 12.752 |
| | 4 | 7.137 | 5.287 | 4.664 | **4.523** | 5.549 | 6.633 | 7.231 |
| | 5 | 2.240 | **2.181** | 2.291 | 2.416 | 2.666 | 2.681 | 2.681 |

TABLE II

The ARL results for $ZIB_{poi}$ MA chart where $\theta = 0.05$ and $ARL_0$ =370, when $\beta$ shifts.

| $\alpha$ | $\beta$ | *w* | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 2 | 3 | 4 | 5 | 10 | 15 | 20 |
| 3 | 10 | 370.398 | 370.398 | 370.398 | 370.398 | 370.398 | 370.398 | 370.398 |
| | 9.99 | 367.560 | 367.533 | 367.505 | 367.477 | 367.340 | 367.205 | **367.072** |
| | 9.98 | 364.635 | 364.525 | 364.416 | 364.306 | 363.765 | 363.233 | **362.710** |
| | 9.97 | 361.627 | 361.382 | 361.138 | 360.895 | 359.694 | 358.518 | **357.365** |
| | 9.96 | 358.537 | 358.106 | 357.678 | 357.252 | 355.151 | 353.104 | **351.107** |
| | 9.95 | 355.370 | 354.705 | 354.044 | 353.388 | 350.166 | 347.043 | **344.017** |
| | 9.94 | 352.129 | 351.183 | 350.245 | 349.315 | 344.767 | 340.392 | **336.185** |
| | 9.93 | 348.817 | 347.547 | 346.289 | 345.044 | 338.988 | 333.213 | **327.706** |
| | 9.92 | 345.437 | 343.802 | 342.185 | 340.587 | 332.865 | 325.571 | **318.681** |
| | 9.91 | 341.994 | 339.954 | 337.943 | 335.958 | 326.433 | 317.532 | **309.210** |
| | 9.90 | 338.491 | 336.011 | 333.571 | 331.170 | 319.728 | 309.161 | **299.392** |
| | 9.8 | 300.950 | 292.706 | 284.888 | 277.466 | 245.421 | 220.069 | **199.681** |
| | 9.7 | 261.339 | 246.697 | 233.518 | 221.604 | 176.100 | 146.000 | **125.066** |
| | 9.6 | 222.711 | 202.939 | 186.179 | 171.817 | 123.209 | 96.034 | **79.437** |
| | 9.5 | 187.159 | 164.306 | 146.087 | 131.271 | 86.281 | 64.641 | **52.936** |
| | 9 | 72.475 | 54.398 | 43.175 | 35.698 | 20.378 | **17.036** | 17.040 |
| | 8 | 13.483 | 9.380 | 7.5708 | **6.740** | 6.871 | 8.417 | 9.842 |

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