Delay-range-dependent Robust Stability for Neutral Systems with Non-differentiable Time-varying Discrete Delay and Nonlinear Perturbations

Peerapongpat Singkibud and Kanit Mukdasai

Abstract—**This paper deals with the problem of delay-range-dependent robust stability for neutral systems with non-differentiable time-varying delay and nonlinear perturbations. By applying a novel Lyapunov-Krasovskii functional approach, Wirtinger-based integral inequality and Peng-Park's integral inequality, decomposition technique of constant matrix, descriptor model transformation, Leibniz Newton formula and utilization of zero equation, new delay-range-dependent robust stability criteria are derived in terms of linear matrix inequalities (LMIs) for the considered systems. Numerical examples are given to illustrate that the presented effective method.**

Index Terms—**neutral system, Lyapunov-Krasovskii functional, linear matrix inequality, model transformation, time-varying delay.**

I. Introduction

TIME delay is frequently a source of instability and a source of generation of oscillation in many dynamic systems such as networked control systems, biological systems, mechanical systems, and chemical or process control systems [23]. Thus, analysis and synthesis problem for systems with time-varying delay have become an important issue and a large variety of problems have been researched since the nineties by several researchers [5], [6], [17].

In some physical system, the system models can be described by functional differential equation of neutral type, which the models depend on the state delay but also depend on the state derivatives, are often encountered in various fields, such as population ecology [8], distributed networks containing lossless transmission lines [1], heat exchangers, robots in contact with rigid environments [18], etc. For interesting research methods, stability criteria for application neutral stochastic systems and neural networks have been discussed in [12], [16], [27], [28], [29], owing to that accurate model cannot be easily obtained, the addressed systems in some existent works always assume that there exist the uncertainties on the parameters or contain both the linear terms and nonlinear ones. On the one hand, in [2], [3], [4], [24] some LMI criteria on robust stability for uncertain ones have been deeply derived. Very recently, improved stability for uncertain systems with time-varying delays were proposed in [21]. However, there are rooms for further improvements in the feasible region of criteria for stability.

Stability criteria for time-delay systems are generally divided into two classes : delay-independent one and delay-dependent one. Delay-independent stability criteria tend to be more conservative, especially for small size delay, such criteria do not give any information on the size of the delay. On the other hand, delay-dependent stability criteria are concerned with the size of the delay and usually provide a maximal delay size. Generally speaking, the latter ones are less conservative than the former ones when the time-delay values are small. Many times and efforts have been put into the development of some techniques and new Lyapunov-Krasovskii functional because how to choose Lyapunov-Krasovskii functional and estimate an upper bound of time-derivative of Lyapunov-Krasovskii functional play key roles to improve the feasible region of stability criteria. Delay-dependent stability criteria for these systems are established in terms of linear matrix inequalities (LMIs).

With above motivations, based on Lyapunov stability theory, an improved stability analysis for neutral systems with non-differentiable time-varying delay and nonlinear perturbations delays is derived by the framework of LMIs which will be introduced in Theorem 9. Some numerical examples are given to illustrate that the presented effective method.

Notation:
The following notations will be accounted in this paper: let $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denotes n-dimensional Euclidean space with vector norm $||.||$ and set of $n \times m$ matrices, respectively. A matrix P is symmetric positive definite, write $P > 0$, if $P^T = P$ and $x^T P x > 0$ for all $x \in \mathbb{R}^n$,$x \neq 0$.

II. Preliminaries

Consider the following neutral system with non-differentiable time-varying discrete delay and nonlinear perturbations of the form

$$\begin{cases} \dot{x}(t) - C\dot{x}(t-r(t)) \\ = Ax(t) + Bx(t-h(t)) + f_1(t,x(t)) \\ +f_2(t,x(t-h(t))) + f_3(t,\dot{x}(t-r(t))), \quad t > 0; \\ x(t) = \phi(t), \qquad\qquad\qquad t \in [-\bar{h}, 0]; \end{cases} \tag{1}$$

Manuscript received December 7, 2017; revised January 30, 2018. The first author was supported by Research Publication Scholarship Fiscal Year of 2018, Graduate School of Khon Kaen University and Department of Mathematics, Faculty of science, Khon Kaen University. The second auther was supported by the Thailand Research Fund (TRF), the Office of the Higher Education Commission (OHEC), Khon Kaen University (Grant number MRG6080042), Research and Academic Affairs Promotion Fund, Faculty of Science, Khon Kaen University, Fiscal year 2017 and National Research Council of Thailand and Khon Kaen University, Thailand (Grant number 600054).

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where $x(t) \in R^n$ is the state variable, $r(t)$ is neutral interval time-varying delays, $h(t)$ is the time-varying delay satisfying

$$0 \le r_1 \le r(t) \le r_2, \quad 0 \le \dot{r}(t) \le r_d, \tag{2}$$
$$0 \le h(t) \le h_M, \tag{3}$$

where r_1, r_2 and h_M are positive real constants. $\phi(t)$ is initial condition function, A, B and C are constant matrices. The uncertainties $f_1(.)$, $f_2(.)$ and $f_3(.)$ represent the nonlinear parameter perturbations with respect to the current state $x(t)$, the delayed state $x(t-h(t))$ and the neutral delayed state $\dot{x}(t-r(t))$, respectively, and are bounded in magnitude

$$f_1^T(t, x(t)) f_1(t, x(t)) \le \alpha^2 x^T(t) x(t), \tag{4}$$
$$\begin{aligned} & f_2^T(t, x(t-h(t))) f_2(t, x(t-h(t))) \\ & \quad \le \beta^2 x^T(t-h(t)) x(t-h(t)), \end{aligned} \tag{5}$$
$$\begin{aligned} & f_3^T(t, \dot{x}(t-r(t))) f_3(t, \dot{x}(t-r(t))) \\ & \quad \le \eta^2 \dot{x}^T(t-r(t)) \dot{x}(t-r(t)), \end{aligned} \tag{6}$$

where α, β and η are given positive real constants.

Lemma 1. (Jensen's Inequality) For any symmetric positive definite matrix Q, positive real number h, and vector function $\dot{x}(t) : [-h, 0] \to R^n$ such that the following integral is well defined, then

$$\begin{aligned} & -h \int_{-h}^{0} \dot{x}^T(s+t) Q \dot{x}(s+t) ds \\ \le \ & -\Big(\int_{-h}^{0} \dot{x}(s+t) ds \Big)^T Q \Big(\int_{-h}^{0} \dot{x}(s+t) ds \Big). \end{aligned}$$

Lemma 2. (Wirtinger-based integral inequality [25]) For any matrix $Z > 0$, the following inequality holds for all continuously differentiable function $x : [\alpha, \beta] \to R^n$:

$$-(\beta - \alpha) \int_{\alpha}^{\beta} \dot{x}^T(s) Z \dot{x}(s) ds \quad \le \quad \omega^T \Theta \omega,$$

where $\omega = [x^T(\beta), x^T(\alpha), \frac{1}{\beta-\alpha} \int_{\alpha}^{\beta} x^T(s) ds]^T$ and $\Theta = \begin{bmatrix} -4Z & -2Z & 6Z \\ * & -4Z & 6Z \\ * & * & -12Z \end{bmatrix}$.

Lemma 3. (Peng-Park's integral inequality [19],[20]) For any matrix $\begin{bmatrix} Z & S \\ * & Z \end{bmatrix} \ge 0$, positive scalars τ and $\tau(t)$ satisfying $0 < \tau(t) < \tau$, vector function $\dot{x} : [-\tau, 0] \to R^n$ such that the concerned integrations are well defined, then

$$-\tau \int_{t-\tau}^{t} \dot{x}^T(s) Z \dot{x}(s) ds \quad \le \quad \omega^T \Theta \omega,$$

where $\omega = [x^T(t), x^T(t-\tau(t)), x^T(t-\tau)]^T$ and $\Theta = \begin{bmatrix} -Z & Z-S & S \\ * & -2Z+S+S^T & Z-S \\ * & * & -Z \end{bmatrix}$.

Lemma 4. [10] For a positive matrix M, the following inequality holds:

$$\begin{aligned} & -\frac{(\alpha-\beta)^2}{2} \int_{\beta}^{\alpha} \int_{s}^{\alpha} x^T(u) M x(u) du ds \\ \le \ & -\Big(\int_{\beta}^{\alpha} \int_{s}^{\alpha} x(u) du ds \Big)^T M \Big(\int_{\beta}^{\alpha} \int_{s}^{\alpha} x(u) du ds \Big). \end{aligned}$$

Lemma 5. [10] For a positive matrix M, the following inequality holds:

$$\begin{aligned} & -\frac{(\alpha-\beta)^3}{6} \int_{\beta}^{\alpha} \int_{s}^{\alpha} \int_{u}^{\alpha} x^T(\lambda) M x(\lambda) d\lambda du ds \\ \le \ & -\Big(\int_{\beta}^{\alpha} \int_{s}^{\alpha} \int_{u}^{\alpha} x(\lambda) d\lambda du ds \Big)^T \\ & \times M \Big(\int_{\beta}^{\alpha} \int_{s}^{\alpha} \int_{u}^{\alpha} x(\lambda) d\lambda du ds \Big). \end{aligned}$$

Lemma 6. [26] For any constant symmetric positive definite matrix $Q \in R^{n \times n}$, $h(t)$ is discrete time-varying delays with (3), vector function $\omega : [-h_M, 0] \to R^n$ such that the integrations concerned are well defined, then

$$\begin{aligned} & -h_M \int_{-h_M}^{0} \omega^T(s) Q \omega(s) ds \\ \le \ & -\int_{-h(t)}^{0} \omega^T(s) ds Q \int_{-h(t)}^{0} \omega(s) ds \\ & -\int_{-h_M}^{-h(t)} \omega^T(s) ds Q \int_{-h_M}^{-h(t)} \omega(s) ds. \end{aligned}$$

Lemma 7. [26] For any constant matrices $Q_1, Q_2, Q_3 \in R^{n \times n}$, $Q_1 \ge 0, Q_3 > 0$, $\begin{bmatrix} Q_1 & Q_2 \\ * & Q_3 \end{bmatrix} \ge 0$, $h(t)$ is discrete time-varying delays with (3) and vector function $\dot{x} : [-h_M, 0] \to R^n$ such that the following integration is well defined, then

$$\begin{aligned} & -h_M \int_{t-h_M}^{t} \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix}^T \begin{bmatrix} Q_1 & Q_2 \\ * & Q_3 \end{bmatrix} \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix} ds \\ \le \ & \begin{bmatrix} x(t) \\ x(t-h(t)) \\ x(t-h_M) \\ \int_{t-h(t)}^{t} x(s) ds \\ \int_{t-h_M}^{t-h(t)} x(s) ds \end{bmatrix}^T \\ & \times \begin{bmatrix} -Q_3 & Q_3 & 0 & -Q_2^T & 0 \\ * & -Q_3 - Q_3^T & Q_3 & Q_2^T & -Q_2^T \\ * & * & -Q_3 & 0 & Q_2^T \\ * & * & * & -Q_1 & 0 \\ * & * & * & * & -Q_1 \end{bmatrix} \\ & \times \begin{bmatrix} x(t) \\ x(t-h(t)) \\ x(t-h_M) \\ \int_{t-h(t)}^{t} x(s) ds \\ \int_{t-h_M}^{t-h(t)} x(s) ds \end{bmatrix}. \end{aligned}$$

Lemma 8. [26] Let $x(t) \in R^n$ be a vector-valued function with first-order continuous-derivative entries. Then, the following integral inequality holds for any constant matrices $X, M_i \in R^{n \times n}, i = 1, 2, \ldots, 5$ and $h(t)$ is discrete time-varying delays with (3),

$$-\int_{t-h_M}^{t}\dot{x}^T(s)X\dot{x}(s)ds \leq \begin{bmatrix} x(t) \\ x(t-h(t)) \\ x(t-h_M) \end{bmatrix}^T \times \begin{bmatrix} M_1+M_1^T & -M_1^T+M_2 & 0 \\ * & M_1+M_1^T-M_2-M_2^T & -M_1^T+M_2 \\ * & * & -M_2-M_2^T \end{bmatrix} \times \begin{bmatrix} x(t) \\ x(t-h(t)) \\ x(t-h_M) \end{bmatrix} + h_M \begin{bmatrix} x(t) \\ x(t-h(t)) \\ x(t-h_M) \end{bmatrix}^T \times \begin{bmatrix} M_3 & M_4 & 0 \\ * & M_3+M_5 & M_4 \\ * & * & M_5 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h(t)) \\ x(t-h_M) \end{bmatrix},$$

where

$$\begin{bmatrix} X & M_1 & M_2 \\ * & M_3 & M_4 \\ * & * & M_5 \end{bmatrix} \geq 0.$$

Remark 1. In Lemma 6, 7 and 8, we have modified the method from [7], [21] and [31], respectively.

III. Main results

In this section, we give our main results. We introduce the following notations for later use:

$$\sum = \left[\Sigma_{(i,j)}\right]_{18\times 18}, \tag{7}$$

where $\Sigma_{i,j} = \Sigma_{j,i}^T, \quad i,j = 1,2,3,...,18,$

$$\begin{aligned}
\Sigma_{1,1} &= \epsilon_1\alpha^2 I + W_1 + W_2 + W_1^T + W_2^T + P_2^T A_1 \\
&\quad + P_2^T B_1 + A_1^T P_2 + B_1^T P_2 + P_3^T + P_3 + Q_1 \\
&\quad + R_1 + h_M^2 Q_2 + M_1 + M_1^T + h_M M_3 - 4Q_4 \\
&\quad - Q_5 + h_M^2 R_4 - R_6 + \frac{h_M^4}{4} Q_6 - 2h_M^2 Q_7 \\
&\quad - \frac{h_M^4}{4} Q_8 + Z_3, \\
\Sigma_{1,2} &= P_1 - P_2^T + A_1^T P_{14} + B_1^T P_{14} + P_{15} + R_2 \\
&\quad + h_M^2 R_5, \\
\Sigma_{1,3} &= A_1^T P_{11} + B_1^T P_{11} + P_{12} - P_4^T - 2Q_4 + S, \\
\Sigma_{1,4} &= -W_1 + P_2^T A_2 + P_2^T B_2 + A_1^T P_5 + B_1^T P_5 \\
&\quad - P_3^T + P_6 + P_4^T - M_1^T + M_2 + h_M M_4 + Q_5 \\
&\quad - S + R_6, \quad \Sigma_{1,5} = -W_2 + L_1, \\
\Sigma_{1,6} &= -W_1 - P_2^T B_1 + P_2^T A_2 - P_3^T + L_2, \\
\Sigma_{1,7} &= -W_2 + L_3, \quad \Sigma_{1,8} = -P_4^T, \\
\Sigma_{1,9} &= \Sigma_{1,10} = 0, \\
\Sigma_{1,11} &= -R_5^T + A_1^T P_8 + B_1^T P_8 + P_9, \\
\Sigma_{1,12} &= \frac{6}{h_M} Q_4, \quad \Sigma_{1,13} = \sqrt{2} h_M Q_7, \\
\Sigma_{1,14} &= \frac{h_M}{2} Q_8, \quad \Sigma_{1,15} = P_2^T C, \\
\Sigma_{1,16} &= \Sigma_{1,17} = \Sigma_{1,18} = P_2^T, \\
\Sigma_{2,2} &= (r_2 - r_1) Q_9 - P_{14}^T - P_{14} + R_3 + h_M Q_3 \\
&\quad + h_M^2 Q_4 + h_M^2 Q_5 + h_M^2 R_6 + \frac{h_M^4}{4} Q_7 \\
&\quad + \frac{h_M^6}{36} Q_8 + h_M^2 Z_1 + r^2 Z_2, \\
\Sigma_{2,3} &= -P_{11} - P_{16}^T, \\
\Sigma_{2,4} &= -P_5 + P_{14}^T A_2 + P_{14}^T B_2 - P_{15}^T + P_{16}, \\
\Sigma_{2,5} &= 0, \quad \Sigma_{2,6} = -P_{14}^T B_1 + P_{14}^T A_2 - P_{15}^T, \\
\Sigma_{2,7} &= 0, \quad \Sigma_{2,8} = -P_{16}^T, \quad \Sigma_{2,9} = \Sigma_{2,10} = 0, \\
\Sigma_{2,11} &= -P_8, \quad \Sigma_{2,12} = \Sigma_{2,13} = \Sigma_{2,14} = 0, \\
\Sigma_{2,15} &= P_{14}^T C, \quad \Sigma_{2,16} = \Sigma_{2,17} = \Sigma_{2,18} = P_{14}^T, \\
\Sigma_{3,3} &= -P_{13}^T - P_{13} - Q_1 - R_1 - M_2 - M_2^T \\
&\quad + h_M M_5 - 4Q_4 - Q_5 - R_6, \\
\Sigma_{3,4} &= P_{11}^T A_2 + P_{11}^T B_2 - P_{12}^T - P_7 + P_{13}^T - M_1 \\
&\quad + M_2^T + h_M M_4^T + Q_5^T - S^T + R_6, \\
\Sigma_{3,5} &= 0, \quad \Sigma_{3,6} = -P_{11}^T B_1 + P_{11}^T A_2 - P_{12}^T, \\
\Sigma_{3,7} &= 0, \quad \Sigma_{3,8} = P_{13}^T, \quad \Sigma_{3,9} = -R_2, \\
\Sigma_{3,10} &= R_5^T, \quad \Sigma_{3,11} = -P_{10}, \\
\Sigma_{3,12} &= \frac{6}{h_M} Q_4, \quad \Sigma_{3,13} = \Sigma_{3,14} = 0, \quad \Sigma_{3,15} = P_{11}^T C, \\
\Sigma_{3,16} &= \Sigma_{3,17} = \Sigma_{3,18} = P_{11}^T, \\
\Sigma_{4,4} &= \epsilon_2\beta^2 I + P_5^T A_2 + P_5^T B_2 + A_2^T P_5 + B_2^T P_5 \\
&\quad - P_6^T - P_6 + P_7^T + P_7 + M_1 + M_1^T - M_2 \\
&\quad - M_2^T + S^T + h_M M_3 + h_M M_5 - 2Q_5 + S \\
&\quad - R_6 - R6^T, \quad \Sigma_{4,5} = 0, \\
\Sigma_{4,6} &= -P_5^T B_1 + P_5^T A_2 - P_6^T - L_2, \quad \Sigma_{4,7} = 0, \\
\Sigma_{4,8} &= -P_7^T, \quad \Sigma_{4,9} = 0, \quad \Sigma_{4,10} = -R_5^T, \\
\Sigma_{4,11} &= R_5^T + A_2^T P_8 + B_2^T P_8 - P_9 + P_{10}, \\
\Sigma_{4,12} &= \Sigma_{4,13} = \Sigma_{4,14} = 0, \quad \Sigma_{4,15} = P_5^T C, \\
\Sigma_{4,16} &= \Sigma_{4,17} = \Sigma_{4,18} = P_5^T, \\
\Sigma_{5,5} &= -L_1^T - L_1 - Z_3 + r_d Z_3, \quad \Sigma_{5,6} = 0, \\
\Sigma_{5,7} &= -L_1^T - L_3, \\
\Sigma_{5,8} &= \Sigma_{5,9} = \Sigma_{5,10} = \Sigma_{5,11} = \Sigma_{5,12} = \Sigma_{5,13} = \Sigma_{5,14} \\
&= \Sigma_{5,15} = \Sigma_{5,16} = \Sigma_{5,17} = \Sigma_{5,18} = 0, \\
\Sigma_{6,6} &= -L_2^T - L_2 - Z_1, \quad \Sigma_{6,7} = 0, \\
\Sigma_{6,8} &= -Z_1, \quad \Sigma_{6,9} = \Sigma_{6,10} = 0, \\
\Sigma_{6,11} &= -B_1^T P_8 + A_2^T P_8 - P_9, \\
\Sigma_{6,12} &= \Sigma_{6,13} = \Sigma_{6,14} = \Sigma_{6,15} = \Sigma_{6,16} = \Sigma_{6,17} \\
&= \Sigma_{6,18} = 0, \\
\Sigma_{7,7} &= -L_3^T - L_3 - Z_2, \\
\Sigma_{7,8} &= \Sigma_{7,9} = \Sigma_{7,10} = \Sigma_{7,11} = \Sigma_{7,12} = \Sigma_{7,13} \\
&= \Sigma_{7,14} = \Sigma_{7,15} = 0, \\
\Sigma_{7,16} &= \Sigma_{7,17} = \Sigma_{7,18} = 0, \\
\Sigma_{8,8} &= -Z_1, \quad \Sigma_{8,9} = \Sigma_{8,10} = 0, \quad \Sigma_{8,11} = -P_{10}, \\
\Sigma_{8,12} &= \Sigma_{8,13} = \Sigma_{8,14} = \Sigma_{8,15} = \Sigma_{8,16} = \Sigma_{8,17} \\
&= \Sigma_{8,18} = 0, \\
\Sigma_{9,9} &= -R_3, \quad \Sigma_{9,10} = \Sigma_{9,11} = \Sigma_{9,12} = \Sigma_{9,13} \\
&= \Sigma_{9,14} = \Sigma_{9,15} = \Sigma_{9,16} = \Sigma_{9,17} = \Sigma_{9,18} = 0, \\
\Sigma_{10,10} &= -Q_2 - R_4, \\
\Sigma_{10,11} &= \Sigma_{10,12} = \Sigma_{10,13} = \Sigma_{10,14} = 0, \\
\Sigma_{10,15} &= \Sigma_{10,16} = \Sigma_{10,17} = \Sigma_{10,18} = 0
\end{aligned}$$

ISBN: 978-988-14048-8-6
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

$$\begin{aligned}
\Sigma_{11,11} &= -Q_2 - R_4,\\
\Sigma_{11,12} &= \Sigma_{11,13} = \Sigma_{11,14} = 0, \quad \Sigma_{11,15} = P_8^T C,\\
\Sigma_{11,16} &= \Sigma_{11,17} = \Sigma_{11,18} = P_8^T,\\
\Sigma_{12,12} &= \frac{-12}{h^2} Q_4, \quad \Sigma_{12,13} = \Sigma_{12,14} = \Sigma_{12,15}\\
&= \Sigma_{12,16} = \Sigma_{12,17} = \Sigma_{12,18} = 0,\\
\Sigma_{13,13} &= -Q_7, \quad \Sigma_{13,14} = \Sigma_{13,15} = \Sigma_{13,16}\\
&= \Sigma_{13,17} = \Sigma_{13,18} = 0,\\
\Sigma_{14,14} &= -Q_6 - Q_8, \quad \Sigma_{14,15} = \Sigma_{14,16}\\
&= \Sigma_{14,17} = \Sigma_{14,18} = 0,\\
\Sigma_{15,15} &= \epsilon_3 \eta^2 I - (r_2 - r_1)(1 - r_d) Q_9,\\
\Sigma_{15,16} &= \Sigma_{15,17} = \Sigma_{15,18} = 0,\\
\Sigma_{16,16} &= \epsilon_1 I, \quad \Sigma_{16,17} = \Sigma_{16,18} = 0,\\
\Sigma_{17,17} &= \epsilon_2 I, \quad \Sigma_{17,18} = 0,\\
\Sigma_{18,18} &= \epsilon_3 I, \quad W_1 = P_1 J, \quad W_2 = P_2 K.
\end{aligned}$$

Theorem 9. For a prescribed scalars $r_2 > 0, r_d \geq 0$ and $h_M > 0$ the system (1) is asymptotically stable, if there exist positive definite symmetric matrices P_1, R_4, R_6, Q_i, Z_j, i=1,2,..,9, j=1,2,3, any appropriate dimensional matrices, S, J, K, R_k, M_l, P_m, k=1,2,..,6, l=1,2,...,5, m=2,3,...,16 and positive real constants ϵ_1, ϵ_2 and ϵ_3 satisfying the following LMIs

$$\begin{bmatrix} R_1 & R_2 \\ R_2^T & R_3 \end{bmatrix} > 0, \tag{8}$$

$$\begin{bmatrix} R_4 & R_5 \\ R_5^T & R_6 \end{bmatrix} > 0, \tag{9}$$

$$\begin{bmatrix} Q_5 & S \\ S^T & Q_5 \end{bmatrix} \geq 0, \tag{10}$$

$$\begin{bmatrix} Q_3 & M_1 & M_2 \\ M_1^T & M_3 & M_4 \\ M_2^T & M_4^T & M_5 \end{bmatrix} \geq 0, \tag{11}$$

$$\sum < 0. \tag{12}$$

Proof. Under the condition of the theorem, we will show the asymptotic stability of system (1). From model transformation method, we rewrite the system (1) in the following system:

$$\begin{aligned}
\dot{x}(t) &= y(t), \qquad (13)\\
0 &= -y(t) + Ax(t) + Bx(t - h(t))\\
&\quad + f_1(t, x(t)) + f_2(t, x(t - h(t)))\\
&\quad + f_3(t, y(t - r(t))) + Cy(t - r(t)). \qquad (14)
\end{aligned}$$

In order to improve of the discrete delay h(t) in (1), let us decompose constant matrix A and B as

$$A = A_1 + A_2, \tag{15}$$

$$B = B_1 + B_2, \tag{16}$$

where A_1, A_2, B_1 and $B_2 \in R^{n\times n}$ are real constant matrices. By utilizing the following zero equation, we obtain

$$0 = Jx(t) - Jx(t - h(t)) - J\int_{t-h(t)}^{t} \dot{x}(s)ds, \tag{17}$$

$$0 = Kx(t) - Kx(t - r(t)) - K\int_{t-r(t)}^{t} \dot{x}(s)ds, \tag{18}$$

where J, $K \in R^{n\times n}$ will be chosen to guarantee the asymptotic stability of the system (1). By (15)-(18), the system and can be represented by the form

$$\begin{aligned}
\dot{x}(t) &= y(t) + Jx(t) - Jx(t - h(t)) - J\int_{t-h(t)}^{t} y(s)ds\\
&\quad + Kx(t) - Kx(t - r(t)) - K\int_{t-r(t)}^{t} y(s)ds, \qquad (19)\\
0 &= -y(t) + (A_1 + B_1)x(t)\\
&\quad + (A_2 + B_2)x(t - h(t))\\
&\quad - (B_1 - A_2)\int_{t-h(t)}^{t} y(s)ds\\
&\quad + f_1(t, x(t)) + f_2(t, x(t - h(t)))\\
&\quad + f_3(t, y(t - r(t))) + Cy(t - r(t)). \qquad (20)
\end{aligned}$$

Construct a Lyapunov-Krasovskii functional candidates for the system (19)-(20) of the form

$$V(t) = \sum_{i=1}^{9} V_i(t), \tag{21}$$

where

$$\begin{aligned}
V_1(t) &= \zeta^T(t) EP\zeta(t),\\
V_2(t) &= \int_{t-h_M}^{t} x^T(s) Q_1 x(s)ds\\
&\quad + \int_{t-h_M}^{t} \begin{bmatrix} x(s) \\ y(s) \end{bmatrix}^T \begin{bmatrix} R_1 & R_2 \\ * & R_3 \end{bmatrix} \begin{bmatrix} x(s) \\ y(s) \end{bmatrix} ds,\\
V_3(t) &= h_M \int_{-h_M}^{0}\int_{t+s}^{t} x^T(\theta) Q_2 x(\theta) d\theta ds\\
&\quad + \int_{-h_M}^{0}\int_{t+s}^{t} y^T(\theta) Q_3 y(\theta) d\theta ds,\\
V_4(t) &= h_M \int_{-h_M}^{0}\int_{t+s}^{t} y^T(\theta) Q_4 y(\theta) d\theta ds\\
&\quad + h_M \int_{-h_M}^{0}\int_{t+s}^{t} y^T(\theta) Q_5 y(\theta) d\theta ds,\\
V_5(t) &= h_M \int_{-h_M}^{0}\int_{t+s}^{t} \begin{bmatrix} x(\theta) \\ y(\theta) \end{bmatrix}^T \begin{bmatrix} R_4 & R_5 \\ * & R_6 \end{bmatrix} \begin{bmatrix} x(\theta) \\ y(\theta) \end{bmatrix} d\theta ds,\\
V_6(t) &= h_M \int_{-h_M}^{0}\int_{t+s}^{t} y^T(\theta) Z_1 y(\theta) d\theta ds\\
&\quad + r_2 \int_{-r_2}^{0}\int_{t+s}^{t} y^T(\theta) Z_2 y(\theta) d\theta ds,\\
V_7(t) &= \frac{(h_M)^2}{2} \int_{t-h_M}^{t}\int_{s}^{t}\int_{u}^{t} x^T(\lambda) Q_6 x(\lambda) d\lambda du ds\\
&\quad + h_M^2 \int_{t-h_M}^{t}\int_{s}^{t}\int_{u}^{t} y^T(\lambda) Q_7 y(\lambda) d\lambda du ds,\\
V_8(t) &= \frac{(h_M)^3}{6} \int_{t-h_M}^{t}\int_{s}^{t}\int_{u}^{t}\int_{\lambda}^{t} y^T(\theta) Q_8 y(\theta) d\theta d\lambda du ds,\\
V_9(t) &= \int_{t-r(t)}^{t} x^T(s) Z_3 x(s)ds\\
&\quad + (r_2 - r_1)\int_{t-r(t)}^{t} y^T(s) Q_9 y(s)ds,
\end{aligned}$$

ISBN: 978-988-14048-8-6
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

with

$$E = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$P = \begin{bmatrix} P_1 & 0 & 0 & 0 & 0 \\ P_2 & P_5 & P_8 & P_{11} & P_{14} \\ P_3 & P_6 & P_9 & P_{12} & P_{15} \\ P_4 & P_7 & P_{10} & P_{13} & P_{16} \end{bmatrix},$$

$$\zeta(t) = \begin{bmatrix} x(t) \\ x(t-h(t)) \\ \int_{t-h(t)}^{t} x(s)ds \\ x(t-h_M) \\ y(t) \end{bmatrix}.$$

The time derivative of $V(t)$ along the trajectory of system (19)-(20) is given by

$$\dot{V}(t) = \sum_{i=1}^{9} \dot{V}_i(t). \tag{22}$$

From (4), (5) and (6), we obtain for any positive real constants $\epsilon_1, \epsilon_2, \epsilon_3$,

$$\epsilon_1\Big(\alpha^2 x^T(t)x(t) - f_1^T(t,x(t))f_1(t,x(t))\Big) \geq 0, \tag{23}$$

$$\epsilon_2\Big(\beta^2 x^T(t-h(t))x(t-h(t)) - f_2^T(t,x(t-h(t)))f_2(t,x(t-h(t)))\Big) \geq 0, \tag{24}$$

$$\epsilon_3\Big(\eta^2 \dot{x}^T(t-r(t))\dot{x}(t-r(t)) - f_3^T(t,\dot{x}(t-r(t)))f_3(t,\dot{x}(t-r(t)))\Big) \geq 0. \tag{25}$$

According to (22)-(25), it is straightforward to see that

$$\dot{V}(t) \leq \zeta^T(t) \sum \zeta(t), \tag{26}$$

where $\zeta^T(t)$=$\Big[x^T(t), y^T(t), x^T(t-h_M), x^T(t-h(t)),$
$x^T(t-r(t)), \int_{t-h(t)}^{t} y^T(s)ds, \int_{t-r(t)}^{t} y^T(s)ds,$
$\int_{t-h_M}^{t-h(t)} y^T(s)ds, y^T(t-h_M), \int_{t-h_M}^{t-h(t)} x^T(s)ds,$
$\int_{t-h(t)}^{t} x^T(s)ds, \int_{t-h_M}^{t} x^T(s)ds, \sqrt{2}\int_{t-h_M}^{t} x^T(u)du,$
$\int_{t-h_M}^{t}\int_{u}^{t} x^T(\lambda)d\lambda du, y(t-r(t)), f_1(t,x(t)),$
$f_2(t,x(t-h(t))), f_3(t,y(t-r(t)))\Big]$. If the conditions (8)-(12) hold, then (26) implies that there exists $\delta > 0$ such that $\dot{V}(t) \leq -\delta \left\|x(t)\right\|^2$. Therefore, system (1) is asymptotically stable.

IV. Numerical Examples

In this section, numerical example is given to present the effectiveness of our main results by comparing the upper bounds of the delays h_M.

Example 10. We consider system (1) of the form

$$\begin{cases} \dot{x}(t) - C\dot{x}(t-r(t)) \\ = Ax(t) + Bx(t-h(t)) + f_1(t,x(t)) \\ +f_2(t,x(t-h(t))) + f_3(t,\dot{x}(t-r(t))), \quad t > 0; \\ x(t) = \phi(t), \qquad t \in [-\bar{h}, 0]; \end{cases} \tag{27}$$

with the parameters

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0.4 \\ 0.4 & 0 \end{bmatrix},$$
$$C = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad \alpha = 0.1, \quad \beta = \eta = 0.05. \tag{28}$$

Decompose matrix A and B as follows : $A = A_1 + A_2$, $B = B_1 + B_2$, where

$$A_1 = \begin{bmatrix} -1.5 & -0.5 \\ -0.1 & -1.5 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -0.5 & 0.5 \\ 0.1 & -0.5 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} -0.6 & 0 \\ -0.5 & -1.1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.6 & 0.4 \\ 0.9 & 1.1 \end{bmatrix}. \tag{29}$$

Table 1 lists the comparison of the upper bounds delays for asymptotic stability of system (27) by different methods. It is clear that our results are superior to those in [4], [14], [15], [22].

Table 1
Maximum allowable upper bounds h_M for the delay is time-varying in Example 10.

Method				
μ_m	0.5	0.9	1.1	Unknown
r_d=0.6				
Lakshmanan et al. (2011) [15]	-	-	-	3.9563
Cheng et al. (2013) [4]	-	-	-	4.6235
Qiu et al. (2015) [22]	-	-	-	4.9423
Ours	-	-	-	8.7375
r_d=0				
Liu et al. (2015) [14]	8.975	8.820	-	-
Qiu et al. (2015) [22]	9.646	9.225	-	-
Cheng et al. (2013) [4]	9.975	9.756	9.685	-
Ours	-	-	-	9.7967

V. Conclusion

The problem of robust stability for neutral systems with mixed interval time-varying delays and nonlinear perturbations was studied. The restriction on the derivative of the discrete time-varying delay is removed. By applying a novel Lyapunov-Krasovskii functional approach, Wirtinger-based integral inequality and Peng-Park's integral inequality, decomposition technique of constant matrix, descriptor model transformation, Leibniz Newton formula and utilization of zero equation, an improved delay-range-dependent stability for considered system are established in terms of linear matrix inequalities (LMIs). Numerical examples have shown significant improvements over some existing results.

Acknowledgment

The authors thank anonymous reviewers for their valuable comments and suggestions.

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