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Empirical Bayes Prediction for Variables Process Mean in Sequential Sampling Plan

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***Abstract*—The objective of this study is to utilize the Empirical Bayes in sequential sampling plan and compare to the traditional approaches, single sampling plan and sequential sampling plan by variables when data are normally distributed under unknown mean but known variance. The probability of acceptance for the lot (P_a) and the average number of sample size (*ASN*) are considered as criteria of comparison. It is shown that the proposed plan yielded the average number of sample size smaller than single sampling plan and sequential sampling plan by variables and provided the probability of acceptance the lot higher than the traditional approach. Moreover, the Empirical Bayes in sequential sampling plan is given the approximation of the P_a and *ASN* more precise than the single sampling plan and sequential sampling plan by variables.**

***Index Terms*—Empirical Bayes, Single Sampling Plan, Sequential Sampling Plan, Variables Process Mean**

I. Introduction

The statistical analysis was applied in the acceptance sampling plan for inspecting large amounts and it was often impossible to inspect all the products in the batch which can be reduced the producer's risk and consumer's risk. The acceptance sampling plan was developed by Dodge and Romig in 1920. It can be classified by three main points as follows. Firstly, there is non-inspection. Next, 100% inspection and thirdly, random inspection, or sampling plan, meaning samples are taken randomly for possible defects of products and are then subject to acceptance or rejection. Advantages for acceptance sampling plan, the majority of the inspectors apply the acceptance sampling plan for testing for destruction, auditing in case of large lots or using suppliers with a good quality history because this method might reduce the damage by less handling of the products, reduced errors, and saves costs and time in the manufacturing process. The acceptance sampling plan by variables is the quantitative data which can be measured by a continuous scale and assumed to be a normal distribution. [1] The advantages are the variables sampling plan, which the production in the lots provide more information than the attributes, with the small sample size and high level of acceptable quality levels (AQL). The types of variable sampling plans can be classified by variables sampling for the process parameters and estimation proportion nonconforming in the lot. Then, the part for process parameters analysis is often assessed by process mean which the average is utilized for comparison in acceptance sampling plan. Balamurali *et al*. [2] considered the repetitive group in the variables sampling plan and assumed normal and lognormal distributions with known and unknown variances. The proposed plan was compared with the single sampling plan, double sampling plan and sequential sampling plan to minimize the average sample number (ASN) at acceptable quality levels (AQL) and at limiting the quality levels (RQL). Sankle and Singh [3] studied the variable single sampling plan when the data were correlated with a known variance. Thus, the sequential sampling plan often provides a small sample size, which is benefit for cost saving. [2] In addition, the sequential sampling plan in variables can be performed either one-tailed or two tailed testing by indicating specification limits for upper testing, lower testing and double testing, respectively. Lam *et al*. [4] studied Bayesian approach in sequential sampling plan by variables assuming normally distributed data with unknown mean but known variance.

The Bayesian approach is applied in parameters estimation. Its principle is to incorporate information in history about parameters, called prior distribution to estimate the unknown parameters when the parameters of prior distribution, called hyper-parameters, are assumed to be known. On the other hand, when the unknown hyper-parameters are estimated from the observed data it is called Empirical Bayes (EB) method. [5] It method is alternative in the manufacturing for planning the product inspection which benefits is to reduce the amount of waste matter and to decrease production costs. Krutchkoff [6] considered the Empirical Bayes for estimating hyper-parameter and considered mean squared error in the construction of the Nike missile, using grain for movement of the rocket. Casella [7] studied the Empirical Bayes in the case of normal distribution with an unknown mean, known variance in prior normal distribution. Gupta and Liang [8] developed the Empirical Bayes method in the acceptance sampling plan based on the minimum Bayes risk in the selection acceptance or the rejection population. Khaledi and Rivaz [9] studied the Empirical Bayes prediction under spatial data which the hyper-parameters were estimated by the maximum likelihood (ML) method and using the EM algorithm in the stages of maximization marginal distribution. Xu *et al*. [10] considered the Bayesian prediction under Gaussian process regression and applied in the mobile sensor networks with the sequential algorithm.

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The purpose in this study is testing upper specification limit (USL) of the process mean using the acceptance sampling plan and compared with the single sampling plan, the sequential sampling plan by variables and Empirical Bayes in sequential sampling plan. In section 2, the variables process mean and steps for simulation are expressed. Next section 3 and section 4, the traditional plans, the single sampling plan and the sequential sampling plan by variables are illustrated respectively. The Empirical Bayes in sequential sampling plan is interpreted in section 5. The final part is the simulation data and conclusion.

II. Variable Process Mean

The testing upper specification limit (USL) for variables process mean is specified by the acceptance control chart which is considered under six-sigma quality level. Also, it has 3.4 defectives per million opportunities (p) then the process mean assume shift to ±1.5σ. The data are assumed normal distribution: $X \sim N(\mu,\sigma^2)$, the process mean has parameters as follows: μ_1 is acceptable process level (*APL*) and μ_2 is rejectable process level (*RPL*). Thus, it can be estimated as follows.

$$\begin{aligned} APL(\mu_1) &= \bar{x} + 1.5\sigma \\ RPL(\mu_2) &= ACL + Z_\beta \sigma_{\bar{x}} \end{aligned} \tag{1}$$

Where *ACL* is acceptance control limits, it is defined by $ACL = APL(\mu_1) + Z_\alpha \sigma_{\bar{x}}$ when $Z \sim N(0,1)$. The producer's risk (α) is the probability of rejection at the APL and the consumer's risk (β) is the probability of acceptance at the RPL. [11]

III. Single Sampling Plan

The single sampling plan by variables in process mean is the classical method in the acceptance sampling plan. This plan can be calculated the sample size (n) and the acceptance limit $(\bar{X}_a)$ as follows.

$$\begin{aligned} n &= \left[\left(Z_\alpha + Z_\beta \right) \sigma / \left(\mu_2 - \mu_1 \right) \right]^2 \\ \bar{X}_a &= \left(Z_\beta \mu_2 + Z_\alpha \mu_1 \right) / \left(Z_\beta + Z_\alpha \right) \end{aligned} \tag{2}$$

The criterion for comparison the results are as follows. 1. Probability of acceptance (P_a) is the probability of acceptance of the process when the quality level is defined. The graphs of probability of acceptance against the process mean are called the operating characteristic (OC) curve. 2. Average sample number (*ASN*) is the average sample size is inspected per lot. The graphs of the average sample number are against the process mean it is called the ASN curve. The making decision for single sampling plan is accept the lot if $\bar{x} \ge \bar{X}_a$ or to reject the lot if $\bar{x} < \bar{X}_a$, the probability of acceptance the lot is defined by $P(z)$ where $z = \left(\bar{x} - \bar{X}_a \right) / \sigma_{\bar{x}}$ and $ASN = n.$ [12]

IV. Sequential Sampling Plan By Variables

The sequential sampling plan by variables can be tested by the specification limits for upper, lower and double respectively. The samples of this method are taken sequentially from the lot. The samples are taken one sample then called and then item-by-item sequential sampling and more than one sample is taken and then called group sequential sampling. The sequential sampling for the item-by-item plan was developed from the sequential probability ratio test: SPRT) by Wald in 1947. [1] The criteria for inspection of the samples are considered by acceptance limit line (Y_1) and rejection limit line (Y_2) which are shown as follows.

$$\begin{aligned} Y_1 &= -h_1 + s \cdot n \\ Y_2 &= h_2 + s \cdot n \end{aligned} \tag{3}$$

Specification h_1 is the intercept of acceptance line, h_2 is the intercept of rejection line, *s* is the slope of the lines and *n* is the sample sizes. Thus, the making decision is accept the lot if $\sum_{i=1}^{n} x_i \le Y_1$, to reject the batch if $\sum_{i=1}^{n} x_i \ge Y_2$ and continue sampling unit if $Y_1 < \sum_{i=1}^{n} x_i < Y_2$. The evaluation the results are the operating characteristic (OC) curve and the average sample number (*ASN*). The Probability of acceptance (P_a) and the average sample number (*ASN*) are calculated as follows. [13]

$$P_a = \frac{\left\{ \left[(1-\beta)/\alpha \right]^w - 1 \right\}}{\left\{ \left[(1-\beta)/\alpha \right]^w - \left[\beta/(1-\alpha) \right]^w \right\}} \tag{4}$$

$$ASN = \frac{\left\{ \dfrac{La\sigma^2}{\mu_2 - \mu_1} + P_a \cdot \left(\dfrac{L\sigma^2 \log\left[\beta/(1-\alpha) \right] - L\sigma^2 a}{\mu_2 - \mu_1} \right) \right\}}{\left(\dfrac{2\mu - \mu_2 - \mu_1}{2} \right)} \tag{5}$$

where $h_1 = Lb\sigma^2 / (\mu_2 - \mu_1)$, $h_2 = La\sigma^2 / (\mu_2 - \mu_1)$, $s = (\mu_2 + \mu_1)/2$, $a = \log((1-\beta)/\alpha)$, $b = \log((1-\alpha)/\beta)$, $w = (\mu_2 + \mu_1 - 2\mu)/(\mu_2 - \mu_1)$ and $L = 2.3026$.

V. Empirical Bayes Prediction in Sequential Sampling Plan

The Bayesian approach is the one of methods for estimating parameters when assuming unknown parameters (θ) but it is to incorporate information in history about

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parameters, called prior distribution function: $\pi(\theta|\eta)$ and are supposed known hyper-parameters (η). Also, the Bayes' theorem can be given by likelihood function: $L(\theta)$ multiple with the prior distribution function called the posterior distribution function: $h(\theta|\underline{x})$ as follows.

$$h(\theta|\underline{x}) = \frac{L(\theta)\cdot\pi(\theta|\eta)}{M(\underline{x}|\eta)} \propto L(\theta)\cdot\pi(\theta)$$

On the other hand, when the unknown hyper-parameters are estimated from the observed data it is called Empirical Bayes (EB) approach. In addition, the hyper-parameters can be obtained from the marginal distribution function with using the maximum likelihood (ML) method as follows.

$$M(\underline{x}|\eta) = \int_{\theta} f(\underline{x}|\theta)\cdot\pi(\theta|\eta)d\theta$$

Assuming the data are continuously characteristics. [5]

However, the Empirical Bayes method is developed for the estimating new observed data or prediction a future observation (x_{n+1}) when based on full dataset $(x_1, x_2, x_3, ..., x_n$ or $\underline{x})$, called the Empirical Bayes prediction approach. Therefore, the prediction a future observation can obtained from posterior predictive distribution function is provided by

$$h(x_{n+1}|\underline{x}) = \int_{\theta} f(x_{n+1}|\theta)\cdot h(\theta|\underline{x})d\theta.$$

Assuming the samples data $(\underline{x})$ are continuous characteristics and independently and $f(x_{n+1}|\theta)$ is the function of the future observation. [14]

In this study, we consider $X \sim N(\mu, \sigma_0^2)$, unknown mean (μ) but known variance (σ_0^2) and assume informative prior on μ: $\mu \sim N(\theta, \tau^2)$ when μ is parameter, θ and τ^2 are hyper-parameters. The steps for finding the hyper-parameter estimators can be accorded as follows: firstly, specification the marginal likelihood distribution function is provided by

$$M(\underline{x}|\theta,\tau^2) = \int_{-\infty}^{\infty} f(\underline{x}|\mu)\cdot\pi(\mu)d\mu$$

$$M(\underline{x}|\theta,\tau^2) = \int_{-\infty}^{\infty} \frac{1}{(2\pi\sigma_0^2)^{n/2}} e^{-\frac{1}{2\sigma_0^2}\sum_{i=1}^{n}(x_i-\mu)^2} \cdot \frac{1}{(2\pi\tau^2)^{1/2}} e^{-\frac{1}{2\tau^2}(\mu-\theta)^2} d\mu$$
$$\propto \frac{e^{-\frac{1}{2(n\tau^2+\sigma_0^2)}\sum_{i=1}^{n}(x_i-\theta)^2 - \frac{n\tau^2}{2\sigma_0^2(n\tau^2+\sigma_0^2)}\left(\sum_{i=1}^{n}x_i^2 - n\bar{x}^2\right)}}{(n\tau^2+\sigma_0^2)^{1/2}} \tag{6}$$

Next, this is not a closed form then can be determined by ML method and then the likelihood function of $\underline{x}$ given θ and τ^2 is given by

$$L(\underline{x}|\theta,\tau^2) = M(\underline{x}|\theta,\tau^2).$$

Then, the hyper-parameters estimators of the θ and τ^2 with ML method are $\hat{\theta} = \bar{x}$ and $\hat{\tau}^2 = \sigma_0^2/n$ respectively. Thus, the both $\hat{\theta}$ and $\hat{\tau}^2$ are replaced in the posterior distribution function. For the stage of the estimation parameter, it can be obtained by the posterior distribution function as follow.

$$h(\mu|\underline{x}) \propto L(\mu)\cdot\pi(\mu)$$
$$= \frac{1}{(2\pi\sigma^2)^{n/2}} e^{-\frac{1}{2\sigma_0^2}\sum_{i=1}^{n}(x-\mu)^2} \cdot \frac{1}{(2\pi\hat{\tau}^2)^{1/2}} e^{-\frac{1}{2\tau^2}(\mu-\hat{\theta})^2}$$
$$\propto e^{-\frac{1}{2\sigma_0^2}\sum_{i=1}^{n}(x-\mu)^2 - \frac{1}{2\hat{\tau}^2}(\mu-\hat{\theta})^2}$$
$$\propto e^{-\frac{(n\hat{\tau}^2+\sigma_0^2)}{2\sigma_0^2\hat{\tau}^2}\left(\mu - \frac{(\hat{\theta}\sigma_0^2+n\bar{x}\hat{\tau}^2)}{(n\hat{\tau}^2+\sigma_0^2)}\right)^2} \tag{7}$$

Thus, the posterior distribution function of $\mu|\underline{x}$ is normal distribution as follows

$$\mu|\underline{x} \sim N(M, H)$$

where $M = \left(\hat{\theta}\sigma_0^2 + n\bar{x}\hat{\tau}^2\right)/\left(n\hat{\tau}^2+\sigma_0^2\right)$ and $H = \sigma_0^2\hat{\tau}^2/\left(n\hat{\tau}^2+\sigma_0^2\right)$.

Finally, determination the posterior predictive distribution function of $x_{n+1}|\underline{x}$ is

$$h(x_{n+1}\,|\,\underline{x}) = \int_{\mu} f\left(x_{n+1}\,|\,\mu\right)\cdot h\left(\mu\,|\,\underline{x}\right)d\mu$$

$$= \int_{-\infty}^{\infty} \frac{1}{\left(2\pi\sigma^2\right)^{1/2}} e^{-\frac{1}{2\sigma_0^2}(x_{n+1}-\mu)^2} \cdot \frac{\left(n\hat{\tau}^2+\sigma_0^2\right)^{1/2}}{\left(2\pi\sigma_0^2\hat{\tau}^2\right)^{1/2}}$$

$$\cdot e^{-\frac{\left(n\hat{\tau}^2+\sigma_0^2\right)}{2\sigma_0^2\hat{\tau}^2}\left(\mu-\frac{\left(\hat{\theta}\sigma_0^2+n\bar{x}\hat{\tau}^2\right)}{\left(n\hat{\tau}^2+\sigma_0^2\right)}\right)^2} d\mu$$

$$= \frac{1}{\left(2\pi(H+\sigma_0^2)\right)^{1/2}} \cdot e^{-\frac{1}{2(H+\sigma_0^2)}\left(x_{n+1}-M\right)^2} \tag{8}$$

Also, the posterior predictive distribution function is normal distribution as follows

$$x_{n+1}\,|\,\underline{x} \sim N(M,\ H+\sigma_0^2).$$

where the mean of posterior predictive distribution function $E(x_{n+1}\,|\,\underline{x}) = M$, the same as the mean of posterior distribution function and the variation of posterior predictive distribution function is $V(x_{n+1}\,|\,\underline{x}) = H+\sigma_0^2$.

Moreover, the Empirical Bayes prediction can apply in the acceptance sampling plan which is utilized in the part of the sequential sampling plan. Then, the mean of posterior distribution function of this method is estimated sequentially and is compared with the acceptance line and the rejection line. The criteria for the making decision is accept the lot if $E(x_{n+1}\,|\,\underline{x}) \le Y_1$, to reject the lot if $E(x_{n+1}\,|\,\underline{x}) \ge Y_2$ and continue sampling unit when $Y_1 < E(x_{n+1}\,|\,\underline{x}) < Y_2$. [15]

VI. Numerical Comparisons of Performance

The data are simulated under normal distribution: $X \sim N(0,1)$ with unknown mean (μ) but known variance (σ_0^2 =1), assuming informative priors $\mu \sim N(\theta,\tau^2)$ which the hyper-parameters θ and τ^2 are estimated by ML method are given by $\hat{\theta}$ = 1.5027 and $\hat{\tau}^2$ = 0.02. The lot size is N = 1,000, the sample size is n = 50 and the number of iteration is t = 1,000. The proportion of defective at *APL* is p = 0.00034 under six-sigma process, α = 0.05 and β = 0.10. In addition, TABLE I show the average $(\bar{x})$ and the mean of posterior predictive distribution $\left(E(x_{n+1}\,|\,\underline{x})\right)$ are specified by the single sampling plan (Single SP), sequential sampling plan by variables (SSP) and EB in sequential sampling plan (EB in SSP) and are classified by the probability of acceptance (P_a) and the average sample number (*ASN*) as follows.

TABLE I
The P_A and *ASN* at μ_1, s and μ_2 for Single SP, SSP and EB in SSP

Mean	Single SP		SSP and EB in SSP	
	P_a	*ASN*	P_a	*ASN*
1.51 (μ_1)	0.10	50	0.95	23.28
1.71 (s)	0.57	50	0.56	37.98
1.92 (μ_2)	0.95	50	0.10	27.74

For the testing single sampling plan, the acceptance limit is $\bar{X}_a$ = 1.6857 refer to (2). Thus, the decision is to accept the lot if $\bar{x} \ge \bar{X}_a$ or to reject the lot if $\bar{x} < \bar{X}_a$. The comparison between mean $(\bar{x})$ and $\bar{X}_a$ = 1.6857 found that are to reject the lot for all of the means. The *ASN* for this plan is 50 for all of the means.

The hypothesis testing USL for the process mean with the sequential sampling plan is given by $H_0: \mu \le \mu_1$ vs. $H_1: \mu \ge \mu_2$, $(\mu_2 > \mu_1)$. The estimation μ_1 (*APL*), μ_2 (*RPL*) and the slope *S* are approximated refer to (1) which are μ_1 = 1.51 and μ_2 = 1.92 in TABLE I. Also, the P_a for single SP, SSP and EB in SSP are classified by mean at *APL*, slope *S* and *RPL*. The P_a for single sampling plan depended on mean. In contrast, the SSP and EB in SSP provided the P_a at the μ_1 equal to 0.95 and at the μ_2 equal to 0.10. The *ASN* of the single sampling plan is 50 per lot for all of averages when *ASN* of the SSP and EB in SSP are a smaller than single sampling plan and the both are equality and peak at the slope S.

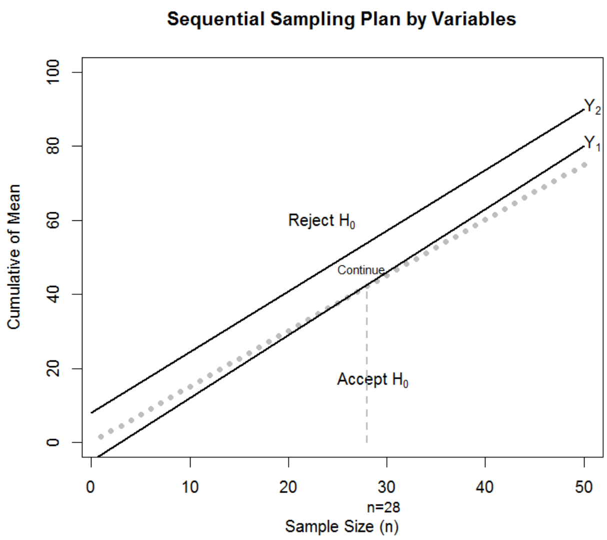

Fig. 1. Comparison of the Cumulative of Mean of SSP with Y_1, Y_2 and classification by the Sample Size (n)

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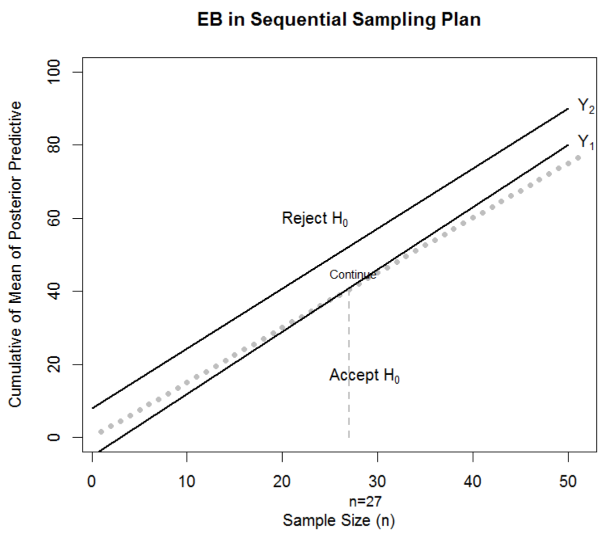

Fig. 2. Comparison of the Cumulative of $E(x_{n+1} | \underline{x})$ with Y_1, Y_2 and classification by the Sample Size (n)

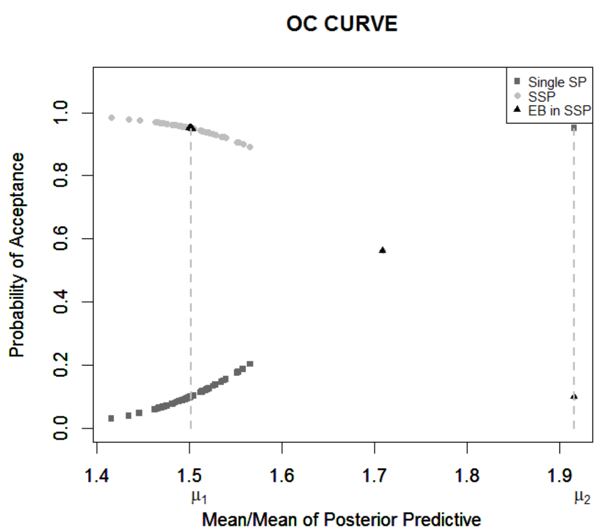

Fig. 3. Comparison P_a of Single SP, SSP and EB in SSP

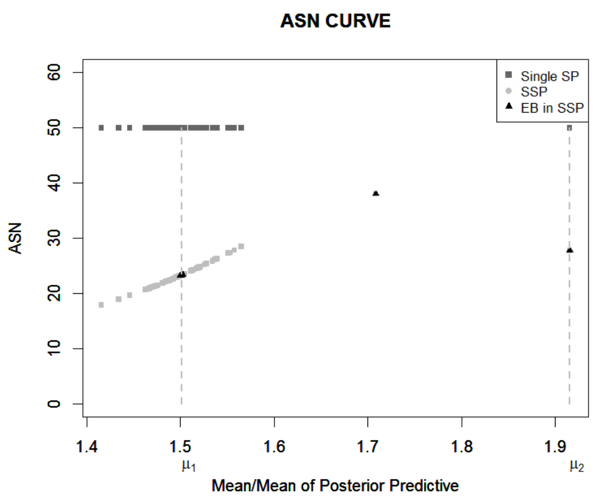

Fig. 4. Comparison *ASN* of Single SP, SSP and EB in SSP

We consider Fig. 1. under $H_0: \mu \le \mu_1$ this method is compared with the cumulative of mean $(\bar{x})$, Y_1 and Y_2 which are classified by sample size (n). When Y_1 and Y_2 can determine from refer to (3) where h_1 = 5.4387, h_2 = 6.9826 and S = 1.7114. Therefore, when the sample size is n = 28 found that the cumulative of mean is less than Y_1 and then to accept H_0 or to accept the lot.

We consider Fig. 2. under $H_0: \mu \le \mu_1$ the cumulative of $E(x_{n+1} | \underline{x})$ compare with the Y_1 and Y_2 when following by sample size (n). At the sample size is n = 27, the $E(x_{n+1} | \underline{x})$ is less than Y_1 and then to accept H_0 or to accept the lot.

From Fig. 3. the P_a is compared with the single SP, SSP and EB in SSP found that the single SP provides the smallest P_a and trend to increase when large the average. However, the P_a of the SSP and EB in SSP trended to decrease when the mean are reduced. The P_a of the SSP is given the high and has a wider range than EB in SSP. The P_a for EB in SSP have nearly 0.95 and are given the narrow range that means the product's risk a smaller than SSP.

We consider Fig. 4. the *ASN* are compared by the three methods with mean/ mean of posterior predictive distribution. The single SP provides the highest *ASN* which is 50 per lot and the *ASN* of SSP has between 20 and 28 per lot. In contrast, the EB in SSP are given the *ASN* around 23 per lot. It can see that the propose plan provides the least *ASN*.

VII. Conclusion

This paper reviewed the testing USL for process mean of the single SP, the SSP and propose plan. The criteria of comparison are the P_a and *ASN*. The majority result of the propose plan are provided the high P_a, small the *ASN* and more precise than the traditional approach. It is clear that the EB in SSP reduce the product's risk and safe cost for inspection products in the lot.

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