An Investigation of Non-cooperative Agents on Asymmetric Optimization Problems

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Abstract—**Coalition of agents is a fundamental problem in multiagent systems. While the coalition structure generation is based on characteristic functions, optimization problems might exist among agents in the background of cooperation. Several cooperation problems can be described as asymmetric constraint optimization schemes that represent the parts of objective functions corresponding to individual agents. When the optimality of the solution is critical, exact solution methods will be employed. However, due to the scalability of the exact solution methods, large scale problems must be approximated ignoring relationships among several agents. In other situations, several agents might not provide the information of their objectives. In such cases, the agents' choices of cooperation or non-cooperation will affect their utilities, and the behaviors of non-cooperative agents can be an issue for the stability of cooperation. As the first investigation, we address the coalition on the asymmetric constraint optimization where several non-cooperative agents independently determine their assignment.**

Index Terms—**Agents, Cooperation, Coalition, Asymmetric Constraint Optimization Problem**

I. INTRODUCTION

Coalition of agents has been studied as fundamental problems in multiagent systems including coalition structure generation problems [1] and mechanism design [2]. With a coalition structure generation problem, a set of agents is partitioned into several groups to maximize the total value of the characteristic functions that describe the values of the groups. Then the agents share the earned values, based on their individual contributions. A share is defined so that no agent has incentives to switch to other groups. In the mechanism design, the share of the utility or the cost is also addressed, and a major issue of the shares is strategy proofness. While the coalition structure generation is based on characteristic functions, optimization problems might exist among agents in the background of cooperation.

Cooperative problem solving, including distributed constraint satisfaction/optimization problems (DCSPs/DCOPs), has also been studied as a different type of cooperation problem [3], [4], [5], [6]. In the standard settings of problems, agents cooperatively optimize the total utility or cost. Several studies have addressed asymmetric problems, where each agent has a set of its individual objective functions. A basic assumption of such problems is that no properties are transferred among agents.

On the other hand, a few studies have addressed the relationship of cooperative constraint optimization and coalition structure generation problems or mechanism design. In [7], the characteristic functions of the coalition structure generation problem are defined as the objective functions of DCOPs. The problem is then solved as a DCOP with approximation algorithms. In [8], a class of DCOPs is defined with a VCG mechanism, where agents exchange their utilities based on their contributions to optimal solutions.

Several cooperation problems can be described as asymmetric constraint optimization schemes that represent the parts of objective functions corresponding to individual agents.

A coalition of agents can be considered from a different view of cooperative problem solving. When the optimality of the solution is critical, exact solution methods will be employed. However, due to the scalability of the exact solution methods, large scale problems must be approximated ignoring relationships among several agents. In other situations, several agents might not provide the information of their objectives. In such cases, the agents that are excluded from the exact solution methods do not belong to a coalition. The agents might also be isolated from a reward-sharing scheme and only earn from its local utility. When these non-cooperative agents make decisions, it will affect the quality of solutions. One of our interests is the influence of such non-cooperative agents under several strategies.

As the first investigation, we address heuristic approaches for agents that partially cooperate to solve a problem, which is based on the representation of asymmetric constraint optimization problems. In this scheme, several agents act based on their selfish strategies, while other agents belong to a group whose optimal solution is cooperatively solved. We address how the behaviors of non-cooperative agents is affected by settings of problems and experimentally evaluate several cases of this problem.

II. BACKGROUND

A. Coalition of agents

As a fundamental coalition problem of multiagent systems, we first address the Coalition Structure Generation (CSG) problem. CSG problems are defined as partitioning problems where agents are partitioned into several groups. A CSG problem consists of a set of agents A and a set of characteristic functions that represents the values for combinations of agents. Coalition S is a subset of agents which represents a partition. The coalitions form coalition structure CS where $S, S' \in CS, S \cap S' = \emptyset$ and $\bigcup_{S \in CS} S = A$.

Value $v(S)$ of each coalition S is evaluated by the corresponding characteristic functions. The goal is to find the optimal coalition structure that maximizes global summation $\sum_{S \in CS} v(S)$ of the characteristic functions.

While standard CSG problems are represented with characteristic functions, several studies address external factors such as relationships among coalitions. In general cases

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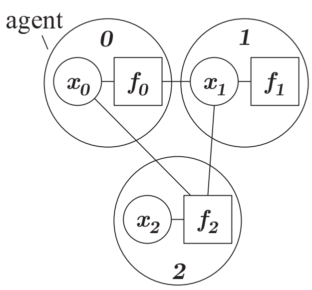

Fig. 1. Factor graph of Asymmetric COP

where complex optimization problems exist as external factors, the relationships of coalitions might not be directly represented due to the size of the problem.

B. Payoff for cooperation

The share of values for each agent in a coalition structure is another CSG problem. Several types of shares restrict the incentives to prefer other coalition structures.

In mechanism design approaches such as the VCG mechanism, the payoff for each agent is a major issue. Here the share of agents is designed for strategy proofness where agents should present their true desires or prices.

A typical way to determine a payoff is based on the contribution of each agent. For example, when solution s is evaluated by $v(s)$, the contribution of agent i is evaluated as $v(s) - v(s_i^-)$. Here $v(s_i^-)$ denotes the evaluation when agent i does not participate in solution s.

C. Asymmetric constraint optimization problem for agents

The Distributed Constraint Optimization Problem (DCOP) is a class of cooperation problems in multiagent systems. A DCOP consists of variables and objective functions that are distributed among agents who cooperatively solve the problem in a decentralized manner to maximize or minimize the global summation of the function values. In the original DCOP, agents share symmetric functions.

To distinguish the individual evaluations of agents, the asymmetric DCOP (ADCOP) [9], [10] has been proposed. With ADCOPs, different objective functions are asymmetrically defined for a set of agents. We address this asymmetric problem structure to represent local problems for individual agents. Here we assume that a problem and its solver are centralized, since we mainly focus on the problem structure.

We employ an asymmetric constraint optimization problem (ACOP) representation that resembles [11], where each agent has its own objective function. An ACOP is defined as follows.

Definition 1 (Asymmetric COP for agents): An asymmetric COP is defined by (A, X, D, F), where A is a set of agents, X is a set of variables, D is a set of domains of variables, and F is a set of objective functions.

The variables and functions are related to the agents in A. Variable $x_i \in X$ takes values from its domain defined by discrete finite set $D_i \in D$.

Agent $i \in A$ has its own local problem on $X_i \subseteq X$. $\exists(i,j)$ s.t. $i \neq j, X_i \cap X_j \neq \emptyset$. F is a set of objective functions $f_i(X_i)$ for all $i \in A$.

1 Agents are categorized into cooperative and non−cooperative ones.
2 Each non−cooperative agent fixes the assignment to its own variable.
3 The optimal solution for cooperative agents is computed under the fixed assignments.
4 Each cooperative agent receives its reward based on its contribution.
5 Each non−cooperative agent takes utility under the optimal solution of cooperative agents and fixed assignments.

Fig. 2. Flow of proposed model

1 Each agent determines an assignment to its own variable for the case of non-cooperation.
2 The globally optimal solution s for all agents is computed.
3 For each agent i{
4 The optimal solution s_i^- for agents excluding i is computed.
5 The reward of i for cooperation is the difference between the utilities of s and s_i^-.
6 The utility u_i^- of i for non−cooperation is computed under s_i^- and i's fixed assignment.
7 Under s_i^-, the existence of other assignments to x_i whose utility is greater than u_i^- is searched.
8 }

Fig. 3. Analysis of single non-cooperative agents

Function $f_i(X_i) : D_{i_0} \times \cdots \times D_{i_k} \to \mathbb{N}_0$ represents the objective value for agent i based on the variables in $X_i = \{x_{i_0}, \cdots, x_{i_k}\}$. Scope X_i of f_i should contain x_i of agent i.

For assignment $\mathcal{A}$ of the variables, global objective function $F(\mathcal{A})$ is defined as $F(\mathcal{A}) = \sum_{i \in A} f_i(\mathcal{A}_i)$. Here $\mathcal{A}_i$ denotes the projection of assignment $\mathcal{A}$ on X_i. The goal is to find assignment $\mathcal{A}^*$ that maximizes the global objective function.

Figure 1 shows a factor graph of an ACOP, where each agent i has variable x_i and function f_i. Since factor graphs directly correspond to n-ary functions, any asymmetric problems are represented.

III. NON-COOPERATIVE AGENTS ON ACOPS

A. Coalition on problem solving

Based on the representation of ACOPs, we define a simple coalition model. In addition to the ACOP elements, each agent has a choice whether the agent cooperates with a group of agents. Namely, in a single group for a problem, there are cooperative agents and other non-cooperative agents.

We only perform an exact optimization method for the cooperative agents. On the other hand, the other non-cooperative agents independently select their assignments based on their strategies. For simplicity, we assume that each non-cooperative agent fixes its own assignment before the optimization process which is performed under fixed assignments.

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B. Payoffs for problem solving

While the cooperative agents share their utilities based on their individual contributions, the non-cooperative agents independently obtain their utilities from their own objective functions. Only cooperative agents deposit their possible utilities and obtain payoffs based on the actual optimal assignments.

For each cooperative agent, its contribution is evaluated as the difference value between the optimal global utility and another one for the problem where the agent does not exist. This scheme resembles a previous work [8]. Here we assume that all non-cooperative agents select their own fixed values in the same manner for simplicity.

On the other hand, the non-cooperative agents obtain their local utilities for their fixed assignments and the optimal assignments of the cooperative agents. Note that here we assume not a mechanism for all agents but an example payoff scheme for cooperative agents.

The computation flow of the proposed model is shown in Fig. 2. Here, as the first investigation, we experimentally analyze the case where one agent is excluded from the cooperation. This is simply evaluated based on the approach in [8] as shown in Fig. 3. To find the optimal solution, we employ a tree search method. Since there are multiple optimal solutions in general cases, we selected one sample solution instead of the statistical aggregation of possible combinations of contributions that require an exhaustive search.

C. Strategies of agents

In our simple model, an agent has two types of choices. One is the decision whether to participate in the group, and another one is the fixed assignment to its variable. We experimentally investigate several heuristics.

Basically, we assume that the strategy of non-cooperative agents is greedy. Each non-cooperative agent knows the lower and upper bound values of its own function. Obviously, the lower bound value is always obtained. Therefore, if an agent is satisfied with the lower bound, it simply selects a corresponding assignment. However, the resulting utility might not be so high. If an agent takes other assignments, the agent might obtain the utilities that are less than its desired value due to the lack of agreements with other agents.

After the optimization process, a non-cooperative agent might find other assignments that improve its utility on the solution for cooperative agents. In such cases, there are risks on stability of solutions.

D. Characteristics of agents

We address several characteristics of agents to investigate the strategies of non-cooperative agents. The function of individual agents has several aspects. The variance of function values is a basic characteristic which is often considered in approximation algorithms. There are upper and lower bounds for all scopes, as described above. The boundaries are also aggregated for each assignment to a variable. The lower bound $\underline{f}_i(x_i)$ for each assignment to own variable x_i of agent i is represented as follows.

$$\underline{f}_i(x_i) = \min_{X_i \setminus x_i} f_i(X_i) \qquad (1)$$

When the lower bounds are different among the assignments, the baseline utility for each assignment can be determined.

A different characteristic is the ratio of function values among agents. The agents whose functions have large values are influential in a group.

The agents might be also affected by their degree. When an agent is related to a number of other agents by functions, the agent's decision will highly affect the solution quality.

E. Selection of non-cooperative agents

Which agents perform as non-cooperative agents will also affect the quality and stability of solutions. We address two heuristics that consider the impact of the variables of non-cooperative agents. Basically, for each agent, the impact of its own variable is evaluated as the difference between the upper and lower bound values of objective functions. The impacts resemble the one shown in [12]. Then the non-cooperative agents are selected based on the ascending order of the impacts.

The first heuristic evaluates the impact for the own function of each agent i. The impact $\hat{f}_i$ for the own function is represented as follows.

$$\hat{f}_i = \max_{x_i} \left(\max_{X_i \setminus x_i} f_i(X_i) - \min_{X_i \setminus x_i} f_i(X_i) \right) \qquad (2)$$

This impact will emphasize the influence of a variable for each individual agent.

The second heuristic evaluates the impact for all functions which relates the own variable of each agent. The impact $\check{f}_i$ for all related functions is represented as follows.

$$\check{f}_i = \max_{x_i} \left(\max_{X_i \setminus x_i} \sum_{k \text{ s.t. } x_i \in X_k} f_k(X_k) - \min_{X_i \setminus x_i} \sum_{k \text{ s.t. } x_i \in X_k} f_k(X_k) \right) \qquad (3)$$

This emphasizes the influence of a variable for all agents related to the variable.

IV. EVALUATION

A. Settings

We experimentally investigated the behavior of the proposed scheme. The problem consists of n variables and n functions. Variable x_i takes a value from its related domain whose size is three.

We employed the following types of function values.

- rnd.: random integer values with the uniform distribution. The range of values is $[0, 10]$ or $[0, 100]$.
- $g92$: random integer values based on the gamma distribution of $(\alpha, \beta) = (9, 2)$. The values are rounded to integer values.

In the basic settings, we employed ternary functions whose arity a is three.

Each non-cooperative agent takes an assignment to its own variable based on one of the following strategies.

- lb: the assignment corresponds to the lower bound of its function values.
- ub: the assignment for the upper bound of its function values.
- dlb: the assignment corresponds to the maximum lower bound value for its own assignments.

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TABLE I
INFLUENCE OF GREEDINESS OF NON-COOPERATIVE AGENTS (n=10)

problem	alg.	coop−single			coop−choice			incentive
		gt	eq	le	gt	eq	le	to change
rnd. [0, 10]	lb	0.776	0.224	0	1	0	0	0.668
	dlb	0.718	0.282	0	0.998	0.002	0	0.592
	ub	0.694	0.306	0	0.262	0.250	0.488	0.582
rnd. [0, 100]	lb	0.796	0.204	0	1	0	0	0.668
	dlb	0.774	0.226	0	0.998	0.002	0	0.64
	ub	0.708	0.292	0	0.270	0.120	0.610	0.588
$g92$	lb	0.794	0.206	0	0.994	0.006	0	0.664
	dlb	0.766	0.234	0	0.996	0.004	0	0.612
	ub	0.708	0.292	0	0.234	0.140	0.626	0.568

TABLE II
TWO GROUPS OF DIFFERENT ARITIES (n=10, $rnd.$ $[0, 10]$)

ratio of agents	alg.	a=5							a=3						
		coop−single			coop−choice			incentive	coop−single			coop−choice			incentive
		gt	eq	le	gt	eq	le	to change	gt	eq	le	gt	eq	le	to change
1 : 9	lb	0.84	0.16	0	1	0	0	0.62	0.796	0.204	0	1	0	0	0.678
	dlb	0.84	0.16	0	1	0	0	0.62	0.756	0.244	0	0.993	0.007	0	0.613
	ub	0.84	0.16	0	0.24	0.3	0.46	0.62	0.731	0.269	0	0.224	0.222	0.553	0.58
5 : 5	lb	0.8	0.2	0	1	0	0	0.644	0.764	0.236	0	1	0	0	0.668
	dlb	0.8	0.2	0	1	0	0	0.644	0.708	0.292	0	0.996	0.004	0	0.588
	ub	0.8	0.2	0	0.272	0.168	0.56	0.644	0.684	0.316	0	0.136	0.216	0.648	0.56

TABLE III
TWO GROUPS OF DIFFERENT LOWER BOUNDS OF FUNCTION VALUES (n=10, $[0, 10]$)

ratio of agents	alg.	$[5, 10]$ for an assignment to own variable							$[0, 10]$ for all assignment						
		coop−single			coop−choice			incentive	coop−single			coop−choice			incentive
		gt	eq	le	gt	eq	le	to change	gt	eq	le	gt	eq	le	to change
1 : 9	lb	0.86	0.14	0	1	0	0	0.8	0.787	0.213	0	1	0	0	0.678
	dlb	0.62	0.38	0	0.98	0.02	0	0.46	0.749	0.251	0	0.998	0.002	0	0.576
	ub	0.66	0.34	0	0.24	0.18	0.58	0.66	0.742	0.258	0	0.193	0.233	0.573	0.544
5 : 5	lb	0.812	0.188	0	1	0	0	0.744	0.804	0.196	0	1	0	0	0.644
	dlb	0.62	0.38	0	0.98	0.02	0	0.368	0.764	0.236	0	1	0	0	0.544
	ub	0.716	0.284	0	0.276	0.252	0.472	0.544	0.696	0.304	0	0.244	0.228	0.528	0.524

TABLE IV
TWO GROUPS OF DIFFERENT SCALES OF FUNCTION VALUES (n=10, $[0, 10]$)

ratio of agents	alg.	$[0, 100]$ for all assignment							$[0, 10]$ for all assignment						
		coop−single			coop−choice			incentive	coop−single			coop−choice			incentive
		gt	eq	le	gt	eq	le	to change	gt	eq	le	gt	eq	le	to change
1 : 9	lb	0.9	0.1	0	1	0	0	0.7	0.773	0.227	0	1	0	0	0.653
	dlb	0.86	0.14	0	1	0	0	0.56	0.704	0.296	0	0.996	0.004	0	0.544
	ub	0.84	0.16	0	0.08	0.24	0.68	0.62	0.66	0.34	0	0.3	0.198	0.502	0.551
5 : 5	lb	0.884	0.116	0	1	0	0	0.696	0.756	0.244	0	0.988	0.012	0	0.632
	dlb	0.848	0.152	0	1	0	0	0.524	0.684	0.316	0	0.968	0.032	0	0.568
	ub	0.84	0.16	0	0.188	0.164	0.648	0.62	0.68	0.32	0	0.452	0.14	0.408	0.512

With the above settings, we evaluated the heuristics for non-cooperative agents. The results were averaged over fifty instances.

B. Single non-cooperative agents

We evaluated the cases where one of the agents performs as a non-cooperative agent. In addition to basic settings shown above, we composed several cases where the type of a part of agents is different from other agents.

- arity: several agents have functions whose arity a is larger than others.
- lower bound: an assignment to the variable of each agent is related to a lower bound of its function value which is the largest one in all assignments. Namely, agent i selects $\mathrm{argmax}_{x_i} \underline{f}_i(x_i)$.
- function value: several agents have a different scale of function values, which is ten times greater than other ones.

Here we set the number of agent n to ten. The number of different types agents is set to one or five. A major criterion of the evaluation is the difference between the utilities of each agent in the cases of cooperation and non-cooperation.

First, we evaluated the cases where non-cooperative agents take non-cooperative assignments based on their strategies. Table I shows the results for the cases of several functions. Each value in the tables is the ratio of the number of agents. Here 'coop−single' denotes the difference between

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TABLE V
MULTIPLE NON-COOPERATEIVE AGENTS ($n = 20$, $[0, 100]$)

num.	sel.	rnd.				own				rel.			
non-coop.	alg.	sum. util.	incnt.	dcr. ratio	dcr util.	sum. util.	incnt.	dcr. ratio	dcr util.	sum. util.	incnt.	dcr. ratio	dcr util.
2	lb	1629	0.29	0.28	62.0	1629	0.34	0.26	31.4	1642	0.34	0.03	3.3
	dlb	1627	0.29	0.24	49.3	1632	0.41	0.33	45.9	1651	0.29	0.01	0.3
	ub	1634	0.21	0.20	32.7	1648	0.23	0.17	31.1	1661	0.25	0.01	1.3
5	lb	1533	0.45	0.34	93.3	1545	0.44	0.32	84.1	1579	0.37	0.17	39.4
	dlb	1544	0.40	0.30	76.8	1546	0.45	0.32	70.3	1577	0.34	0.16	26.9
	ub	1580	0.27	0.22	67.4	1581	0.32	0.26	91.3	1600	0.25	0.12	18.3
10	lb	1367	0.55	0.38	100.0	1368	0.52	0.36	96.5	1421	0.47	0.29	61.2
	dlb	1404	0.48	0.34	74.4	1419	0.46	0.33	63.5	1463	0.42	0.25	49.8
	ub	1434	0.44	0.29	82.2	1461	0.40	0.29	98.5	1482	0.34	0.21	49.9

TABLE VI
MULTIPLE NON-COOPERATEIVE AGENTS ($n = 20$, $g92$)

num.	sel.	rnd.				own				rel.			
non-coop.	alg.	sum. util.	incnt.	dcr. ratio	dcr util.	sum. util.	incnt.	dcr. ratio	dcr util.	sum. util.	incnt.	dcr. ratio	dcr util.
2	lb	494	0.43	0.32	12.99	496	0.43	0.35	13.27	502	0.53	0.11	2.67
	dlb	495	0.35	0.29	9.56	497	0.38	0.28	9.31	503	0.36	0.03	0.68
	ub	501	0.18	0.13	6.76	497	0.35	0.26	9.57	506	0.29	0.04	0.72
5	lb	473	0.46	0.35	20.09	472	0.46	0.33	21.15	483	0.42	0.19	8.84
	dlb	474	0.42	0.29	14.57	479	0.39	0.28	16.88	487	0.38	0.17	6.11
	ub	479	0.30	0.21	13.19	483	0.32	0.22	15.81	494	0.28	0.12	6.68
10	lb	428	0.51	0.33	16.83	426	0.54	0.36	20.30	442	0.48	0.27	14.97
	dlb	441	0.46	0.29	18.27	446	0.47	0.33	16.67	453	0.44	0.22	13.58
	ub	449	0.38	0.24	17.52	451	0.41	0.26	20.15	468	0.32	0.18	11.46

the reward of cooperation and the actual utility of non-cooperative case. 'coop−choice' denotes the difference between the reward of cooperation and the boundary value corresponding to the selected non-cooperative assignment. 'gt', 'eq' and 'lt' denote 'greater than', 'equals' and 'less than', respectively. 'incentive to change' denotes the cases where there are other assignments to a non-cooperative agent that improves its utility on the solution of cooperative agents.

The result shows that in all cases where an agent cooperates, the rewards of the agents are not decreased (i.e. 'coop−single' is never less than zero). On the other hand, the values of 'coop−choice' depend on the cases. Obviously, in the case of 'lb', the actual utility does not decrease, since the selected assignment corresponds to the lower bound.

In the case of 'ub', the actual utility can decrease from the corresponding upper bound in the cases of over requirement. For a greedy non-cooperative agent that prefers an upper bound, its requirement is not easily satisfied due to a mismatch of the decisions among different agents. On the other hand, the utility of 'ub' can also increase from the upper bound of a function when the corresponding agent contributes the cooperation.

In both differences, several agents obtained the same reward regardless cooperation or non-cooperation. In these situations, the non-cooperation of one of the agents did not affect the results. The ratio of such agents was relatively large in average when non-cooperative agents prefer the assignments of locally larger utility. The incentive to change an assignment is relatively large in the cases where an agent takes the assignment of lower bounds. Such a choice leaves the margin to locally improve the assignment of non-cooperative agent from one side, while that increases the risk of peer agents.

Next we examined the cases where several agents have functions whose arity is relatively high. Table II shows the results. Except 'coop−choice', the result of the agents of higher arity (a=5) is almost the same regardless their strategies. This is considered that the higher arity restricts the behaviors of agents.

Table III shows the results where several agents have a characteristic structure of lower bounds. Such an agent has an assignment that assures higher lower bound than ones for other assignments. This setting is compatible with some parts of 'dlb' that selects the assignment corresponding to the maximum lower bound in all assignments. The result shows that the incentive to change an assignment is relatively small in the case of 'dlb'. Similarly, the cases of 'gt' of 'coop−single' is also smaller than that of other strategies. Such assignment might be more influential than other assignments, and might improve the stability of solutions.

We finally evaluated the cases where several agents have the functions whose scale of values is larger than others. The results shown in Table IV slightly resemble the previous case where the incentive to change is relatively small in 'dlb'. A possible consideration is that the higher function values have a relatively large lower bound value for an assignment which is selected by 'dlb'. On the other hand, the result of 'coop−single' is different from the previous case.

C. Multiple non-cooperative agents

We also evaluated several cases with multiple non-cooperative agents. The problem consists of twenty agents including one, five or ten non-cooperative agents. Here we employed the heuristics to select non-cooperative agents as follows.

- rnd.: a random selection based on the uniform distribution.
- own: the impact for own function shown in III-E.

- rel.: the impact for related functions shown in III-E.

Table V shows the results for the functions of $[0, 100]$. 'sum. util.' denotes the total utility for all agents. It decreases when the number of non-cooperative agents increases. It is also relatively large in the case of 'rel' which considers the impact of an agent for all corresponding agents. 'incnt.' denotes the incentive to change an assignment. It resembles the previous results, while it also relatively small in the case of 'rel'.

'dcr. ratio' denotes the cases where the non-cooperative agents switch to other assignments to improve their utility after the optimization process, and that decreases the total utility for all agents. The ratio decreases when the required utility (i.e. lb, dlb and lb) increases. It also relatively small in the case of 'rel'.

'dcr. util.' denotes the decreased amount of total utility corresponding to 'dcr. ratio'. It is relatively small in several cases of 'dlb' in comparison to other boundaries in the same heuristic to select non-cooperative agents.

Table VI shows the results for the functions of $g92$. The results resemble the case of $[0, 100]$. Basically, 'rel' decreased the risk of cooperative agents more than 'own'. On the other hand, in several cases, 'dlb' decreased the possible loss of utility in comparison to other boundaries.

The results reveal the possibility of heuristics to evaluate agents which cannot participate in the cooperation. The solutions of cooperative agents should be stable for non-cooperative agents even if that cannot be fully controlled. In such situations, the heuristics and criteria of risks for non-cooperative agents might be an issue.

V. CONCLUSION

In this work, we addressed non-cooperative agents on asymmetric constraint optimization problems where the non-cooperative agents independently select its assignment. In such a situation, strategies of non-cooperative agents and problem settings affected the quality and risk of solutions. The experimental results revealed the possibility of heuristics to evaluate agents which cannot participate the cooperation. Our future work will include more detailed analysis, and methods to determine the appropriate heuristics that reduce the unsatisfactory and risk of non-cooperative agents.

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