Redesign of Water Distribution Networks Systems – Linear Case

Nagib Ghaleb N. Mohammed, *Member, IAENG*

***ABSTRACT*— Simulation is a useful technology, which can be applied in different areas including water distribution systems. Different variables can, be used to predict the behavior of the systems. Computer simulation can be, used to assist the modelling and analysis of the water distribution networks systems. The application of simulation into networking area such as simulation water distributions networks systems is relatively new.**

The paper deals with the simulation of water distribution network systems remodeling and demonstrate how virtual distortion in a selected pipes (e.g. in the pipe No.4) can simulate the network modification due to modification of the cross-section of one pipe. The resultant state of flow redistribution is calculated from the formulas superposing linear response of the original network configuration and the component induced by unknown virtual distortion modelling the modification. Then making use of the analytical network model (cf.Refs.3,1) of this installation and using the so-called Virtual Distortion Method (VDM), simulation of water distribution networks systems remodeling can be conducted.

***Index Terms*— redesign, simulation, water networks, VDM based design**

I. Introduction

Water distribution systems are designed to adequately satisfy the municipal water requirements of the community, which include, domestic, commercial, industrial and fire demands as well as adequate quality. The main components of water distribution system are storage, pumps and water distribution network. The hydraulic parameters such as pressure at each node, flow in each pipe and velocity of flow in each pipe have to be satisfied. In case of inadequate pressure at nodes, or to overcome the major losses of energy due to friction in the pipes, pump has to be used to satisfy required pressure. The continuity flow condition, relating internal flow and external flow at each node have to be satisfied (i.e. the sum of internal flow and external flow must be equal zero). One thing has to be noticed in water distribution systems in comparison with truss structure and electrical networks systems is that, the constitutive equation relating the pressure head and flowrate in the pipe is non-linear.

Water supply systems are becoming more important, since water demand has increased rapidly in the developing countries as a result of high population growth, improvement of living standards, rapid urbanization, industrialization and improvement of economic conditions while accessible sources of water keep decreasing in number and capacity [4]. During the last two decades, there have been increasing requirements to improve operation of water supply systems while improving the environment so that their behaviour can be fully understood and the total process is optimised [5]. These are exerting increasing pressure on local water authorities and water planners to satisfy the growing water demands [6, 7].

Global demand for water is continuously increasing due to population growth, industrial development, and improvements of economic conditions, while accessible sources keep decreasing in number and capacity, moreover, the applications involving manipulation and transport of water and fluids in general demand high power consumption. The optimal use of such water supply networks seems to be the best solution for the present and thus it is necessary to carefully manage water transfer [9, 10].

The proposed approach is based on continuous observation of the pressure distribution in nodes of the water network. Having a reliable (verified versus field tests) numerical model of the network and its responses for determined inlet and outlet conditions, any modifications to the normal network response (pressure distribution) can be detected. Then, applying proposed bellow numerical procedure, the correction of water supply can be determined.

The proposed methodology is based on the so-called Virtual Distortion Method (VDM) approach, applicable also in the problem of damage identification through monitoring of piezo-generated elastic wave propagation [2, 4].

II. FORMULATION OF THE REDESIGN PROBLEM

The so called Virtual Distortion Method (VDM, used originally in redesign of structural systems) [3] is the basis of analytical formulation of the simulation problem.

The main advantage of the proposed VDM approach is it's numerical efficiency in modelling of non-linearities and modifications of primary design.

For water distribution system in a steady state flow, let me define the following set of equations. The equilibrium of the system relating the internal flow distribution in the

Manuscript received Oct. 10, 2017; revised Nov. 13, 2017.
Nagib Ghaleb N. Mohammed, Civil Engineering Department, University of Bahrain – Bahrain, (email: nnasher@uob.edu.bh).

network's branches Q [m^3/s] with the external inlet/outlet q [m^3/s] is expressed as follows:

$$q = N\,Q \qquad (1)$$

Where,
q = external inlet/outlet flow,
Q = internal flow in element and
N = incidence matrix defining topology of the network (storing only three values 0, -1, 1) connecting branches to nodes and showing the direction of flow in the network. The -1 (inlet flow) and 1 (outlet flow) entries are incident to the direction of flow.

The relation between water head and head losses can be presented as follows:

$$h_i = N^T H_i \qquad (2)$$

Where,
h = energy loss,
H = water head and
N = incidence matrix.

The constitutive relation for water networks relates the pressure head h with the flow Q in the elements can expressed as:

$$\boldsymbol{Q_i^2 = R_i\, h_i} \qquad (3)$$

Where,
Q_i = flow in that element
h_i = energy loss in element i
R_i = constant depending on pipe diameter, length, type.

Substituting Eqs. (3) and (2) into (1), the following formula can be obtained:

$$\boldsymbol{N\,(R\,N^T\,H)^2 = q} \qquad (4)$$

The relation (4) is non-linear, nevertheless, let me temporarily assume the linearity of this relation i.e.

$$\boldsymbol{N\,R\,N^T\,H = q} \qquad (5)$$

Let me consider an example of the water distribution network consisting of 4 nodes, 5 branches and 2 loops, presented in Fig. 1.

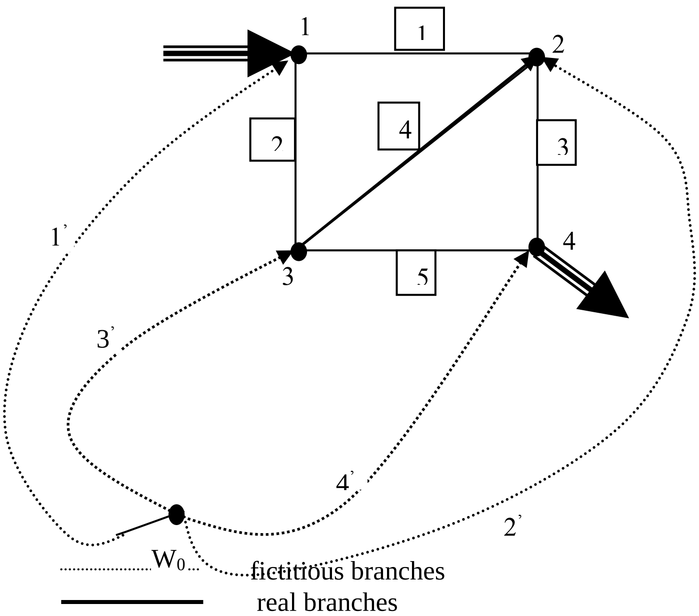

Fig. 1 Oriented *graph* modelling a 2-loop water network

Describing the water network shown in Fig.1, the set of equations (5) takes the following form

$$\begin{bmatrix} R_1+R_2 & -R_1 & -R_2 & 0 \\ -R_1 & R_1+R_3+R_4 & -R_4 & -R_3 \\ -R_2 & -R_4 & R_2+R_4+R_5 & -R_5 \\ 0 & -R_3 & -R_5 & R_3+R_5+R_4' \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \end{bmatrix} = \begin{bmatrix} q_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (6)$$

where:

$$q_4 = R_4'(H_0 - H_4), \qquad R = \frac{K^2}{l},$$

K- the characteristic of the element, l - the element's length,
H –water head in the node
q –flow in the branch,
And it was assumed that the network is supplied only through the node No.1 (inlet with intensity q_1) and the only outlet is through the node No.4 (the coefficient R_4' =1).
$R_2' = R_3' = 0$, what means, that the outlets in nodes No.2 and 3 vanish.

III. VDM-BASED SIMULATION OF PARAMETER MODIFICATION

Analogously to the Virtual Distortion Method (VDM) applicable to the truss structures (cf.[3]) let me postulate that local modification of a network parameter can be introduced into the system through the *virtual distortion* ε^0 , incorporated into the formula (4):

$$\mathbf{N\,R(N^TH - \varepsilon^0) = q} \qquad (7)$$

ISBN: 978-988-14048-8-6
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

The virtual distortion ε_i^0 is of the same character as the water head h_i (see Fig. 2) and its physical meaning is an additional water head externally forced in branch “ i “ (e.g. due to a locally installed pump).

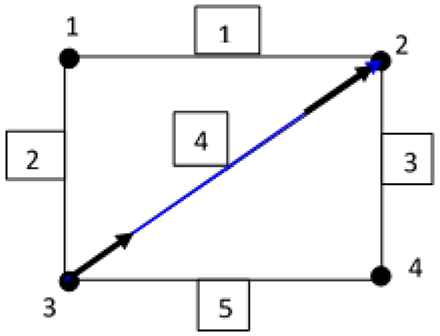

Fig. 2 Distortion simulating water flow (water head modification) in branch No. 4

The influence of virtual distortions on the resultant flow redistribution can be calculated using the so-called *influence matrix* $\mathbf{D}_{ij}$ collecting *i* responses (row-wise) in terms of water heads $H_i^{@\varepsilon^0=1}$ induced in the network by imposing the unit virtual distortion $\varepsilon^0_j=1$ generated consecutively in each network branch *j*. Thus each *influence vector* $H_i^{@\varepsilon^0=1}$ can be calculated on the basis of the following equation obtained from Eq. (7):

$$N\,R\,N^T H^{@\varepsilon^0=1} = q^* + N\,R\,I \tag{8}$$

The vector q* disregards the external inlet and outlet (the flow is now provided by the imposition of virtual distortion), and it accounts for the water flow distribution in the closed network (cf. Eq. (6)). There is a set of j (j the number of branches) equations (8) to be solved in order to create the full influence matrix D. Each time the right hand-side changes as the unit virtual distortion is applied to another branch. In practice this can be realised by applying a pair of inlets-outlets N_{ik} $R_{kj}\,\varepsilon_j^0$ corresponding to each branch (cf. Eq. (7)) – it is the so-called *compensative charge*.

So, the parameter modification in the system is accounted for by superposing the so-called *linear response* of the original network and the so-called *residual response* due to imposition of the virtual distortion. Therefore, the resultant water head distribution can be expressed as:

$$H_i = H_i^L + H_i^R = H_i^L + \sum_j D_{ij}\varepsilon_j^0 \tag{9}$$

and the resultant water flow as:

$$Q_j = Q_j^L + Q_j^R = Q_j^L + R_j N_{ij}^T \sum_j \left(D_{ij} - \delta_{ij}\right)\varepsilon_j^0 \tag{10}$$

The analogous set of relations governs the VDM based approach to modifications of truss structure system [3].
Coming back to the example shown in Fig. 2, let me generate the unit virtual distortion in branch No. 4, connecting the nodes Nos. 2 & 3. The corresponding set of equations (8), accounting for boundary conditions (i.e. outlet in node No.4), takes the following form:

$$\begin{bmatrix} R_1+R_2 & -R_1 & -R_2 & 0 \\ -R_1 & R_1+R_3+R_4 & -R_4 & -R_3 \\ -R_2 & -R_4 & R_2+R_4+R_5 & -R_5 \\ 0 & -R_3 & -R_5 & R_3+R_5+R_4 \end{bmatrix} \begin{bmatrix} H_1^{@\varepsilon^0=1} \\ H_2^{@\varepsilon^0=1} \\ H_3^{@\varepsilon^0=1} \\ H_4^{@\varepsilon^0=1} \end{bmatrix} = \begin{bmatrix} 0 \\ -R_4\varepsilon_4^0 \\ R_4\varepsilon_4^0 \\ 0 \end{bmatrix} \tag{11}$$

where $\varepsilon_4^0 = 1$. Assuming the following data: K_1=0.2 m^3/s, $K_2=K_3=K_4=K_5$=0.4 m^3/s, $l_1=l_2=l_3=l_5$=10.000 m, l_4=14.142 m, q_1=0.050 m^3/s, H_0=0.000 m, the above set of equations leads the following water head distribution:

$H^{@\varepsilon^0=1}$ = $[0.151,\ -0.251,\ 0.251,\ 0.000]^T$ constitutes the 4th column of the influence matrix D. Continuing this procedure for virtual distortions generated in other branches, and taking into account relation (3) and applying it consecutively to each influence vector $H^{@\varepsilon^0=1}$, the influence matrix D^ε can be created, collecting the response to unit virtual distortions in terms of the pressure head $\varepsilon^{@\varepsilon^0=1}$.

$$D^\varepsilon = \begin{bmatrix} 0.314 & 0.686 & -0.284 & -0.044 & 0.284 \\ 0.172 & 0.828 & 0.071 & 0.101 & -0.071 \\ -0.071 & 0.071 & 0.678 & -0.173 & 0.322 \\ 0.142 & -0.142 & -0.355 & 0.503 & 0.355 \\ 0.071 & -0.071 & 0.322 & -0.101 & 0.678 \end{bmatrix} \tag{12}$$

IV. SIMULATION OF NETWORK REMODELLING (MODIFICATION OF THE CROSS-SECTION OF ONE BRANCH) LINEAR CASE

Comparing internal flaw distribution Q modeled by virtual distortions and modified flaw distribution Q', as follows:

$$Q = Q' \tag{13}$$

That is :

$$R\,(\varepsilon - \varepsilon^0) = R'\,\varepsilon \tag{14}$$

Where

$$\varepsilon_j = \varepsilon_j^L + \sum D_{ij}^\varepsilon\,\varepsilon_j^0 \tag{15}$$

and introducing the parameter:

$$\mu = \frac{R'}{R}, \tag{16}$$

ISBN: 978-988-14048-8-6
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

we get the following set of equations:

$$[\sum_j D_{ij}^{\varepsilon}(\delta_{ij}-\mu_i)-\delta_{ij}]\,\varepsilon_j^0=\varepsilon_i^L(\mu_i-\delta_{ij}) \qquad (17)$$

where ε^0 denotes virtual distortions modelling change of hydraulic compliance in branch " j ".
The above set of equations should be solved with respect to the unknown virtual distortions ε^0.

Coming back to our small example shown in Fig. 1 assuming $\mu = 0.8$ (i.e. making the modification in the element No.4) and substituting to (16) one can get:

$$\mu = [\ 1.0 \quad 1.0 \quad 1.0 \quad 1.0 \quad 0.8 \quad 1.0\ ]^T$$

Substituting to (17) we can get the virtual distortion modelling the modification as follows:

$$\varepsilon_4^0 = -\,0.148$$

and substituting to (15), the modified pressure head can be obtained as the following:

$\varepsilon^m = [3.093 \quad 2.352 \quad 1.192 \quad -0.742 \quad 1.933]^T$ (18)

Solving set of equations (7)(just for check) taking into consideration the modification of the element No.4, the following pressure head distribution can be obtained:

$\varepsilon^m = [3.095 \quad 2.350 \quad 1.190 \quad -0.745 \quad 1.933]^T$ (19)

Finally one can notice that the states (18) and (19) are the same.

V. Conclusion

The main advantages of the VDM based approach to the water network analysis WATNET-M are the reduction of numerical costs and avoidance of iteration due to incremental approach in the analysis of water network.
The numerical cost of linear analysis consists (in WATNET-M) of solving the linear problem (7) and composing the influence matrix D^ε (12).

References

[1] Biedugnis S., Smolarkiewicz M. Model matematyczny sieci wodociągowej na potrzeby projektowania i analizy jej działania, praca doktorska, Warszawa, 2001.

[2] Cross H. (1936) Analysis of Flow in Networks of Conduits or Conductors, *University of Illinois Engineering Experiment Station Bulletin*, No. 286

[3] Holnicki-Szulc J., Gierlinski J.T. (1995) *Structural Analysis, Design and Control by the Virtual Distortion Method*, J. Wiley & Sons, Chichester, U.K.

[4] W. A. Abderrahman, Urban water management in developing arid countries, Water Resour Dev., Vol. 16, no.1, pp 7-20, 2000

[5] İ. Eker, T. Kara, Modelling and simulation of water supply systems for feedback control, in: Proceedings of UPEC'2001: 36th Universities Power Engineering Conference, Power Utilisation, Part-5C, University of Wales Swansea, 12-14 September, Swansea, U.K., 2001.

[6] D.S. Brookshire, D. Whittington, Water resources issues in the developing countries, Water Resour Res., Vol. 29, No. 7, pp 1883-1888, 1993.

[7] M.A. Brdys, B. Ulanicki, Operational Control of Water Systems, Prentice Hall International Ltd, U.K., 1994.

[8] Mays, L. W., ed. (2000). *Water Distribution Systems Handbook*. McGraw-Hill, New York, New York.

[9] C. Biscos, M Mulholland, M-V Le Lann, CA Buckley, and CJ Brouckaert, " Optimal operation of water distribution networks by predictive control using MINLP" water S. A., Vol. 29, No. 4, PP. 393 – 404, 2003

[10] Ilyas Eker,* Tolgay Kara, "Operation and control of a water supply system", ISA Transaction, Vol. 42, PP. 1 – 13, 2003.