Application of Optimal Policies for a Two-stage Product Supply Chain Management Inventory System

Enahoro A. OWOLOKO, *Member, IAENG,* and Imekela D. EZEKIEL

***Abstract*—The study examines extended two-echelon newsvendor problem for single product manufacturer, who is also the supplier, and having multiple distributors for the product in a supply chain system under specific (or not) service level requirements optimisation problems. This paper studies two specific supply chain management systems of a single product: a decentralised or restricted supply chain management system; where the product supplier keeps separate stocks for each distributor; and centralised or combined inventory system where the product supplier holds one central warehouse of stocks shared by the distributors. The product supplier makes the inventory decisions based on the random demand of the distributors and the retailers. The distributors, who in turn make inventory policies sell directly to the retailers. We found that regardless of whether the distributors impose service level requirement or not, the profits of the supplier and the distributors always increase after combining inventory when product wholesale price is fixed and random demands of the distributors and the retailers are assumed normally distributed. The results of the findings opened other areas for more complete simulation study in order to capture full understanding of relative performance of supply chain management system.**

***Index Terms*— combined inventory, customer service level, restricted inventory, supply chain**

I. Background to the Study

THIS paper studies two different supply chain management systems for a single manufacturer who is also the supplier. These are the traditional or the restricted supply chain management system; where the supplier holds, separately, restricted inventories for the distributors and combined inventory system where the supplier holds one central inventory or single distribution centres which is shared by the distributors. The paper examines a supply chain system in which an upstream supplier maintained N physically separated inventories of cements products for N competing distributors. Since product demands is highly uncertain coupled with its shortened product life cycles, the procurement trend in this factory is to delay ownership until after the demand for the product is known. The focus of this paper is on the advantage, value and benefits of combined inventory supply chain management system on the optimal profit and optimal order quantity for the supplier and the distributors when compared to the traditional supply chain management system. It is assumed that the distributors' and retailers' random demands are independently normally distributed.

The paper therefore considers a Cement Producing Factory (CPF), which holds inventory of cement for N cement distributors. The current existing inventory decision by the company is to keep each company's inventory physically separate in their warehouses and distribution centres for their various distributors. This paper determines the most profitable inventory policy for both the supplier and the distributors by examining the value of combined versus the restricted inventory policies. The study also determines which of the inventory policies will give maximum profit to both the supplier and the competing distributors. Additionally, how about when the cement factory agreed to maintain its reputational cost by accepting the distributors service level agreements? The study explores these statements and question above for a dual-echelon supply chain management system using descriptive design to address the question and the statement raised above with a cement producing company who is also the supplier with N distributors. In this work, it will be assumed that $N \geq 2$.

The aim of this paper therefore concentrates on the value and advantage of combining inventory system as against the traditional restricted or dedicated inventory system. Therefore, the objectives of this work are to prove, discuss and compare the benefits of combined inventory system over the restricted inventory policy for product supply chain management system when random demands are normally distributed.

II. Literature Review

The single-item single period newsvendor problem is a classical problem in operations management literature used in handling optimization problems involving supply chain management systems. The newsvendor model, which starts as newsboy and Christmas tree problems can be traced back to [7]. Eppen [8] pioneered the benefits of inventory pooling

Manuscript received Dec. 12, 2016; revised Jan.,09, 2017.
This work was supported in part by Covenant University, Nigeria.
E.A.O. and E.D.I. (alfred.owoloko@covenantuniversity.edu.ng) are with the Department of Mathematics, Covenant University, Canaan land, Ota, Nigeria.

for profit maximization and the effects of risk pooling on safety stocks due to shortened product life cycles, procurement trend and demand uncertainty.

Literature on inventory pooling encompasses a wide range of supply chain management system involving product component commonality (PCC), inventory transshipment in single echelon supply chain and pricing decisions in single-period models and so on. Product Component Commonality (PCC) for instance encourages the use of a common component to replace several distinguishable components of the same products so that safety stock costs can be significantly reduced by pooling together inventories of various distributors to a single warehouse while still maintaining service level requirements. Product component commonality (PCC) is one common chain strategies used in handling inventory challenges. Among the significant amount of literature on PCC are the works in [3], [6], [1], [9] and [5] which show that by introducing component commonality the total number of units in stock is reduced. However, a major drawback of component commonality is that it centred heavily on changes in safety stocks to the neglect on the value and gains of inventory pooling both to the supplier and the distributors in a supply chain management system.

Inventory transshipment in Single Echelon Supply Chains system on the other hand involves transferring stocks from one distributor who has surplus stocks as left over to another distributor of the same echelon of a supply chain who is stocked-out provided the associated cost of transferring the stocks is not too high. A significant amount of literature in inventory transshipments assumed that complete pooling policy is to be applied. In the works of [17], [2] and [15], they suggested lateral transshipments among warehouses by centralizing the warehouses in an echelon in the same echelon. However, there is no inventory pooling and their results only apply to determining the transshipment rate among warehouses.

A significant amount of literatures investigates the benefits of inventory pooling in product supply chain system with more than one echelon. Netessine and Rudi [11], in their work concentrated on the gains and the values of inventory pooling problem for substitutable products. Substitutable products occur when full substitution without stock-out is allowed. The authors consider cases when the distributors hold inventory purchased from the supplier when the supplier holds inventory. However, they do not give any result on the total inventory level of the items being combined but submits that optimal inventory levels of the substituted items increase for some items after pooling. In Sukun et al [14], they considered two echelon supply chain system with two retailers and one common supplier who bears the inventory risk. Their focus is particularly essential because of the benefits of pooling on the ability to find optimal inventory levels. However, they only considered cases in which the wholesale price is fixed without service level requirement. In [6], they assumed multivariate normal demands and based their findings on the benefits of inventory pooling over reserved inventory system, by using copulas to model dependence structure between demands. However, Xiaoli et al [16] considered a two-echelon supply chain policy with single product single period supplier and two retailers where the supplier takes all the supply and inventory management decisions. Our work extends the single supplier and two retailers' model proposed in Xiaoli et al [16] to single supplier and N distributors with and without service level constraints.

In Ozsen et al [12], they analyzed a centralized coordination system in which a single company owns the production facility and the set of distributors with established warehouses and distribution centres that will replenish the retailers' inventories. Their model is formulated with an objective function that is neither concave nor nonconvex.

Sukun et al [14] considered a supply chain system consisting of suppliers, distribution centres and retailers. Their objective is to minimize the location cost, transportation and routing costs and inventory costs through allocation and assignment process. Kumar and Tiwari [10] considered inventory location, production, distribution centres and inventory systems and design a model for a supply chain to determine facility locations and their capacity to minimize total network cost for profit maximization. Because demands are stochastic, the model considers risk pooling effect for both safety stock and inventory running costs. In their work, Patel and Gor [13] considered a lost sale recapture model in a newsvendor framework where they assume the lost sales to be lost finally. However, there may be an opportunity to backlog the lost sales. This is done by offering some rebate that will maximize its expected profit.
However, the objective of this study is to determine the most profitable inventory policy for both the supplier and the distributors by examining different inventory systems. This work therefore, proves, discusses, and compared the advantage of combined inventory system over the restricted inventory system for product supply chain management system using cement producing company as our case study when the wholesale price is fixed.

III. MODEL ASSUMPTIONS AND FORMULATION

In this paper, it is assumed that the supplier bears the inventory risk. It is also assumed that the supplier, which is the cement factory, has only one chance to produce before the selling season and that sales are lost when stock-out occurs at the supplier's end. Furthermore, sales of produced products occur during or at the end of the single selling period. We assume that each distributor has its own customers' domain. The objective of the supplier is to maximize her single-product single-period profit. Profits of the distributors are maximized when they receive their full order. However, the distributors' profits depend only on expected sales since they do not have control over inventory decisions.

A. Model Formulation for the Inventory Systems without Service Level Agreement

In this section, we consider a situation where wholesale price is fixed for restricted inventory system when the factory stocks $x_i, (i = 1, 2, \dots, N)$ units of cement products for the distributors at production unit cost c. Let D_i be the random demands of the distributors or the retailers. The distributors buy from the supplier after they receive orders from retailers or the consumers and pay a unit wholesale price $w_i > c > 0$ for each unit received. The distributors sell directly to the retailers or the consumers at a unit retail price $p_i = w_i + m_i$ with a unit mark-up price m_i at distributor i. For each stock

x_i of the supplier, if $x_i \geq D_i$, then $x_i - D_i$ are the left over at the end of the selling season. The left over is the units remaining at the end of the selling season which can be salvaged at per unit cost v_i. It is assumed that $v_i < c_i$. In this study, we assume that v_i is the holding cost. Similarly, if $x_i < D_i$, then $D_i - x_i$ units represent lost sales which will be assumed to be lost finally. Note that $i, (i = 1, \dots, N)$

In a similar manner, for the combined inventory case, the supplier stocks x_{p} units of the products for the distributors at manufacturing cost c per unit. Let $\mathrm{D_p}$ be the joint random demands of the retailers or the consumers for combined inventory case. The joint distributors buy from the supplier after they receive orders from their customers and pay a unit wholesale price $w_p > c > 0$ for each unit received. The distributors sell directly to the retailers or the consumers who in turn pay a unit retail price of $p_p = w_p + m_p$ with a unit mark-up price m_p. For each stock x_p from the combined inventory, if $x_p \geq D_p$, then $x_p - D_p$ are the left over at the end of the selling season. The left over represents units remaining at the end of the selling season and salvaged at per unit cost v_p, where for the purpose of this paper, it is assumed that $v_p < c_p$. We further assume that v_p is the unit holding cost for the combined inventory system. Similarly, if $x_p < D_p$ then $D_p - x_p$ units represent lost sales which we assumed to be lost finally.

For mathematical simplicity, continuity of demand will be assumed; simplify the analysis since in practice demand is discrete. At the beginning of the one selling season, the single supplier is interested in determining optimal profit and a stocking policy of inventory x_i to satisfy each distributor's demand. For each distributor, the retailer or customer's demand D_i is assumed to be stochastic and characterized by a random demand with probability density function (PDF) $f_i(.)$ and cumulative density function (CDF) $F_i(.)$.

In order to make the distributor happy the supplier commits a reputational cost. The supplier tries to maximize his expected profit while satisfying a minimum service level agreement for each distributor. However, the distributors determine the reputational cost for the supplier in this study. Throughout this paper, we assume the minimum service level agreement is measured by the probability of no stock-out. We shall denote by τ_{i} the minimum acceptable probability of no stock-out for the distributors in the restricted inventory case and τ_{p} for the combined inventory system.

In examining the advantage of combined inventory system over restricted system when wholesale prices are fixed, we further assume that the customers' demands are independent, nonnegative, and continuous random variables with CDF $F_i(.)$ and PDF $f_i(.)$ respectively. We let D_p be the demand for the joint distributors with PDF $f_p(.)$ and CDF $F_p(.)$. Let the joint continuous random demand be $D_P = D_1 + D_2 + \cdots + D_N$ and the joint cumulative density function is $F_P(.) = F_1(.) + F_2(.) + \cdots + F_N(.)$.

With these assumptions, we analyse the decisions of the supplier for both restricted and combined inventory policies. We also compare the effect of the two policies on the expected profits of the supplier and their total expected sales and profit of the distributors when demands are normally distributed random variables. For each of the policies, we consider cases when distributors have and does not have service level constraint.

For the restricted inventory case, when the distributors do not impose service level agreements on the supplier, the supplier keeps separate inventories for each distributor. The objective of the supplier is to maximize his expected profit, which is defined as the expected revenue less the expected holding and the production (or manufacturing) costs.

Let $\Pi^{ri}(x_1, x_2, \dots, x_N)$ denote supplier's expected profit for any chosen inventory levels $x_1, x_2, \dots, x_N$ for the restricted inventory system defined as:

$$\Pi^{ri}(x_1, \dots, x_N) := E[w_i \min(x_i, D_i) - v\,(x_i - D_i)^+ - cx_i], (i = 1, 2, \dots, N) \quad (1)$$

where E is the expectation operator taken over the random variables, D_i. The first term $\min(x_i, D_i)$ represents units sold at supplier wholesale price w_i, the second term $(x_i - D_i)^+ = \max(x_i - D_i; 0)$ corresponds to units salvaged at unit holding cost v , and the third term is the unit manufacturing cost. Because of continuity of demand, the expected sale volume is defined as

$$\int_0^\infty \min(x_i, D_i) f(u)du = \int_0^{x_i} uf(u)du + \int_{x_i}^\infty f(u)du \quad (2)$$

and the expected units salvaged is given by

$$\int_0^{x_i} \min\,(x_i - D_i)^+ \mathrm{f(u)du} = \int_0^{x_i} x_i \mathrm{f(u)du} - \int_0^{x_i} \mathrm{D_i f(u)du} \quad (3)$$

Since $D_i, (i = 1, 2, \dots, N)$ are independent random variables, $\Pi^{ri}(x_1, x_2, \dots, x_N)$ can be decomposed as

$$\Pi^{ri} = (x_1, x_2, \dots, x_N) = \Pi^{r1}(x_1) + \cdots + \Pi^{rN}(x_N)$$

Where

$$\Pi^{ri}(x_i) := E[w_i \min(x_i, D_i) - v\,(x_i - D_i)^+ - cx_i],\ (i = 1, 2, \dots, N) \quad (4)$$

Equation (4) can be expressed as

$$\Pi^{ri}(x_i) = w_i \int_0^\infty \min(x_i, D_i) f_i(u)du - v \int_0^{x_i} \min(x_i - D_i) f_i(u)du - cx_i \quad (5)$$

since D_i are continuous random variables. On substituting (2) and (3) in (5), we have:

$$\Pi^{ri} = w_i \left(\int_0^{x_i} uf_i(u)du + x_i \int_{x_i}^\infty f_i(u)du \right) - v \int_0^{x_i} x_i f_i(u)du + v \int_0^{x_i} D_i f_i(u)du - cx_i \quad (6)$$

But $\int_{x_i}^\infty f_i(u)du = F_i(\infty) - F_i(x_i) = 1 - F_i(x_i)$

From Equation (6) and the definition of CDF above, we have the following profit function upon simplification:

$$\Pi^{ri}(x_i) = w_i \int_0^{x_i} uf_i(u)du + w_i x_i - w_i x_i F_i(x_i) - vx_i F_i(x_i) - vD_i F_i(x_i) - cx_i \quad (7)$$

Thus, the optimization problem for the cement factory becomes

$$\max_{x_1 \geq 0, \dots, x_N \geq 0} \Pi^{ri}(x_1, \dots, x_N),\ (i = 1, \dots, N) \quad (8)$$

By the Fundamental Theorem of Calculus, equation (7) becomes

$$\Pi^{ri'}(x_i) = w_i - c - (w_i + v)F_i(x_i) \quad (9)$$

By first order necessary condition (FONC) of (9), we have

$$\Pi^{ri} = F_i^{-1}\left(\frac{w_i - c}{w_i + v}\right) = F_i^{-1}(t_i), t_i := \frac{w_i - c}{w_i + v} \quad (10)$$

Since $v \leq c$ and $w > c > 0$, then $0 < t_i < 1$ which indicates that $F_i^{-1}(t_i)$ is well-defined.

ISBN: 978-988-14047-7-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

Since $\Pi^{ri}(x_i)$ is continuous, and twice differentiable in x_i, then from second order sufficiency condition (SOSC) of $\Pi^{ri}(x_i)$ in x_i, and by the concavity of $\Pi^{ri}(x_i)$ we have

$$\Pi^{ri''}(x_i) = -(w_i + v)f_i(x_i) < 0 \quad (11)$$

Since for every $x_i \geq 0$ the supplier's profit function is a real bounded random variable, by Wierestrass Theorem, the continuous function $\Pi^{ri}(x_i)$ attain its own extremum point. Hence the supplier's objective function given by $\Pi^{ri}(x_i)$ is a non-increasing concave function in x_i since F is non-decreasing.

The optimization problem for the supplier in equation (8) can be separated into N separate problems, one for each distributor, since the D_i are independent random variables. From the above analysis, we see that for distributor i, the supplier sets the inventory level to maximize his profit by solving the following optimisation problem:

$$\max_{x_i \geq 0} \Pi^{ri}(x_i), (i = 1, \dots, N) \quad (12)$$

Let Π^{ri^*} be the optimal solution of problem (12) Since the random demand is assumed continuous and there is no service level agreement, the analysis of the inventory decisions and the profit of the supplier are like a single location single period newsvendor problem. Hence the optimal profits levels for the supplier are given as:

$$\Pi^{ri^*} = F_i^{-1}(t_i), (i = 1, \dots, N) \quad (13)$$

Since distributors do not hold inventory, their expected profits are equivalent to the mark-up price times the expected sales.

Let $\pi^{ri}(x_i)$ be the profit of distributors when supplier keeps inventory level x_i i.e.

$$\pi^{ri}(x_i) = m_i E[\min(x_i, D_i)], \ (i = 1, \dots, N) \quad (14)$$

For combined inventory system, we let $\Pi^p(x_p)$ to be the supplier's expected profit when inventory level is x_p. We have

$$\Pi^p(x_p) = E\left[w_p \min(x_p, D_p) - v(x_p - D_p)^+ - cx_i\right] \quad (15a)$$

$$\Pi^p(x_p) = w_p \int_0^{x_p} u f_p(u) du + w_p x_p \, w_p x_p F_p(x_p) - v x_p F_p(x_p) - v D_p F_p(x_p), \quad (15b)$$

Applying the fundamental theorem of calculus on $(15b)$, we have

$$\mathrm{x}^p = F_p^{-1}(t_p); \text{ where } t_p := \frac{w_p - c}{w_p + v} \quad (16)$$

In similar manner, thus given the profit function $\Pi^p(x_p)$, the order quantity is given in (16) above.

Thus, for every $x_p \geq 0$, the profit functions $\Pi^p(x_p)$ is real bounded random variable. To maximize his profit, the supplier sets the optimal inventory level to x_p^* by solving the unconstrained optimization problem given by

$$\max_{x_p \geq 0} \Pi^p(x_p) \quad (17)$$

The optimal stocking policy of (17) is given by

$$\Pi^{p*} = F_p^{-1}(t_p). \quad (18)$$

Since demand follows normal distribution , we standardize $N(\mu, \sigma^2)$ with mean μ and the standard deviation σ. Let $\Phi(.)$ and $\phi(.)$ be the CDF and PDF of $N(0,1)$respectively. Since the optimal solutions of (6) and (17) can be reduced to the standard normal demand, we set

$$x_i = \mu_i + \sigma_i \Phi_i^{-1}(t_i), (i = 1, \dots, N) \quad (19)$$

and the optimal total base-stock level is given by

$$x_{ri}^* = \sum_{i=1}^N \mu_i + \sum_{i=1}^N \sigma_i \Phi_i^{-1}(t_i) \quad (20)$$

for restricted inventory case.

Similarly, for the combined inventory case, the optimal base-stock level is given by

$$x_p^* = \mu_i + \sqrt{\sigma_i^2} \Phi_i^{-1}(t_i), \quad (21)$$

and the total optimal base-stock level for the combined is given by

$$x_p^* = \sum_{i=1}^N \mu_i + \sqrt{\sum_{i=1}^N \sigma_i^2} \, \Phi_i^{-1}(t_i) \quad (22)$$

Since D_i are independent and normally distributed random variables with means μ_i and standard deviations σ_i, let $\Phi(.)$ and $\phi(.)$ denote the CDF and the PDF of the standard normal distribution. In addition, we denote by $L(z)$ the general standard lost sale function when inventory level is represented by z, that is

$$L(z) = \int_z^\infty (u \, z) \, \phi(u) du \quad (23)$$

where $\phi(u)$ is the standardized normal density function and $Z = \frac{z-\mu}{\sigma}$ is the standardized variate. From definition,

$$L(z) = \int_z^\infty (u - z) \, \phi(u) du = \int_z^\infty u\phi(u) du - z \int_z^\infty \phi(u) du$$

$$= \int_z^\infty u\phi(u) du - z[1 - \phi(z)]$$

$$= \phi(z) - z + z\phi(z) \quad (24)$$

But $\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-0.5z^2}$ is nonnegative and non-increasing function in z. On applying first derivative on (23), we have

$$L'(z) = \frac{-z}{\sqrt{2\pi}} e^{-0.5z^2} - 1 + \Phi(z) + \frac{z}{\sqrt{2\pi}} e^{-0.5z^2}$$

$$= \Phi(z) - 1 \leq 0 \quad (25)$$

Since $L(z)$ is nonnegative and non-increasing function of z, then

$$L(z) + z \geq 0 \text{ for any real z} \quad (26)$$

B. Comparison of Inventory Systems without Service Level Agreement

From the basic assumptions and definitions above, we characterize the benefits of combined inventory over restricted inventory to both the supplier and the distributors below by stating the following theorems:

Theorem 1 Given unpredictable independently and normally distributed distributors' demands D_i assuming the distributor's wholesale prices for both restricted and combined inventory policies are identical then $\Pi^{p^*} \geq \Pi^{ri^*}$.

Proof

From optimal base levels for restricted inventory system of (20), we have

$$x_i^* = \sum_{i=1}^N \mu_i + \sum_{i=1}^N \sigma_i \Phi^{-1}(t)$$

Similarly, the optimal inventory level for combined inventory system given in (22) is

ISBN: 978-988-14047-7-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

$$x_p^* = \sum_{i=1}^{N} \mu_i + \sqrt{\sum_{i=1}^{N} \sigma_i^2}\, \Phi^{-1}(t)$$

However, the excess stock is given by

$$(x_i^* - D_i)^+ = x_i^* - \min(x_i^*, D_i) \quad (27)$$

For the optimal base level for restricted inventory system, the profit function is given by

$\Pi^{ri^*} = E[w(\min(x_i^*, D_i) - v\,(x_i^* - D_i)^+ - cx_i)]$.

On substituting (27) into the profit function above, we have

$$\Pi^{ri^*} = E\left[\left(w \min(x_i^*, D_i) - v\left(x_i^* - \min(x_i^*, D_i)\right)^+ - c{x_i}^*\right)\right],$$

$$= E[(w+v)\textstyle\sum_{i=1}^N \min(x_i^*, D_i) - (v+c)\sum_{i=1}^N x_i^*]$$

For normally distributed random demands, we have

$$\Pi^{ri^*} = E\left[(w+v)\textstyle\sum_{i=1}^N \mu_i - (\sum_{i=1}^N \sigma_i) L\left(\Phi^{-1}(t)\right) - (v+c)\left(\sum_{i=1}^N \mu_i + (\sum_{i=1}^N \sigma_i)\left(\Phi^{-1}(t)\right)\right)\right].$$

Similarly, for the combined inventory system we have

$$\Pi^{p^*} = E\left[w \min\left(x_p^*, D_p\right) - v\left(x_p^* - D_p\right)^+ - cx_p^*\right]$$

$$\Pi^{p^*} = E\left[\left(w \min\left(x_p^*, D_p\right) - v\left(x^{p*} - \min\left({x_p}^*, D_p\right)\right)^+ - c{x_p}^*\right)\right]$$

$$\Pi^{p^*} = E\left[(w+v)\min\left(x_p^*, D_p\right) - (v+c)x_p^*\right]$$

Since the random variables are normally distributed, then

$$\Pi^{p^*} = E\left[(w+v)\left(\textstyle\sum_{i=1}^N \mu_i - \sqrt{\sum_{i=1}^N \sigma_i^2}\right)L\left(\Phi^{-1}(t)\right) - (v+c)\left(\sum_{i=1}^N \mu_i + \sqrt{\sum_{i=1}^N \sigma_i^2}\,\Phi^{-1}(t)\right)\right]$$

$\therefore\ \Pi^{ri^*} - \Pi^{p^*}$ becomes

$$(w+v)\left(\sum_{i=1}^{N} \mu_i - \sum_{i=1}^{N} \sigma_i\right) L\left(\Phi^{-1}(t)\right)$$

$$-(v+c)(\textstyle\sum_{i=1}^N \mu_i + \sum_{i=1}^N \sigma_i)\Phi^{-1}(t)$$

$$-(w+v)\left(\textstyle\sum_{i=1}^N \mu_i - \sqrt{\sum_{i=1}^N \sigma_i^2}\right)L\left(\Phi^{-1}(t)\right) + (v+c)\left(\sum_{i=1}^N \mu_i + \sqrt{\sum_{i=1}^N \sigma_i^2}\right)\Phi^{-1}(t)$$

$$\leq (c+v)\left(\sqrt{\textstyle\sum_{i=1}^N \sigma_i^2} - \sum_{i=1}^N \sigma_i\right) L\left(\Phi^{-1}(t)\right) + (v+c)\left(\sqrt{\sum_{i=1}^N \sigma_i^2} - \sum_{i=1}^N \sigma_i\right)\left(\Phi^{-1}(t)\right)$$

$$= (c+v)\left(\sqrt{\textstyle\sum_{i=1}^N \sigma_i^2} - \sum_{i=1}^N \sigma_i\right)\left[\, L\left(\Phi^{-1}(t)\right) + \left(\Phi^{-1}(t)\right)\right] \leq 0$$

Since $w > c$ and $L\left(\Phi^{-1}(t)\right) > 0$ from the definition of loss function, the first inequality $\sqrt{\sum_{I=1}^N \sigma_i^2} < \sum_{i=1}^N \sigma_i$ holds for any positive σ_i. Since $L\left(\Phi^{-1}(t)\right) + \left(\Phi^{-1}(t)\right) \geq 0$, the second inequality from (26) holds. Hence $\Pi^{p^*} \geq \Pi^{ri^*}$

Theorem 2 Given unpredictable independently and normally distributed distributors' demands $D_i, (i = 1, 2, \ldots, N)$, if the distributors' wholesale and mark-up prices for both restricted inventory and combined inventory systems are identical, then $\pi^{p^*} \geq \pi^{ri^*}$.

Proof

For the distributors', their optimal profit in the restricted case is given by

$$\pi^{ri^*} = mE[\min(x^{i*}, D_i)],$$

While total distributors' optimal profit for the combined inventory system is given by

$$\pi^{p^*} = mE\left[\min\left(x^{p*}, D_p\right)\right]$$

Since the unpredictable variables are normally distributed, the restricted inventory system becomes

$\pi^{i^*} = m\left(\mu_i - \sigma_i L\left(\Phi^{-1}(t)\right)\right), (i = 1, 2, \ldots, N)$. Thus

$$\pi^{ri^*} = m\left(\sum_{i=1}^{N} \mu_i - \sum_{i=1}^{N} \sigma_i\right) L\left(\Phi^{-1}(t)\right)$$

Similarly, for normally distributed demands for the combined inventory case, we have

$$\pi^{p^*} = m\left(\sum_{i=1}^{N} \mu_i - \sqrt{\sum_{i=1}^{N} {\sigma_i}^2}\right) L\left(\Phi^{-1}(t)\right)$$

$$\therefore \pi^{p*} - \pi^{ri*} = \pi^{p*} - \textstyle\sum_{i=1}^N \pi^{ri*}$$

$$= m\left(\textstyle\sum_{i=1}^N \sigma_i - \sqrt{\sum_{i=1}^N {\sigma_i}^2}\right)L\left(\Phi^{-1}(t)\right)$$

Since $L\left(\Phi^{-1}(t)\right)$ is a nonnegative and $L\left(\Phi^{-1}(t)\right) > 0$ from definition of loss function, the first inequality $\sqrt{\sum_{I=1}^N \sigma_i^2} < \sum_{i=1}^N \sigma_i$ holds for any positive $\sigma_i, (i = 1, \ldots, N)$. Thus $\pi^{p*} \geq \pi^{ri*}$. Hence the distributors' total expected profit is increased after combining inventory.

Theorem 3 Given unpredictable independently and normally distributed distributors' demands $D_i, (i = 1, 2, \ldots, N)$ and the critical ratio $t \geq 0.5$, if the distributors' wholesale and mark-up prices in restricted and combined inventory cases are identical, then $x_p^* \leq \sum_{i=1}^N x_{ri}^*$. Otherwise $x_p^* > \sum_{i=1}^N x_{ri}^*$.

Proof

For the restricted inventory system, the optimal total base inventory level is given by

$$\sum_{i=1}^{N} x_{ri}^* = \sum_{i=1}^{N} \mu_i + \sum_{I=1}^{N} \sigma_i\, \Phi^{-1}(t).$$

The combined optimal total base inventory level is similarly given by

$$x_p^* = \sum_{i=1}^{N} \mu_i + \sqrt{\sum_{i=1}^{N} \sigma_i^2}\, \Phi^{-1}(t)$$

Since $\Phi^{-1}(0.5) = 0$ and the fact that Φ^{-1} is monotone nondecreasing function then $\Phi^{-1}(t) \geq 0$ provided the critical ratio $t \geq 0.5$. Hence $x_p^* \leq \sum_{i=1}^N x_{ri}^*$ follows from the inequality $\sum_{I=1}^N \sigma_i \geq \sqrt{\sum_{i=1}^N \sigma_i^2}$. Otherwise when $t < 0.5$ we have $\sum_{i=1}^N x_{ri}^* < x_p^*$.

Since $\mathrm{F}^{-1}(t)$ is a normal distribution where $= \frac{w-c}{w+v}$, the number of standard deviation depends on the relationship between w and v. As v is increasing $\mathrm{F}^{-1}(t)$ is decreasing thus x_{ri}^* is decreasing. As w is increasing then $\mathrm{F}^{-1}(t)$ is increasing whereby x_{ri}^* is increasing. Thus, for normally distributed random variable, the trade-off is generally between the holding and the stock-out costs. Thus, for both combined and restricted inventory systems, the number of standard deviation depends on the relationship between w and v.

ISBN: 978-988-14047-7-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

C. Model Formulation for Inventory Systems with Service Level Agreement

Since customers' demands $D_i, D_2, \dots, D_N$ are independent random variables with CDF, $F_i(.)$, we further assume that that the N distributors impose a minimum service level requirements donated by $\tau_i, (i = 1, 2, \dots, N)$ respectively. The service level requirement is given by the probability of no stock-out. Thus, given the inventory levels x_i, the probability of no-stock out at distributor i is given by

$$P(D_i \leq x_i) = F_i(x_i),\ (i = 1, \dots, N) \quad (28)$$

Since the objective of the supplier is to maximise his expected profit while keeping the minimum service level agreements at the distributors' region using as in (5) above, the suppliers' maximisation problem becomes

$$\underset{x_i \geq 0}{\text{maximize}}\ \Pi^{ri}(x_i) \quad \text{subject to} \quad (x_i) \geq \tau_i, (i = 1, 2, \dots, N) \quad (29)$$

where τ_i is the minimum acceptable probability of no-stock-out for distributor i. Since distributors do not hold any inventory, their expected profit denoted by $\pi^{ri}(x_i)$ is equivalent to mark-up price times expected sales.

$$\pi^{ri}(x_i) = E[\min(x_i, D_i)], \quad (i = 1, 2, \dots, N) \quad (30)$$

Let π^{ri*} denote the total optimal profit of distributor i when the supplier holds x_i product for distributor i. For the problem with service level constraints, the profit function is given by

$$E[w_i \min(x_i, D_i) - v\,(x_i - D_i)^+ - cx_i]$$

From (29), $F_i(x_i)$ is nondecreasing in x_i. The profit function is a convex function of inventory x_i. Thus the optimal inventory level x_i^* is given by

$$x_i^* = F_i^{-1}\big(\max(\tau_i,\ t_i)\big) \quad (31)$$

Let the optimal stocking level be represented by $F_i^{-1}\big(\max(\tau_i,\ t_i)\big)$ when service level constraints are present and the total safety stock level the supplier must hold is given as

$$\sum_{i=1}^{N} F_i^{-1}\big(\max(\tau_i,\ t_i)\big) \quad (32)$$

The service level constraints $\tau_i, (i = 1, 2, \dots, N)$ is necessary and binding on the distributors whenever $\tau_i \geq t_i$ since the distributors are assumed to set the reputational cost for the supplier. The supplier maximizes total expected profit by satisfying the distributors' minimum service level agreement. For $\tau_i \in (0, t_i)$, the supplier provides more than needed service level to the distributors. However, for $\tau_i > t_i$, the distributors are well off to the detriment of the supplier and this does not necessarily imply higher stock. Therefore, for increasing holding cost v the supplier is forced to provide higher than required service to the distributors. Hence for $\tau_i \in (0,\ t_i)$, the distributors expected sales are more than or equal to what their service level guarantees them.

With service level requirement, we next consider supply chain system when supplier pools inventory but must satisfy a joint service level requirement of the distributors. We will keep the same notation as in previous section. In this case the objective of the supplier is to maximize his profit, subject to satisfying all the distributors' demand including probability of no-stock out. The supplier sets his inventory level by solving the following optimisation problem

$$\underset{x_p \geq 0}{\text{maximize}}\ \Pi_p(x_p) \quad \text{subject to} \quad F_p(x_p) \geq \tau_p \quad (33)$$

where the supplier expected profit is given by

$$\Pi_p = E\left[w_p \min(x_p, D_p) - v\,(x_p - D_p)^+ - cx_p.\right]$$

The optional inventory level is $F_p^{-1}(\ t_i)$ for the combined inventory case in the absence of a service level constraint.

The convexity of $E\big[w_p \min(x_p, D_p) - v\,(x_p - D_p)^+ - cx_p\big]$ being a linear function and the fact that the joint CDF $F_P(.)$ is non-decreasing implies the optimal inventory level given by

$$x_p^* = F_p^{-1}\big(\max(\tau_i,\ t_i)\big) \quad (34)$$

Given the supplier inventory level x_p, the total expected profit of the retailer is given by

$$\pi_p = m_p \min(x_p, D_p) \quad (35)$$

We use π^{p*} to denote the optimal total expected profit of the retailer when the supplier inventory level is x^{p*}.

D. Comparison of the Inventory Systems with Service Level Agreement

We now examine the benefits of inventory pooling policy on restricted inventory system with regards to the profits of the supplier and the distributors. For normally distributed random demands with higher inventory levels, the expected service level provided to the distributors and their expected sales will also increase. If the required service level exceeds the critical ratio, the supplier loses money by providing a higher service level. We therefore show the results for the case when the distributors have same service level requirements. For the case of normally distributed demands, we provide a detailed comparison of restricted and combined inventory cases. We assume that D_i are independently and normally distributed random variable with means μ_i and the standard deviations σ_i respectively. In addition, if assume that all the service level requirements by the distributors to the supplier are the same, i.e. $\tau_1 = \tau_2 = \dots = \tau_N = \tau_p = \tau$, then we have the following theorems.

Theorem 4 Given an unpredictable independently and normally distributed distributors' demands $D_i, (i = 1, 2, \dots, N)$, assuming the distributor's wholesale prices and no stock-out constraints for each distributor in both restricted and combined inventory cases are the same, then $\Pi^{p*} \geq \Pi^{r*}$.

Proof

Under the restricted inventory scenario, the optimal inventory levels are given by

$$\Pi^{ri^*} = \mu_i + \sigma_i \Phi^{-1}\big(\max(t, \tau)\big), (i = 1, \dots, N)$$

$$\Pi^{ri^*} = \sum_{i=1}^{N} \mu_i + \sum_{i=1}^{N} \sigma_i \Phi^{-1}\big(\max(t, \tau)\big)$$

For the combined inventory scenario,

$$D_p \sim N\left(\sum_{i=1}^{N} \mu_i\,, \sqrt{\sum_{i=1}^{N} \sigma_i^2}\right)$$

Similarly, the optimal base inventory level for the combined system gives

ISBN: 978-988-14047-7-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

$$\Pi^{p*} = \sum_{i=1}^{N} \mu_i + \sqrt{\sum_{i=1}^{N} \sigma_i{}^2}\, \Phi^{-1}(\max(t,\tau))$$

The excess stock is given by

$\Pi^{ri^*} = E[(w \min(x_i^*, D_i) - v\,(x_i^* - D_i)^+ - cx_i^*)]$

$\Pi^{ri^*} = E\left[\left(w \min(x_i{}^*, D_i) - v\,(x_i{}^* - \min(x_i{}^*, D_i))^+ - cx_i{}^*\right)\right], i = 1, 2, \dots, N$

$\Pi^{ri^*} = (w+v)E[\sum_{i=1}^{N} \min(x_i^*, D_i)] - (v+c)\sum_{i=1}^{N} x_i^*$

Since the random variables are normally distributed, we have

$\Pi^{ri^*} = (w+v)E\left[\sum_{i=1}^{N}\mu_i - (\sum_{i=1}^{N}\sigma_i)L\left(\Phi^{-1}(\max(t,\tau))\right) - (v+c)\left(\sum_{i=1}^{N}\mu_i + (\sum_{i=1}^{N}\sigma_i)\left(\Phi^{-1}(\max(t,\tau))\right)\right)\right]$

and for the combined, we have

$$\Pi^{p^*} = wE\left[\min(x_p^*, D_p) - v\,(x_p^* - D_p)^+ - cx_p^*\right]$$

$\Pi^{p^*} = E\left[\left(w \min(x_p^*, D_p) - v\,\left(x_p^* - \min(x^{p*}, D_p)\right)^+ - cx_p^*\right)\right]$

$\Pi^{p^*} = (w+v)E\left[\min(x_p^*, D_p)\right] - (v+c)x_p^*$

For the normally distributed random demands, we have

$\Pi^{p^*} = \left[(w+v)\sum_{i=1}^{N}\mu_i - (\sum_{i=1}^{N}\sigma_i{}^2)L\left(\Phi^{-1}(\max(t,\tau))\right) - (v+c)\left(\sum_{i=1}^{N}\mu_i + (\sum_{i=1}^{N}\sigma_i{}^2)\left(\Phi^{-1}(\max(t,\tau))\right)\right)\right]$

Now $\Pi^{ri^*} - \Pi^{p^*}$ becomes

$(w+v)\left(\sum_{i=1}^{N}\mu_i - \sqrt{\sum_{i=1}^{N}\sigma_i}\right)L\left(\Phi^{-1}(\max(t,\tau))\right) - (v+c)\left(\sum_{i=1}^{N}\mu_i + \sqrt{\sum_{i=1}^{N}\sigma_i}\right)\Phi^{-1}(\max(t,\tau))$

$-(w+v)\left(\sum_{i=1}^{N}\mu_i - \sqrt{\sum_{i=1}^{N}\sigma_i^2}\right)L\left(\Phi^{-1}(\max(t,\tau))\right) + (v+c)\left(\sum_{i=1}^{N}\mu_i + \sqrt{\sum_{i=1}^{N}\sigma_i^2}\right)\Phi^{-1}(\max(t,\tau))$

$\leq (c+v)\left(\sqrt{\sum_{i=1}^{N}\sigma_i^2} - \sum_{i=1}^{N}\sigma_i\right)L\left(\Phi^{-1}(\max(t,\tau))\right) + (v+c)\left(\sqrt{\sum_{i=1}^{N}\sigma_i^2} - \sum_{i=1}^{N}\sigma_i\right)\left(\Phi^{-1}(\max(t,\tau))\right)$

$= (c+v)\left(\sqrt{\sum_{i=1}^{N}\sigma_i^2} - \sum_{i=1}^{N}\sigma_i\right)\left[L\left(\Phi^{-1}(\max(t_i,\tau))\right) - \Phi^{-1}(\max(t,\tau))\right] \leq 0$

Since $w > c$, $L\left(\Phi^{-1}(\max(t_i,\tau))\right) > 0$ from the general definition of loss function, the inequality $\sqrt{\sum_{i=1}^{N}\sigma_i^2} < \sum_{i=1}^{N}\sigma_i$ holds for any positive σ_i.

Since $L(x) + x \geq 0$, for any x, the second inequality from (26) holds. Hence, the supplier's profit is increased after pooling the inventory, i.e., $\Pi^{p^*} \geq \Pi^{r^*}$.

Theorem 5 Given unpredictable independently and normally distributed distributors' demands $D_i, (i = 1, 2, \dots, N)$, assuming the distributor's wholesale and mark up prices and no stock-out constraints for each distributor under restricted inventory and combined inventory cases are the same, then $\pi^{p^*} \geq \pi^{r^*}$.

Proof

The expected profits of the N distributors in the restricted inventory system are given by

$$\pi^{ri^*} = mE\left[\min(\pi^{ri^*}, D_i)\right], (i = 1, 2, \dots, N)$$

The distributors' total expected profit for combined inventory system is given by

$$\pi^{p^*} = mE\left[\min(\pi^{p^*}, D_p)\right]$$

Since the random variables are normally distributed, we have for the restricted system as:

$$\pi^{ri^*} = m\left(\mu_i - \sigma_i L\left(\Phi^{-1}(\max(t_i,\tau))\right)\right), (i = 1, 2, \dots, N)$$

$$\pi^{ri*} = m\left(\sum_{i=1}^{N}\mu_i - \sum_{i=1}^{N}\sigma_i\right)L\left(\Phi^{-1}(\max(t_i,\tau))\right)$$

and for the normally distributed demands for the combined inventory case, we have

$\pi^{p^*} = \left(\sum_{i=1}^{N}\mu_i - \sqrt{\sum_{i=1}^{N}\sigma_i{}^2}\right)L\left(\Phi^{-1}(\max(t_i,\tau))\right)$

Therefore, $\pi_p^* - \pi^{ri^*} = \pi_p^* - \sum_{i=1}^{N}\pi^{ri^*}$,

$\pi^{p^*} - \pi^{ri^*} = m\left(\sum_{i=1}^{N}\sigma_i - \sqrt{\sum_{i=1}^{N}\sigma_i{}^2}\right)L\left(\Phi^{-1}(\max(t_i,\tau))\right)$

Since, $L\left(x\Phi^{-1}(\max(t_i,\tau))\right) > 0$ from the general definition of loss function and the inequality $\sqrt{\sum_{i=1}^{N}\sigma_i^2} < \sum_{i=1}^{N}\sigma_i$ holds for any positive $\sigma_i, (i = 1, \dots, N)$.

Thus $\pi^{p^*} \geq \pi^{r^*}$. Hence the distributors' total expected profit is increased after combining inventory of the distributors for restricted inventory case and joint service level constraint for combined inventory case.

Theorem 6 Given unpredictable independently and normally distributed distributors' demands $D_i, (i = 1, 2, \dots, N)$ and critical ratio $t \geq 0.5$, if the wholesale price and no stock-out constraints of each distributor under restricted and combined inventory cases are identical, then $\Pi^{p^*} \leq \sum_{i=1}^{N} x_i^*$. Otherwise, $\Pi^{p^*} > \sum_{i=1}^{N} x_i^*$.

Proof

For the restricted inventory system, optimal base total inventory level, we have

$$\sum_{i=1}^{N} x_i^* = \sum_{i=1}^{N}\mu_i + \sqrt{\sum_{I=1}^{N}\sigma_i}\,\Phi^{-1}(\max(t,\tau))$$

The combined inventory system optimal base total inventory level is given by

$$x_p^* = \sum_{i=1}^{N}\mu_i + \sqrt{\sum_{i=1}^{N}\sigma_i^2}\,\Phi^{-1}(\max(t,\tau)).$$

Since $\Phi^{-1}(0.5) = 0$, and the fact that $\Phi^{-1}(.)$ is monotone non-decreasing function then $\Phi^{-1}(\max(t,\tau)) \geq 0$, provided the critical ratio $\max(t,\tau) \geq 0.5$.

Hence $x_p^* \leq \sum_{i=1}^{N} x_i^*$ follows from the inequalities $\sqrt{\sum_{i=1}^{N}\sigma_i{}^2} \leq \sum_{i=1}^{N}\sigma_i$. Otherwise, when $\max(t,\tau) < 0.5$ we have $\sum_{i=1}^{N} x_i^* < x_p^*$.

In the absence of service level requirement, the total optimal inventory level and the mean value of the demands is decreased when the inventory is combined and increased when inventory is restricted. Because of the advantages of inventory pooling over the restricted inventory, the distributors and the supplier will prefer combined inventory policy.

ISBN: 978-988-14047-7-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

IV. NUMERICAL APPLICATION OF THE MODEL TO A PRODUCTION COMPANY

We now examine the benefits of combining inventory as this section gives practical application of the entire work. The primary data used in the study represents daily demand of cement products by three major distributors in a cement factory who buy from the factory directly and sell to the retailers and the consumers on weekly basis for one year. The choice of the three distributors was informed by the fact that the cement factory has only three "gold" distributors. The data obtained from each distributor covers a period of 52 weeks (one year). The total production capacity as at the time of the study is 600 bag of cements products per truck with expected trucks of 500 per day. Thus, the expected production capacity is 300, 000 bags per day. We use the parameter set listed below as the base parameters.

Table 1: The Base Parameter Decision Variables

Total stock of inventory level of cements	x
Random demand of cements	D
Manufacturing cost price per bag of cement	c
Distributor wholesale price per bag of cement	w
Distributor retail price per bag of cement	p
Distributor holding cost per bag of cement	v

In analysing the data, we let $R_i, (\ i = 1, 2\ and\ 3)$ represents the three major distributors in the cement factory for the restricted inventory. R_p represents combined inventory case for the joint distributors. From the collected and collated data, we obtain the following classifications through MATLAB 2010a programme.

Table 2: Parameters and Decisions Variables

Var.	R_1	R_2	R_3	R_p
N	52	52	52	R
x	101000	151000	51000	300000
D	80038	120057	40019	240115
Min.	70200	105300	35100	210600
Max.	89800	134700	44900	269400
μ	80038.46	120057.69	40019.23	240115.38
σ	5879.447	8819.17	2939.72	17638.34
Profit	7756360	12079540	3433180	25940300

V. DISCUSSION OF RESULTS

The unit of measurement used in this study are naira and kobo. Since 25940300.00 < 23269080.00, we conclude that the total profit for the combined inventory is higher than the profit of the joint restricted inventory case. Also, from the Table 2 above, we obtain the following probability density functions for the various distributors based on the assumptions of the model applied to collected and collated data. From Table 2 above, we have:

The Normal Function for R_1

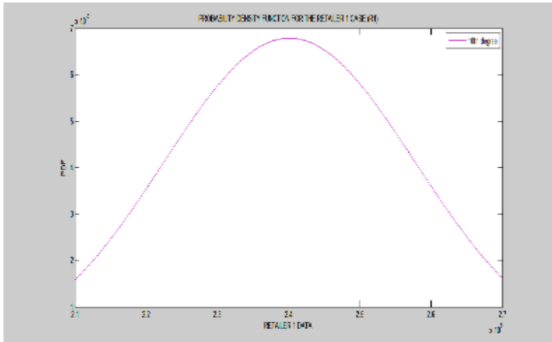

Fig. 1: Graph of the probability distribution function for R_1

The Normal Function for R_2

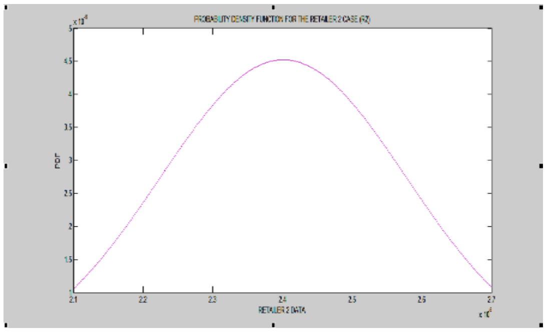

Fig. 2: Graph of the probability distribution function for R_2

The Normal Function for R_3

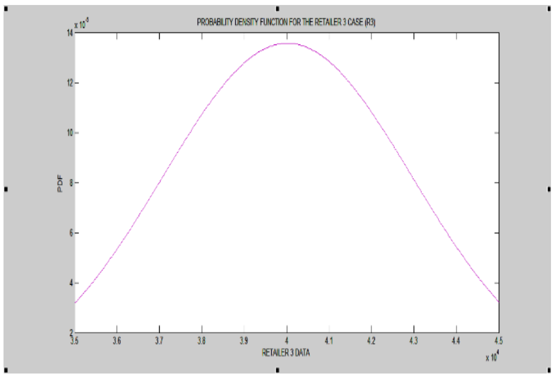

Fig. 3: Graph of the probability distribution function for R_3.

ISBN: 978-988-14047-7-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

Graph of Normal Function for R_p

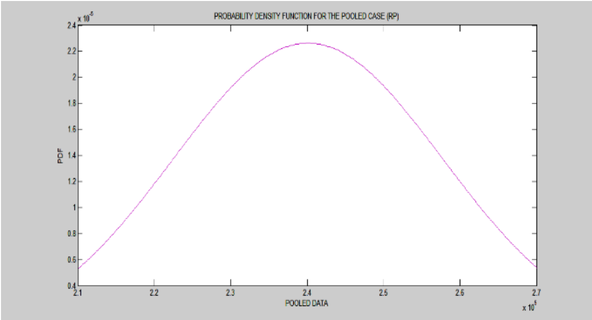

Fig. 4: Graph of the probability distribution function for R_p.

The graph above is the probability distribution function when inventory is combined.

Based on the displayed data on Table (2) above, we discussed and compared the profits of the suppliers against other decision variables and parameters.

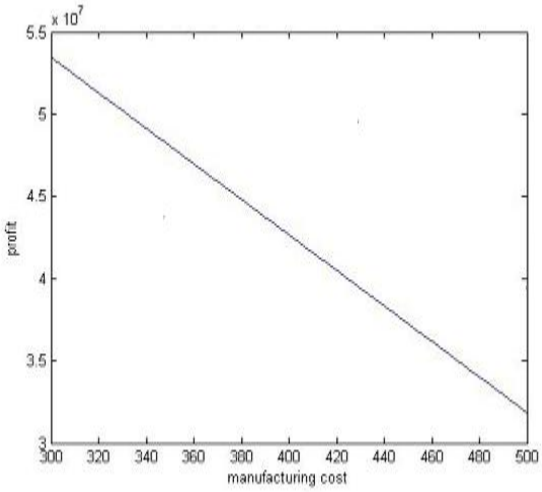

Fig. 5: The Profit Function versus the Manufacturing Cost.

Fig. 5 above shows the graph of the profit function against manufacturing cost. Manufacturing cost was varied from 300 to 500 and as observed from the graph, manufacturing cost is inversely proportional to profit of the manufacturer (supplier) while other variables (wholesale price, holding cost, etc.) are being held constant.

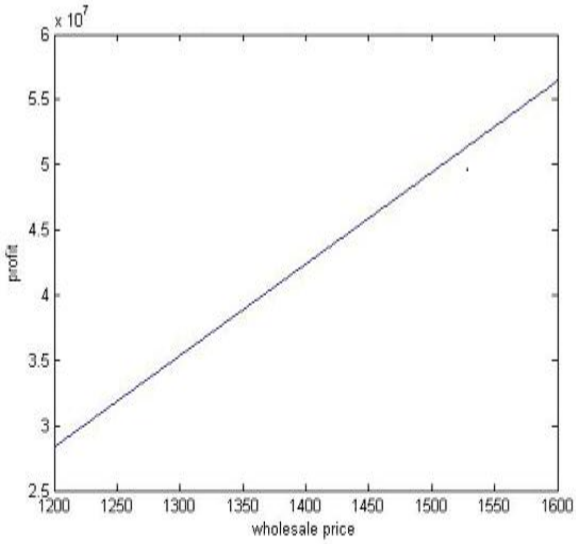

Fig. 6: Profit and wholesale plot

Fig. 6 above shows the graph of profit function plotted against distributors' wholesale price when the wholesale price was varied from 1200 to 1600, as observed from the same graph above, the distributors' wholesale price is directly proportional to the profit of the supplier when other decision variables (manufacturing cost, holding cost, etc.) are being held constant.

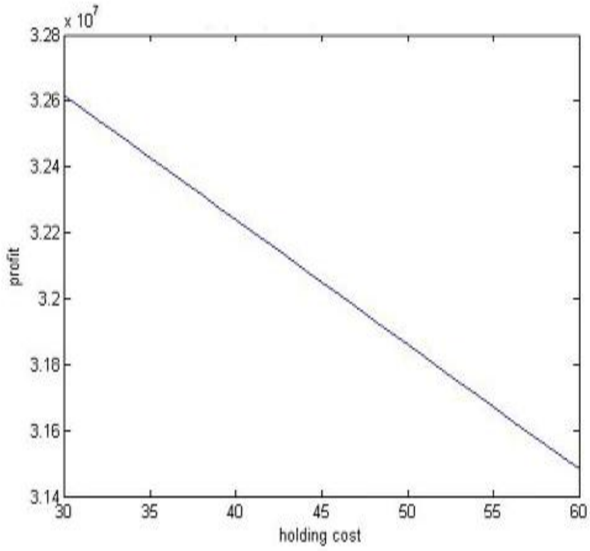

Fig. 7: The Profit Function versus the Suppliers' Holding Cost.

The graph above is the graph of the profit function against the supplier's holding cost. The holding cost is ranging from 30 to 60 and as observed from the graph above, the profit function is inversely proportional to the holding cost when other decision variables are being held constant.

ISBN: 978-988-14047-7-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

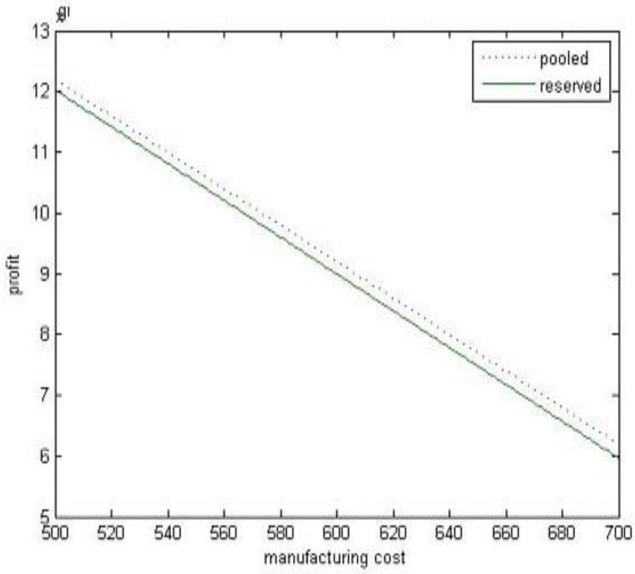

Fig. 8: Graph of Profit Function for Inventory Systems versus Manufacturing Cost.

Fig. 8 above shows the relationship between the profit function and manufacturing cost for the two-inventory system. From the graph above, the profit from combined inventory of the distributors is slightly higher than the profit function from the restricted inventory. This is observed since total safety stock for the distributors in the restricted inventory case is higher than total safety stock for the combined inventory case. Hence it is beneficial and of high advantage for the supplier (cement factory) to operate combined inventory system rather than restricted inventory system.

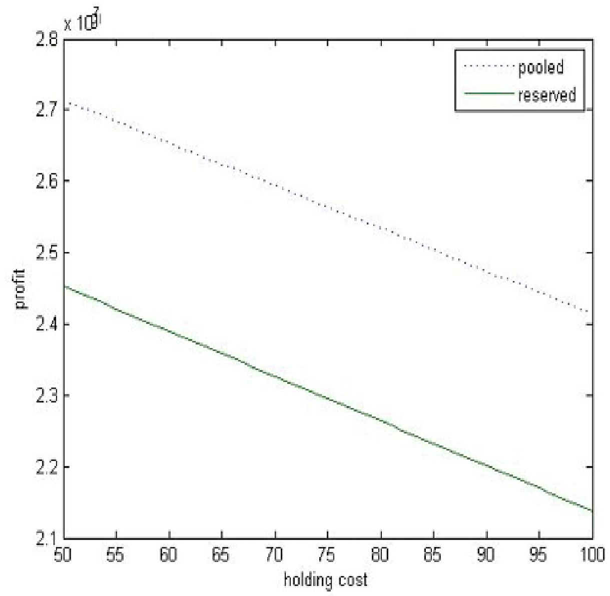

Fig. 9: The profit Function for the Inventory Systems versus the Holding Cost.

The graph above indicates a situation where the profit function is plotted against the supplier holding cost. The profit from the combined inventories of the distributors is much higher than the profit function from separate distributors when profit is compared with supplier holding cost. Because of pooling, the supplier (cement factory) does not need to keep as much stock as in the restricted inventory case. For equal distributors' wholesale price, the distributors will prefer combined rather than restricted inventory case.

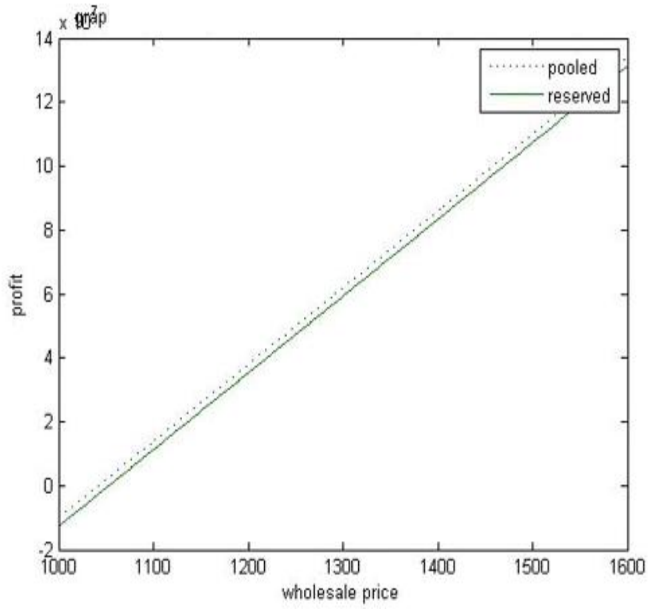

Fig.10: Graph of Function of the Inventory Systems versus the Wholesale Price.

The graph above shows the relationship between the profit function and the wholesale price for combined versus restricted inventory systems. As observed from the graph above, the profit function is slightly higher in the case of combined inventory than in the case of restricted distributors. For equal distributors' wholesale price, the distributors prefer combined rather than restricted (restricted) inventory case. Hence it is beneficial and of high advantage for the cement factory which is the supplier and the distributors to operate combined inventory system rather than restricted inventory system.

VI. CONCLUSION AND RECOMMENDATIONS

The graphs above showed the relationship between the profit of the supplier when the other decision variables and other parameter sets like manufacturing costs, holding cost and wholesale price are being held constant for normally distributed random demands. For example, the graph of fig. 8 above shows the relationship between the profit functions when the manufacturing cost for both the combined and restricted inventory systems are varied from 500 to 700. As can be observed from the graphs, the profit function for combined inventory is higher than the profit function for restricted inventory. This applies to the graph of fig.9 and fig. 10 where the supplier's holding cost and the distributors' wholesale prices are varied for both combined and the restricted inventory systems. It is observed that the total stock for all the distributors operating restricted inventory policy is higher than the total stock for combined inventory system because$\sqrt{\sum_{i=1}^{N}\sigma_i^2} < \sum_{i=1}^{N}\sigma_i$ for any positive σ_i.

Thus, for both combined and restricted inventory systems, the number of standard deviation depends on the relationship between w and v. For equal distributors' wholesale prices,

ISBN: 978-988-14047-7-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

the distributors prefer combined rather than restricted inventory system. Therefore, as a result of inventory pooling, the supplier (cement factory) does not need to keep as much stock as observed in the restricted inventory case. Hence, in all cases, the graphs of the analysis showed that the single product supplier, the cement factory, and the multiple distributors will always prefer combined inventory system rather than restricted inventory system. However, for identically and normally distributed random variables, with same service level requirement the supplier gets more benefit from combining inventory than the distributors. In the absence of service level requirement, the total optimal inventory level and the mean value of the demands is decreased when the inventory is combined and increased when inventory is restricted. Due to the benefits of inventory pooling, the distributors and the supplier will always prefer combined inventory policy.

From the analyses and graphs of the functions above, a lot of results were encountered in establishing the superiority and benefits of combined inventory policy over the restricted inventory policy. A more complete simulation study of the two supply chain systems is required in the future to capture full understanding of the relative performance of supply chain management policies.

Acknowledgment

The authors sincerely acknowledge Covenant University for financial support.

References

[1] Alfaro, J. A., and Corbett, C., (2003). The value of SKU rationalization in practice (the pooling effect under suboptimal inventory policies and non-normal demand). Production and Operations Management, Vol. 12, No. 1, 12-29.

[2] Balaji, K. Reddy I, Narayanan, S., and Pandian P., (2011). Single-echelon supply chain two stage distribution inventory optimization model for the confectionery industry. Applied Mathematical Sciences, Vol. 5, 2011, no. 50, 2491 – 2504.

[3] Baker, K. R., Magazine, M. J., and Nuttle H. L. W. (1986). The effect of the commonality of safety stock in a simple inventory model. Management Science, Vol. 32, No. 8, 982-988.

[4] Barthodli, J.J. and Kemahioglu E. (2003). Using Shapley value to allocate savings in a supply chain. Working paper, School of Industrial and Systems Engineering, Georgia Institute of Technology.

[5] Clifford, S. (2010). Nordstrom Links Online Inventory to Real World. The New York Times.

[6] Corbett, C. J., and Rajaram, K. 2006. A Generalization of the Inventory Pooling Effect to Non-Normal Dependent Demand. Manufacturing Service Operation Management, 8(4), 351– 358.

[7] Edgeworth, F. (1888): The mathematical theory of banking. Journal of the Royal Statistical Society, Vol. 51, No. 1, 113-127.

[8] Eppen, G., (1979). Effects of centralization on expected costs in a multi-location newsboy problem, Management Science, 25.5, 498-501 (1979).

[9] Jack C. P. Su, Yih-Long C., and Johnny C. H., (2004). Evaluation of Component Commonality Strategies in Supply Chain Environment. Asia Pacific Management Review 9(5), 801-821 801.

[10] Kumar, S. and Tiwari, M. K., (2013). Supply chain system design integrated with risk pooling. Computers and Industrial Engineering, 64.2, 580-588.

[11] Netessine, S. and Rudi N., (2003). Supply chain choice on the internet. Working paper, University of Rochester.

[12] Ozsen, L., Coullard, C. R., and Daskin, M. S., (2008). Capacitated warehouse location model with risk pooling, Naval Research Logistics (NRL), 55.4, 295-312.

[13] Patel, A., and Gor, R., (2014). A single period model where the lost sales recapture is a function of (r/p). Applied Mathematical Sciences, Vol. 7, 2013, No. 103, 5103 – 5114.

[14] Sukun, P., Lee, T. E., and Sung, C. S., (2010). A three-level supply chain network design model with risk pooling and lead times, Transportation Research Part E: Logistics and Transportation Review, 46.5, 563- 581.

[15] Tai A. H. (2015). A continuous-time multi-echelon inventory model with transshipment. International Journal of Inventory Research Volume 2, Issue 4 2015.

[16] Xiaoli, L., Shu-Cheng, F., Henry, L.W. N., and Xiuli C. (2005): Pooled versus restricted inventory policies in a Two-echelon supply chain international. Journal of operations research. l2 No1, 59-76.

[17] Zhao, H., Deshpande, V., Jennifer K., and Ryan, J.K., (2006). Emergency Transshipment in Decentralized Dealer Networks: When to Send, and Accept Transshipment Requests, Naval Research Logistics, Vol.53, 547–567, ISSN: 0894-069X.