

# Proposal of New Ant System Based on Consistency and Discrepancy of Subjective Ranking

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**Abstract**—It is known that the Ant Colony Optimization (ACO) inspired from the collective behavior of real ants, and it is effective to find a better solution for the Traveling Salesman Problem (TSP). Rank based Ant System  $AS_{rank}$  has been proposed as a developed version of basic Ant System. In the algorithm of  $AS_{rank}$ , each agent in Ant System is ranked from the viewpoint outside the system as to the participation in pheromone update. Then, in spite of the fact that the collective behavior of real ants has inspired in constructing the algorithm of Ant System,  $AS_{rank}$  as a developed version includes the viewpoint outside the system that does not exist in the actual ants' swarm. Furthermore, there is a problem that it tends to be easy to fall into a local solution. In our study, we introduce the behavior observed in real ants' experiments in order to construct a new algorithm of Ant System. That is, each ant agent in Ant System estimates its own rank by interaction with encountered agents to determine whether it should contribute to pheromone deposition. Therefore, we carried out exploring simulations in several TSP datasets, and we will show some analysis results that indicate the proposed model has superiority than  $AS_{rank}$ .

**Index Terms**—Ant Colony Optimization, behavior of forager ants, self evaluation, local solution, irregular time distribution.

## I. INTRODUCTION

ANT Colony Optimization (ACO) proposed by inspiring from the collective behavior of real ants has been known as one of metaheuristics to solve combinatorial optimization problems. It is known that ACO is an effective method to find short tours in the Traveling Salesman Problem (TSP). Many ACO models have been developed by extending the Ant System (AS) proposed by Dorigo et al. [1].

Rank based Ant System ( $AS_{rank}$ ) is one of the representative models extending original AS [2]. This model has a feature at the procedure for pheromone update in the exploring simulation to find solution of TSP. At the end of each tour, ant agents are ranked by their tour length in shorter order. Then, only top-ranked agents are allowed to deposit pheromone. This function improves the convergence of the system by adding pheromones on only the edge toured by top-ranked agents. However, although the system can converge faster through concentrating pheromone on specific edges, the system falls into local solution easily. Falling into

local solution is a situation in which a new better solution compared with the current best solution is never found. In addition, there is also a question in decision making method of pheromone deposition.  $AS_{rank}$  introduces a third party's point of view outside the system in order to rank agents, such a mechanism does not exist in the habit of actual ants' swarm.

By the way, an interesting behavior of forager ants to search for feeding site is reported. For instance, the recent experimental research showed a behavior in the tandem running where the follower ant is inducted to a feeding site through contacting with the leader ant [3]. In this experiment, when the follower ant lost leader ant, some follower ants give up looking for a leader and succeed in reaching a feeding site without the inducement by the leader ant. It is estimated that these succeeded ants had information that they experienced own once they reached a feeding site, and they acted based on own knowledge after losing leader ant. That is, although some follower ants have the cues of feeding site they have experienced, they do not use those information while the follower ants are being conducted by leader ant in the tandem running. Accordingly, ants seem to decide on behaviors by switching superiority or inferiority between information obtained from leader ant and information based on their own experiences. Further, through acting as the follower ant, the follower ant could be guided by the leader ant to the feeding site where the experienced ant has never visited. And then, the follower ant can memorize new feeding sites that she has never experienced. From this, it is possible to avoid sticking to a single feeding site so that it can improve the profit of the ants' swarm.

In other studies, it is suggested that living things may estimate the entire state of group from information they have experienced individually and make a decision to take adaptive actions for a swarm [4].

In the case of considering the mentioned above, we should incorporate the behavior of living things for constructing a new AS model. Therefore, inspired by the information interaction with other individuals in the actual ants' swarm, we propose  $AS_{multi}$  as the new ACO model. In the  $AS_{multi}$ , agents estimate by themselves whether or not they should use their own information to solving TSP problem. In other words, ant agents determine whether contribute to pheromone deposition throughout their information interaction with other individuals encountering in a tour. Through voluntary behaviors based on self-evaluation of individual agents, we expect the diversity of solutions and the avoidance to fall into a local solution.

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## II. BACKGROUND

### A. Ant System (AS)

First of all, we explain original Ant System (AS) proposed by Dorigo et al [1]. Here, we define parameters as follows:

$t$	iteration counter
$k$	agent identification number
$c$	the number of cities
$\tau_{ij}$	intensity of pheromone trail on an edge between cities $i$ and $j$
$\eta_{ij}$	visibility of city $j$ from city $i$ , i.e., the reciprocal number of the distance between cities $i$ and $j$
$\alpha$	weight of $\tau_{ij}$
$\beta$	weight of $\eta_{ij}$
$N_k$	set of cities agent $k$ has never visited in one tour
$d_{ij}$	distance between cities $i$ and $j$
$\rho$	parameter to regulate the reduction of $\tau_{ij}$
$L_k$	tour length of solution the agent $k$ found
$T_k$	set of edges included in the solution agent $k$ found

AS is one of metaheuristics for solving optimization problems by imitating group intellectual behavior of social insects, especially ants' foraging behavior. In practice of AS, each ant agent is individually allocated to an initial city and begins destination selection to make a tour. One city among unvisited cities in current tour is determined stochastically as the next destination by using value of the pheromone amount and the reciprocal number of the distance between cities. Here, the probability  $P_{ij}$  that the city  $j$  is chosen as the next destination from the current city  $i$  is given as follows:

$$P_{ij}(t) = \frac{[\tau_{ij}(t)]^\alpha [\eta_{ij}(t)]^\beta}{\sum_{\ell \in N_k(t)} [\tau_{i\ell}(t)]^\alpha [\eta_{i\ell}(t)]^\beta}, \forall j \in N_k(t).$$

After creating one tour, the value of pheromones on all edges regulated at a certain rate, and ant agents deposit pheromones on the edges that each agent has passed. Then, the pheromone deposition amount depends on the length of the traveling tour. The pheromone update process is defined as follows:

$$\tau_{ij}(t+1) = \begin{cases} \rho\tau_{ij}(t) + \sum_k \frac{1}{L_k}, & \text{if } (i, j) \in T_k(t), \\ \rho\tau_{ij}(t), & \text{otherwise.} \end{cases}$$

These operation are repeated in order to find shorter traveling tour. When the search is terminated, the shortest tour solution becomes the result of one trial.

### B. Rank based Ant System ( $AS_{rank}$ )

We describe the Rank based Ant System ( $AS_{rank}$ ).  $AS_{rank}$  is one of the ACO models proposed by Bullnheimer et al. and it is extended from the original AS [2]. The way of determining the probability of next destination selection in  $AS_{rank}$  is same as that of AS. However, there is difference in pheromone update method from AS.

In pheromone update process of  $AS_{rank}$ , only the upper-ranked agents who have shorter tour-length solutions among all agents can contribute to pheromone deposition. In addition, also the virtual agent who has found the best solution until current iteration can deposit pheromones. These upper-ranked agents and the virtual agent are defined as the elitist agents, hence only these elitist agents can contribute to pheromone deposition. Here, we define parameters as follows:

$Q$	weight for pheromone
$\mu$	ranking index
$\Delta\tau_{ij}^\mu$	increase of trail level on edge $(i, j)$ caused by $\mu$ -th best ant
$L_\mu$	tour length of solution the $\mu$ -th best ant found
$T_\mu$	set of edges the $\mu$ -th best ant passed
$\Delta\tau_{ij}^*$	increase of trail level on edge $(i, j)$ caused by elitist ants
$\sigma$	number of elitist ants
$L^*$	tour length of best solution found
$T^*$	set of edges included in the best solution found

Then, pheromone update process is defined as follows:

$$\tau_{ij}(t+1) = \rho\tau_{ij}(t) + \Delta\tau_{ij}(t) + \Delta\tau_{ij}^*(t),$$

$$\Delta\tau_{ij}(t) = \sum_{\mu=1}^{\sigma-1} \Delta\tau_{ij}^\mu(t),$$

$$\Delta\tau_{ij}^\mu(t) = \begin{cases} (\sigma - \mu) \frac{Q}{L_\mu(t)}, & \text{if } (i, j) \in T_\mu(t), \\ 0, & \text{otherwise,} \end{cases}$$

$$\Delta\tau_{ij}^*(t) = \begin{cases} \sigma \frac{Q}{L^*(t)}, & \text{if } (i, j) \in T^*(t), \\ 0, & \text{otherwise.} \end{cases}$$

In this method, only the agent from the top to  $(\sigma - 1)$ th can participate in the pheromone deposition, and pheromones are deposited only on edges where elitist ants have passed in one tour. In contrast, pheromones on edges where elitist ants have never passed through in one tour are evaporated more and more. As a result, pheromones concentrate on specific edges rapidly, so that many agents become to trace out same tour. It is an advantage of  $AS_{rank}$  that a shorter tour length solution can be found shortly after the search begins. However, at the same time, it is true that the diversity of solutions is instantly lost and it tends easily to fall into local solution. Additionally, an external viewpoint that does not exist in the actual ants' swarm is introduced in the process to select elitist agents.

## III. PROPOSAL

In this section, we propose Multi-Rank based Ant System ( $AS_{multi}$ ) as a new model to improve  $AS_{rank}$ . In  $AS_{multi}$ , the way to determine the probability of next destination selection is same to the way in AS and  $AS_{rank}$ , but a new method to regulate agents who contribute to deposit pheromones is adopted. We explain the new method of pheromone update in  $AS_{multi}$  in detail here. Firstly, we define parameters as follows:

$s$	step count in one tour
$\gamma$	threshold for tour step in agent competition
$Z$	value calculated in agent competition
$\theta$	threshold for $Z$ in agent competition
$F_k$	binary parameter whether agent $k$ survived the competition
$a$	the number of agents (equal to $c$ : the number of cities)
$C_x(s, t)$	city where the agent $x$ is in at step $s$
$C_y(s, t)$	city where the agent $y$ is in at step $s$
$\ell_x(s, t)$	accumulated tour length of agent $x$ at step $s$
$\ell_y(s, t)$	accumulated tour length of agent $y$ at step $s$

If following two conditions are satisfied, then the competition among agents occurs:

- The step count  $s$  satisfies the relationship of  $\frac{s}{c} > \gamma$ .
- There is other agents in the same city.

When one agent  $x$  encounters other agent  $y$ , i.e.,  $C_x(s, t) = C_y(s, t)$ , agents compares mutually the accumulated tour lengths at step count  $s$  in one tour. This process purposes to evaluate whether there is enough difference between their accumulated tour lengths. An evaluation value  $Z$  is calculated using their accumulated tour lengths  $l_x(s, t)$  and  $l_y(s, t)$ . If  $Z$  exceeds threshold in agent competition  $\theta$ , the agent having a longer length solution is defeated. At this time, 0 is assigned to the parameter  $F_k$  of the defeated agent. Here, the competition process is defined as follows:

$$Z = \frac{|l_x(s, t) - l_y(s, t)|}{(l_x(s, t) + l_y(s, t))},$$

$$\begin{cases} F_x \leftarrow 0, & \text{if } Z > \theta \cap l_x(s, t) \geq l_y(s, t), \\ F_y \leftarrow 0, & \text{if } Z > \theta \cap l_x(s, t) \leq l_y(s, t). \end{cases}$$

Simultaneously, please note following prerequisites:

- All parameter of  $F_k$  are initialized to 1 at the beginning of one tour.
- If an agent  $x$  is defeated even once in the middle of a tour,  $F_x$  changes to 0 and the condition of  $F_x = 0$  is kept until the end of one tour.
- If three or more agents are exist at the same time in the same city, comparison is carried out in all the pairs.

Finally, agents who have never been defeated after completing one tour (who has value 1 of  $F_k$ ) can contribute to pheromone deposition. In this method, it can be said that each agent makes a decision by oneself as to whether or not to take part in pheromone deposition using the information from contacted agents. Since this method does not require the viewpoint of the external third party for summarizing the agent's information like  $AS_{rank}$ , it is faithful to the actual habits of actual ants' swarm. After creating one tour, the pheromone update process is executed as follows:

$$\tau_{ij}(t+1) = \rho\tau_{ij}(t) + \begin{cases} \sum_k \frac{1}{L_k}, & \text{if } (i, j) \in T_k(t) \cap F_k = 1, \\ 0, & \text{otherwise.} \end{cases}$$

#### IV. EXPERIMENT

We carried out an experiment of tour exploration on TSP benchmark datasets and compared these simulation results of  $AS_{multi}$  (proposed model) with  $AS_{rank}$  (conventional model). In this experiment, we used three TSP datasets, i.e.,  $eil51.tsp(c = 51)$ ,  $eil76.tsp(c = 76)$ ,  $berlin52.tsp(c = 52)$  [5]. The TSP model used here is classified as Symmetric Traveling Salesman Problem that the edge length between two cities is the same in each opposite direction. Further, parameters used for each ACO model are shown in Table I. The values of parameters in  $AS_{rank}$  were set as those used in the proposal paper [2]. On one hand, the values of parameters in  $AS_{multi}$  were selected through preliminary experiments. The trials of 10 times were performed for each TSP dataset.

TABLE I  
PARAMETER SETTINGS

ACO model	$AS_{multi}$	$AS_{rank}$
$(\alpha, \beta)$	(1,1)	(1,5)
$\rho$	0.5	
Loop tour per 1trial	1000	
Simulation trial	10	
Other Parameters	$\gamma = 0.9$ $\theta = 0.001$	$Q = 100$ $\sigma = 6$

TABLE II  
BEST SOLUTION AVERAGE IN 10 TRIALS

TSP	$AS_{multi}$	$AS_{rank}$	p-value	(optimal)
<i>eil51</i>	433.2	441.8	2.46E-03	(426)
<i>ell76</i>	552.6	558.5	2.92E-03	(538)
<i>berlin52</i>	7638	7700	0.185	(7542)

##### A. Best solution

First of all, we compared the best solution obtained from each trial between two ACO models. We show the averaged best solution tour length from 10 trials in Table II. The p-value in Table II is the probability accepting the hypothesis that there is no significant difference between the data of both models under the U test, and 'optimal' in Table II indicates the tour length of the optimal solution in each TSP dataset. As shown in here, the proposed  $AS_{multi}$  has better (shorter) results on average in all datasets than  $AS_{rank}$ . In addition, the result of  $AS_{multi}$  shows significant superiority in both *eil51.tsp* and *eil76.tsp* datasets.

##### B. Relationship between the best solution update and variance of tour length

We analyzed how the best solution improved and how each agent traveled during each tour. The Figure 1 shows the averaged tour length and the best solution found by all agents on tour iteration  $t$ . These results are obtained from one trial for each benchmark dataset of each model as an example, but please note that similar results can be obtained from other trials.

In Figure 1 (b) for  $AS_{rank}$ , both the best solution and the averaged tour length fall down to a constant value shortly after beginning of exploration. It means the feature that  $AS_{rank}$  has the high convergence. On the other hand, in Figure 1 (a) for  $AS_{multi}$ , although the best solution gradually decreases as time elapses, the averaged tour length of each iteration oscillates at a relatively high position. The latter is derived from the feature that  $AS_{multi}$  does not loose shortly the diversity of solutions. Furthermore, it is found that the best solution in  $AS_{multi}$  is favorer comparing with the best solution in  $AS_{rank}$ . This result means that  $AS_{multi}$  found favorer tour by keeping on the diversity of solutions. In contrast, it is seen that  $AS_{rank}$  fallen into a local solution by the convergence ability by oneself.

##### C. Number of the best solution updated

We compared the number of updates of the best solution found up to each step. If the tour length of a solution found at a certain iteration time  $t$  was shorter than the best solution

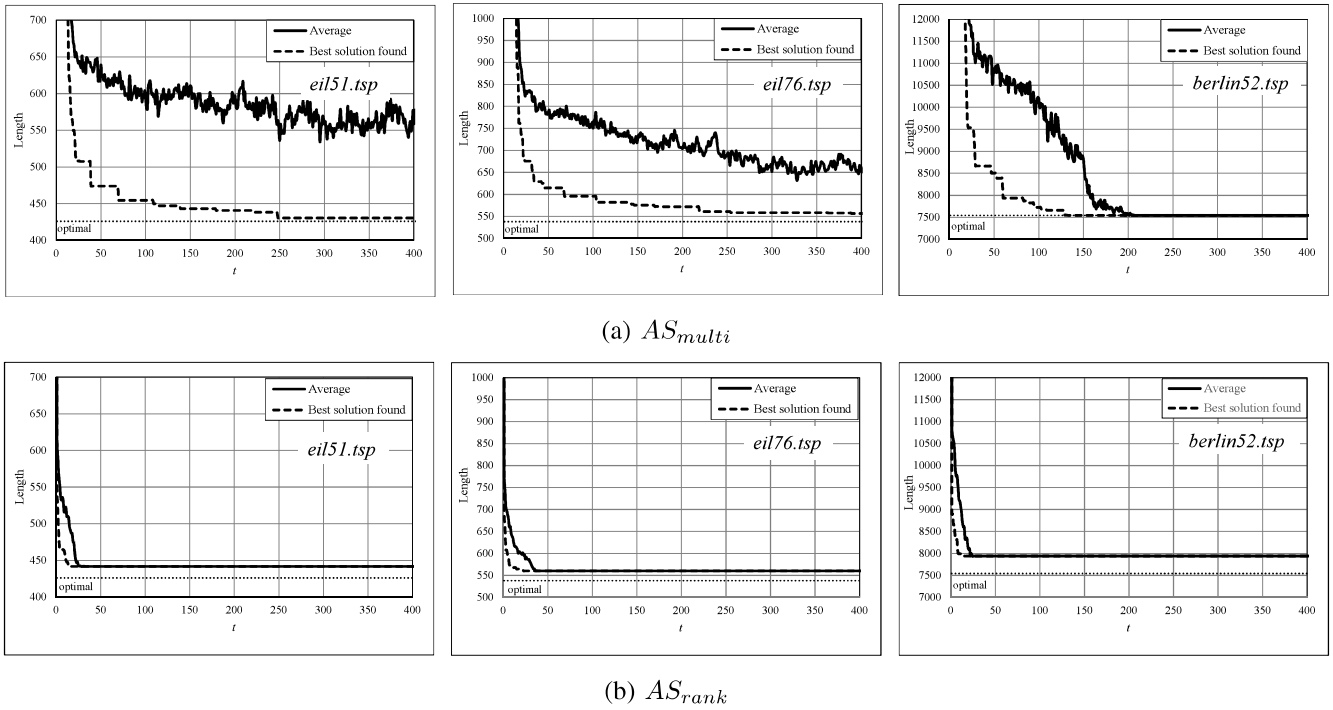


Fig. 1. Transition of average and best solution tour length

TABLE III  
 THE AVERAGE OF PHEROMONE UPDATE TIMES

TSP	$AS_{multi}$	$AS_{rank}$	p-value
<i>eil51</i>	26.4	11.7	1.79E-04
<i>ell76</i>	33.2	11.6	1.66E-04
<i>berlin52</i>	25.4	11.3	1.73E-04

found up to that iteration, it is considered that the best solution has been updated. Then, we calculated the average number of solution updates in 10 trials, and these are shown in Table III. From Table III, it can be seen that  $AS_{multi}$  has about 2 to 3 times the number of updates in any TSP datasets. This result means also that  $AS_{multi}$  found favorer tour by keeping on the diversity of solutions comparing with  $AS_{rank}$ .

#### D. Interval of pheromone deposition

We analyzed the interval data of pheromone deposition to check the peculiarity of agents' behavior. Then, we evaluated which of exponential distribution and power distribution is appropriate for the distribution on the interval data in  $AS_{rank}$  and  $AS_{multi}$  [6]. Figure 2 shows the both logarithmic graph of relationship between that interval and its cumulative distribution. From the graph, it seems that interval data of  $AS_{multi}$  in Figure 2(a) has more linearity than interval data of  $AS_{rank}$  in Figure 2(b).

However, it is difficult to determine exactly which distribution is suitable from only these graph information. Hence, for evaluating the model suitability, we analyzed those data in more detail with AIC (Akaike Information Criterion), and calculated AIC weight that determines which distribution is appropriate [7]. Results in Table IV show that the  $AS_{multi}$  appears to produce power distribution in every TSP benchmark dataset while the  $AS_{rank}$  appears to produce

exponential distribution. Here, it is confirmed that similar results are obtained in the interval data of other agents.

To follow exponential distribution means that the average can be calculated. Therefore, the agent in  $AS_{rank}$  where the pheromone deposition interval follows exponential distribution, it is estimated that the agent is experiencing pheromone update on average, in other words, participating to pheromone update with a certain rhythm. It means that the ranking of agents are switched alternately between higher rank and lower rank at a constant rate. As a result, although it appears that only the upper-ranked agents participate in pheromone deposition in  $AS_{rank}$ , it can be seen that every agent in  $AS_{rank}$  seems to be traveling the best solution tour averagely. Accordingly, every agent in  $AS_{rank}$  participates to pheromone deposition in practice.

In contrast, power distribution is an irregular distribution that calculated average has no meaning, and difficult to predict. When a system has an irregular time distribution, it means that a specific agent occasionally deviates from a tour near the best solution in the long term, but at that time, it is thought that the best solution is being updated. In consequence, it is considered that agents in  $AS_{multi}$  can search new solutions efficiently compared with  $AS_{rank}$ .

#### E. The behavior of agents in simulations

Figure 3 shows a diagram that you can visually understand how tours are produced in the *eil51.tsp* map. In this figure, the best solution at that time is represented by the bold line. In  $AS_{rank}$ , all agents are traveling on the same tour between  $t = 100$  and 400 in Figure 3 (b). On the other hand, in  $AS_{multi}$ , agents travels through various tours until  $t = 400$  in Figure 3 (a), the best solution is updated as time goes by. It shows the diversity of solutions and the flexibility of the solving ability of  $AS_{multi}$ .

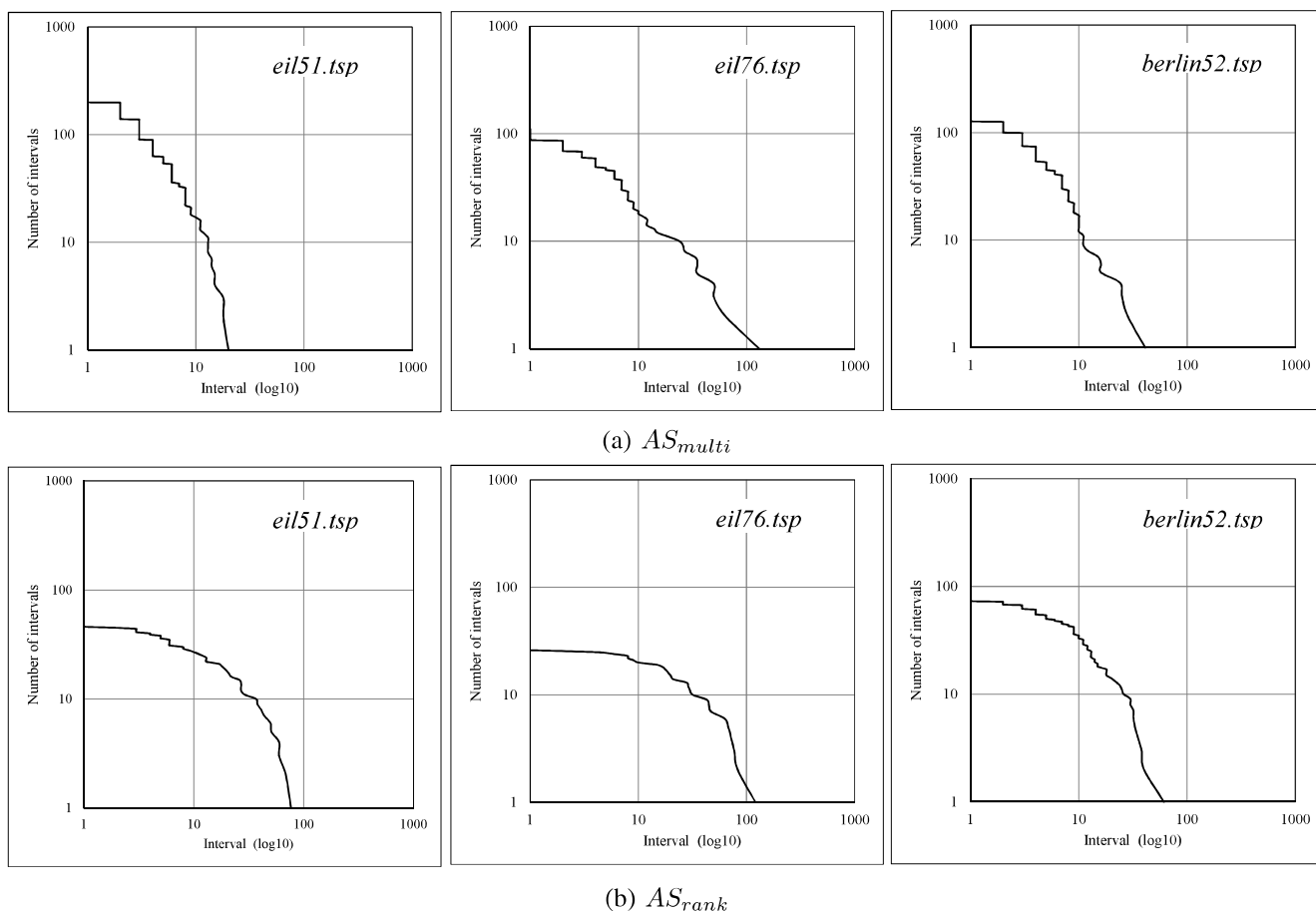


Fig. 2. Pheromone desposition interval (logarithm 10)

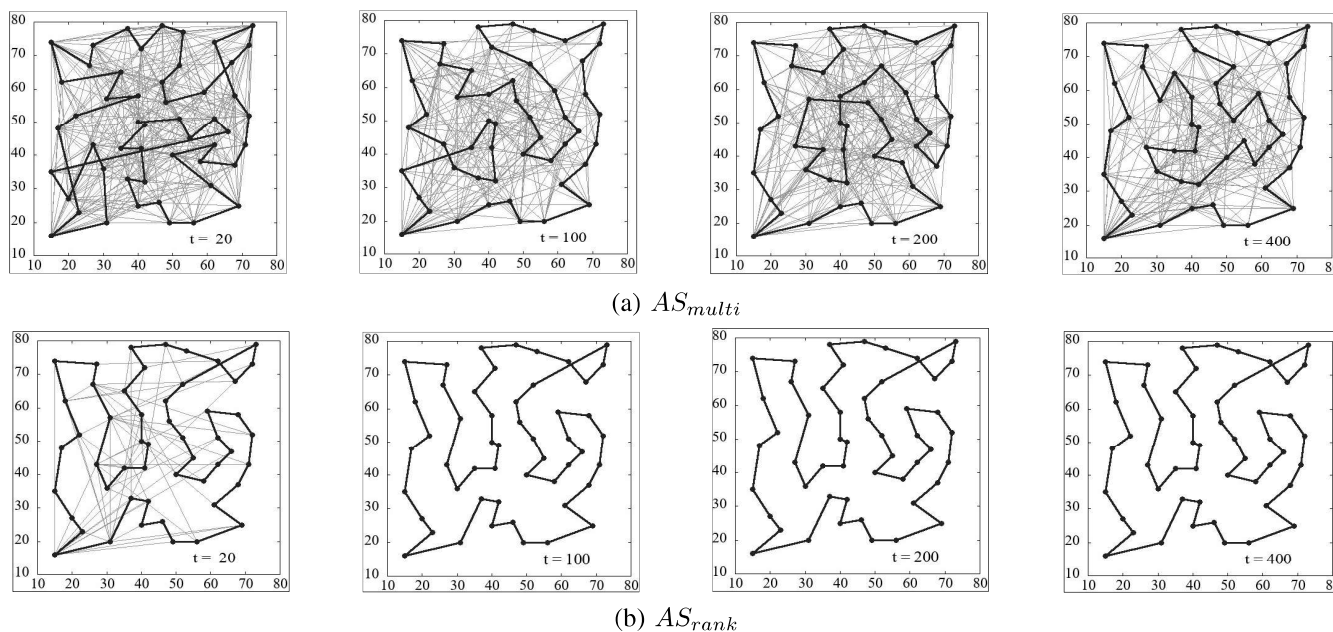


Fig. 3. Examples of agents behavior in time transition

#### F. Relationship between pheromone deposition frequency and agent ranking

We analyzed the relationship between frequency of pheromone deposition and agents' ranking. Figure 4 shows the averaged result obtained from 10 trials. In  $AS_{rank}$ , agents ranked higher than a certain rank can participate in

pheromone deposition at every iteration tour. In contrast, the number of pheromone deposition of agents ranked below a certain rank is zero.

On the other hand, because of the competition of  $AS_{multi}$  described in this paper, the probability of lower ranked agents' participation is not zero (Figure 4). Since the dif-

TABLE IV  
 ANALYSIS RESULT OF PHEROMONE DEPOSITION INTERVAL

(a) $AS_{multi}$				
Interval of pheromone deposition				
TSP	The number of data	AIC weight of power-law against exp-law	$\mu$	$\lambda$
<i>eil51</i>	281	0.999	2.076	0.397
<i>eil76</i>	111	0.999	1.723	0.136
<i>berlin52</i>	172	0.999	1.910	0.285

(b) $AS_{rank}$				
Interval of pheromone deposition				
TSP	The number of data	AIC weight of exp-law against power-law	$\mu$	$\lambda$
<i>eil51</i>	47	0.999	1.403	0.052
<i>eil76</i>	26	0.999	1.323	0.030
<i>berlin52</i>	77	0.999	1.486	0.091

ference of opportunities in pheromone deposition between higher rank agents and lower rank agents is not so extreme,  $AS_{multi}$  can keep the diversity of the solution. As a result, by avoiding to fall into the local solution and maintaining the diversity of solutions, the proposed  $AS_{multi}$  obtains the better solution than  $AS_{rank}$ .

By the way, only in *berlin52.tsp*, convergence of behavior was weak since the pheromone deposition frequency of lower ranked agents is high. That is why the difference of best solutions average between  $AS_{multi}$  and  $AS_{rank}$  was not clear in *berlin52.tsp* result.

## V. CONCLUSION

In this paper, a Multi-rank based Ant System  $AS_{multi}$  which is more faithful to biological behavior and less likely to fall into a local solution than  $AS_{rank}$  model has been proposed. In  $AS_{multi}$ , decisions are made as to whether ants take part in pheromone deposition through contacting with other agents, so that agents participating in pheromone update are not concentrated on upper ranked agents. From the experiment, we found that the number of updates of the best solution is greater and the diversity of the solution among all agents kept high level in longer term compared to those of  $AS_{rank}$ . These results shows  $AS_{multi}$  has the avoidance to fall into local solution. Next, we analyzed interval of opportunities that the agent participate in pheromone deposition. We found that the behavior of agents in  $AS_{multi}$  has the property that the data of pheromone deposition interval follows the power distribution. Hence, it is considered that  $AS_{multi}$  enables to avoid a local solution. As a result, we found that the  $AS_{multi}$  performed better averaged best solution than the  $AS_{rank}$  from simulation experiments. It is seen that  $AS_{multi}$  can keep balance between convergence and dissociation of agents behavior.

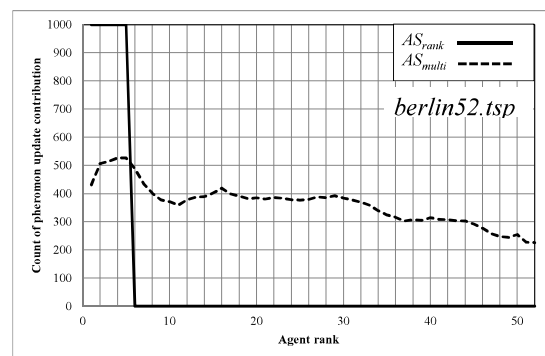
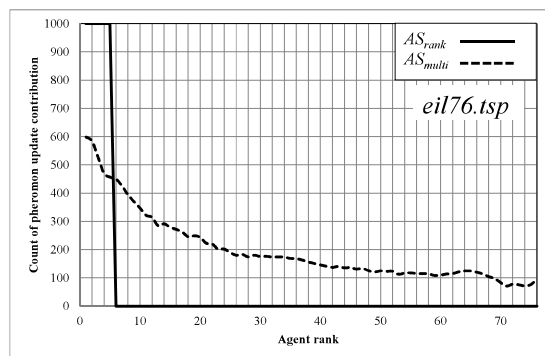
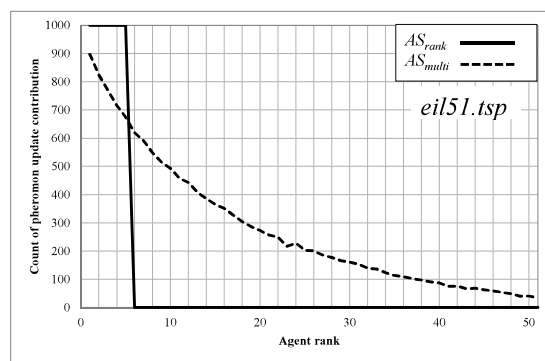


Fig. 4. Relationship between pheromone deposition frequency and agent ranking

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