Minimizing the Conditional Value-at-Risk for a Single Operating Room Scheduling Problem

Mari Ito, Fumiya Kobayashi, and Ryuta Takashima

Abstract—**We introduce a stochastic programming model for scheduling a single operating room using the conditional value-at-risk (CVaR) as a criterion. This criterion expresses the risk-averse attitude of the scheduler to the risk event that the expected end time of a surgery presumed by a surgeon can be considerably delayed. One of the important advantages of the CVaR is that the stochastic programming problem can be treated as a linear programming problem. Owing to this characteristic, the CVaR is more practical than traditional expectation based approaches. In this paper, we evaluate the effectiveness of the proposed model using numerical experiments.**

Index Terms—**Operations research in health services, Operating room scheduling, Conditional value-at-risk.**

I. Introduction

OPERATING room management is important for improving patient treatment quality and reducing hospital costs. Fujiwara [5] suggested that efficient operating room management is an emerging solution for improving medical services in Japanese hospitals, where many patients experience long waiting times before undergoing operations owing to an insufficient number of surgeons, anesthesiologists, and operating rooms. Such problems are also persistent in Europe owing to aging populations; therefore, providing efficient medical services has increasingly become important [2]. Many hospitals operate at a low level of efficiency; however, few analytical studies have reported on efficient medical services.

Similar to the case in most of the other countries, surgeries are the most important source of hospital revenue in Japan and are estimated to generate approximately two-thirds of the total revenues [6]. Thus, improving operating room management and effectiveness makes operating rooms an increasingly important hospital resource. However, Macario et al. [9] showed that operating rooms account for 40 % of the overall hospital costs, thus making them the most significant single cost source. Moreover, delayed operation starting times increase the probability of overtime, which further increases the costs [4].

The quality of operating room schedules is one of the most important factors in operating room management; improved operating room scheduling not only decreases patient waiting times but also reduces both the workload of surgeons and anesthesiologists and the required overtime. In addition, it increases the effectiveness of operating room utilization, which may partially resolve the problem of operating room shortages. Actually, the created schedules are not always suitable for operating room management and are often not closely followed. One reason for this is that required operation times are self-reported by the surgeons; however, these time requirements are sometimes underestimated. This delays surgeries and makes the operating room unavailable for the subsequent scheduled surgeries.

The operating room scheduling problem has been studied by multiple researchers. For example, Lamiri et al. [8] formulated an operating room scheduling method to reduce overtime costs using a stochastic model by proposing a Monte Carlo optimization method comprising Monte Carlo simulations and mixed integer programming. Denton et al. [3] proposed a two-stage stochastic programming model for operating room scheduling. There are also multiple case studies that consider individual hospital characteristics. Blake and Donald [1] solved the operating room scheduling problem at Mount Sinai Hospital using an integer programming model. This approach greatly influenced the scheduling process at the hospital; it fairly allocates operation times to surgeons, reduces operating room manager workloads, and avoids conflicts between surgeons and operating room managers. Ito et al. [7] proposed an estimation method to calculate operation durations using regression analysis and a mixed integer programming model for the operating room scheduling problem. Moreover, they developed an operating room scheduling system to use their proposed method automatically and applied it to operating room scheduling at the Aichi Medical University Hospital in Japan. It is important to prepare a detailed schedule of the operating room and to operate the operating room efficiently. In particular, stochastic approaches capture uncertainties that are frequently encountered.

The stochastic programming problem for operating room scheduling is generally formulated to minimize the expected value of the total delay of surgery from the expected end time. However, this approach cannot adequately address the risk facing a surgery with a very large delay from the expected end time. In other words, minimizing the expected value does not adequately express the risk-averse attitude of the scheduler to the risk that the expected end time of a surgery presumed by a surgeon can be considerably delayed. In this study, we incorporate the CVaR as a criterion in the operating room scheduling problem. To date, there have been no examples of applying the CVaR in the operating room scheduling problem.

In this paper, our purpose is to determine the optimal sequences of surgeries in a single operating room by considering the risk event that the end time of a surgery presumed by a surgeon can be considerably delayed. We show the effectiveness of our model using numerical experiments. The remainder of this paper is organized as follows. Section 2 presents a detailed explanation of the CVaR. Section 3

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provides the motivation and describes the proposed model of single operating room scheduling. Section 4 shows the results obtained by numerical experiments. Section 5 summarizes the paper and identifies the areas for future work.

II. Conditional value-at-risk

Variance is a well-utilized risk measure. Using variance in a risk measure has several disadvantages. First, the case in which a loss is lower than its expected value is also evaluated as a risk. Second, a stochastic programming model that incorporates variance into its mathematical programming model becomes nonlinear. The value at risk (VaR) is another well-utilized risk measure. The VaR is defined using an α-quantile, which is a value that divides the discrete probability distribution into a certain percentage, α and $1-\alpha$. Figure 1 shows an image of the VaR. The vertical axis indicates the probability density function, and the horizontal axis indicates the loss. This criterion can adequately address the risk facing a very large loss. However, a stochastic programming model that incorporates the VaR into its mathematical programming model becomes nonlinear because there is no convexity. The CVaR is a coherent risk measure and is also considered as a measure to improve the disadvantages of the VaR. The CVaR is suitable for incorporation into mathematical programming problems because the coherent risk measure is convex. $F_{\tilde{\eta}}(z)$ defines the cumulative distribution function of a random variable $\tilde{\eta}$ for the loss. When the cumulative distribution function $F_{\tilde{\eta}}(z)$ is continuous, we can represent the following:

$$\text{CVaR}_\alpha(\tilde{\eta}) = \mathbb{E}_{\tilde{\eta}}[\tilde{\eta} | \tilde{\eta} \geq \text{VaR}_\alpha(\tilde{\eta})], \tag{1}$$

$$= \frac{1}{1-\alpha} \int_{\text{VaR}_\alpha(\tilde{\eta})}^{\infty} z dF_{\tilde{\eta}}(z), \tag{2}$$

where $\alpha \in \{0,1\}$.

The CVaR is defined as the form of the expected loss for the range exceeding the VaR. Figure 1 shows an image of the VaR and CVaR. Particularly, the CVaR can be formulated as an optimum value for the following minimization problem. The optimal solution of the minimization problem is the VaR.

$$\text{CVaR}_\alpha(\tilde{\eta}) = \min_v v + \frac{1}{1-\alpha} \mathbb{E}_{\tilde{\eta}}[(\tilde{\eta} - v)^+]. \tag{3}$$

When the CVaR is adopted as a risk measure, it can be formulated as follows using the weighting factor $\beta \in \{0,1\}$:

$$\min_x (1-\beta)\mathbb{E}_{\tilde{\xi}}[V(x,\tilde{\xi})] + \beta \text{CVaR}_\alpha(V(x,\tilde{\xi})), \tag{4}$$

subject to

$$Ax = b, \tag{5}$$

$$x \geq 0, \tag{6}$$

where $V(x,\tilde{\xi})$ is the cost function with a variable x of an n_1 dimensional vector and a random variable $\tilde{\xi}$, $\mathbb{E}_{\tilde{\xi}}[V(x,\tilde{\xi})]$ is the expected value of the cost, A is an $m_1 \times n_1$ matrix, and b is an m_1 dimensional vector.

Assume that the distribution of $\tilde{\xi}$ has a finite support $\Theta = \{\xi^1, ..., \xi^K\}$ and the probability p^k is given to ξ^k (k is the scenario) when the random variable follows the discrete probability distribution. To simplify the expressions, we introduce a new variable $u(\xi^k) > 0$, $k = 1, ..., K$. This problem can be expressed as follows using definition (2) of the CVaR:

$$\min_{x, y(\xi^1),...,y(\xi^K), v, u(\xi^1),...,u(\xi^K)} \tag{7}$$
$$(1-\beta)\left(c^{\mathrm{T}}x + \sum_{k=1}^{K} p^k q(\xi^k)^{\mathrm{T}} y(\xi^k)\right)$$
$$+\beta\left(v + \frac{1}{1-\alpha}\sum_{k=1}^{K} p^k u(\xi^k)\right),$$

subject to

$$c^{\mathrm{T}}x + q(\xi^k)^{\mathrm{T}} y(\xi^k) - v \leq u(\xi^k), k = 1, ..., K, \tag{8}$$

$$u(\xi^k) \geq 0, k = 1, ..., K, \tag{9}$$

$$Ax = b, \tag{10}$$

$$T(\xi^k)x + Wy(\xi^k) = h(\xi^k), k = 1, ..., K, \tag{11}$$

$$x \geq 0, \tag{12}$$

$$y(\xi^k) \geq 0, k = 1, ..., K, \tag{13}$$

where c is a parameter of an n_1 dimensional vector, $q(\xi^k)$ is a parameter of an n_2 dimensional vector, $y(\xi^k)$ is a variable of an n_2 dimensional vector, $T(\xi^k)$ is an $m_2 \times m_1$ matrix, and W is an $m_2 \times n_2$ matrix.

The above results in a linear programming problem when the CVaR is incorporated as a risk measure in a two-stage programming problem under a discrete probability distribution. Sarin et al. [10] introduced the use of the CVaR as a criterion for stochastic scheduling problems. They demonstrated its application for the single or parallel machine scheduling problem and exhibited the use of the CVaR and the effectiveness of minimizing it in that context.

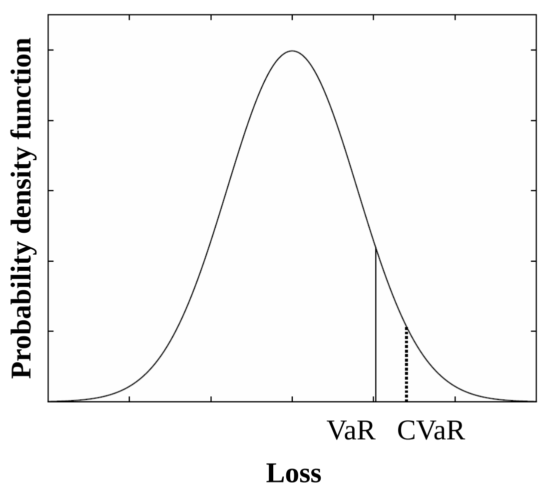

Fig. 1. Image showing the VaR and CVaR.

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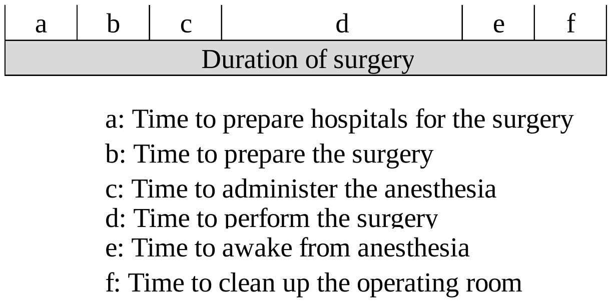

Fig. 2. Stages in a single surgery.

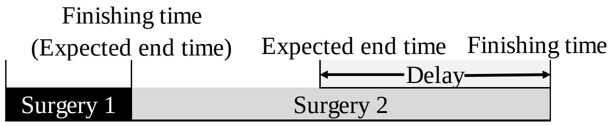

Fig. 3. Example of a schedule in a single operating room.

III. Single operating room scheduling

A. Motivation

For most hospitals in Japan, departments have a limited amount of available time in an operating room according to the day of the week and the time of the day. The surgeon and the patient decide on the desired surgery date and starting time of the surgery within the time available for the operating room. Surgeons propose the durations of their own surgeries and the desired starting times to the scheduler of the operating room. Then, the scheduler creates a schedule using the proposed durations of the surgeries. These operating room schedules are often created manually. In general, the scheduler assigns surgeries from the same department sequentially when several departments use a single operating room. This is because surgeries from the same department are likely to use the same medical equipment. In addition, time adjustments for surgeries within one department are easier to complete than the adjustments between multiple departments.

Figure 2 shows the stages in a single surgery in detail. We define the duration of a surgery as a total of the time required for preparing the operating room for the surgery, the time required for performing the surgery, and the time required for cleaning the operating room (i.e., from time a to time f in Fig. 2). The patient receives several treatments during times b, c, d, and e. Figure 3 shows an example of a schedule for a single operating room. We define the delay in a surgery as the length of time between the expected end time presumed by a surgeon to finishing time determined as requested. In Fig. 3, surgery 1 has the same time as the expected end time and the finishing time determined as requested. For Surgery 2, the expected end time and the finishing time determined as requested are different. Therefore, the surgery delay is the difference between the expected end time and the finishing time.

B. The model

We consider a model of a single operating room scheduling problem that determines the sequences of surgeries in a single operating room. We assume that the durations of the surgeries are the only random variables in this problem. Consider the following notation.

Notation

Index sets

- J: Set of surgeries
- S: Set of scenarios
- D: Set of departments
- E_d: Set of surgeries belonging to the same department d, $d \in D$

Parameters

- w_j: Weight of surgery j, $(\forall j \in J)$
- p_{js}: Duration of surgery j under scenario s, $(\forall j \in J, \forall s \in S)$
- d_j: Expected end time of surgery j presumed by a surgeon, which is defined as $d_j = b_j + \mathbb{E}_s[\sum_{j \in J} p_{js}]$, where b_j is defined as the starting time of surgery j presumed by a surgeon, $(\forall j \in J)$
- π_s: Probability of scenario s, $(\forall s \in S)$
- α: Probability level, $\alpha \in (0, 1)$
- β: Weighting factor, $\beta \in (0, 1)$

Variables

- c_{js}: Finishing time of surgery j under scenario s, $(\forall j \in J, \forall s \in S)$
- t_{js}: Delay of surgery j from the expected end time under scenario s, $(\forall j \in J, \forall s \in S)$
- η: A threshold value (equal to the VaR when an optimal solution is obtained)
- μ_s: Amount of delay of surgery t_{js} exceeding the threshold value η under scenario s, $(\forall s \in S)$
- z_{ij}: Surgery precedence binary variable, where, $z_{ij} = 1$ if surgery i is processed before surgery j, $z_{ij} = 0$ otherwise, $(\forall i, j \in J, i \neq j)$

Formulation

$$\text{minimize} \quad (1-\beta)\mathbb{E}_s[\sum_{j \in J} w_j t_{js}] + \beta(\eta + \frac{1}{1-\alpha}\sum_{s \in S} \pi_s \mu_s) \tag{14}$$

subject to

$$\eta + \mu_s \geq \sum_{j \in J} w_j t_{js}, \qquad \forall s \in S, \tag{15}$$

$$\sum_{j \in J \setminus \{j\}} p_{is} z_{ij} + p_{js} \leq c_{js}, \qquad \forall s \in S, j \in J, \tag{16}$$

$$t_{js} + d_j \geq c_{js}, \qquad \forall s \in S, j \in J, \tag{17}$$

$$z_{ij} + z_{ji} = 1, \qquad \forall i \neq j \in J, \tag{18}$$

$$z_{ij} + z_{jk} + z_{ki} \leq 2, \qquad \forall i \neq j \neq k \in J, \tag{19}$$

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$$|\sum_{j\in J} z_{ij} - \sum_{j\in J} z_{i'j}| = 1, \qquad \forall i \neq i' \in E_d, \forall d \in D, \tag{20}$$

$$c_{js} \geq 0, \qquad \forall s \in S, j \in J, \tag{21}$$

$$t_{js} \geq 0, \qquad \forall s \in S, j \in J, \tag{22}$$

$$\mu_s \geq 0, \qquad \forall s \in S, j \in J, \tag{23}$$

$$z_{ij} \in \{0, 1\}, \qquad \forall i \neq j \in J. \tag{24}$$

In the formulation above, objective function (14) comprises two terms, each having weights 1-β and β. The first term of the objective function minimizes the expected value of the total weighted delay. The second term of the objective function minimizes the CVaR. For each scenario s, constraint (15) determines μ_s to be the amount of total weighted delay that exceeds the threshold value of η (if at all). Constraint (16) bounds the surgery finishing times according to the surgery sequencing relationships. Constraint (17) determines the delay of the surgeries. Constraints (18) and (19) ensure the feasibility of the surgery sequence by eliminating cyclic sequences. Constraint (20) sequentially allocates surgeries i and i' because surgeries i and i' are in the same department. Constraints (21), (22), and (23) are non-negative constraints. Constraint (24) is a binary constraint.

These scenarios are derived either from discrete approximations of the underlying distributions of the problem parameters or from a scenario generation procedure, with the probability value π_s being associated with a scenario s, $\forall s \in S$.

IV. Numerical experiments

A. Data

In this section, we present numerical experiments to evaluate the performance of our proposed formulation. In general, two to five surgeries are scheduled in a single operating room [7]. As an example, we consider a five-surgery problem with 100 scenarios, i.e., $J = 5$ and $S = 100$. The durations of the surgery are assumed to follow left-truncated log-normal distributions that are truncated at zero to ensure non-negativity. Scenario-wise values of p_j were generated via Monte Carlo sampling. Figure 4 shows histograms of the scenario-wise values of the duration of surgery j (p_j). The surgery parameter values are summarized in Table 1. These parameters are generated by means of a simulation based on the parameters of a previous study [7]. w_j has a value of 1. Surgeries 1 and 2 belong to the same department. Surgeries 3, 4, and 5 belong to different departments, i.e., $D = 4, E_1 = 1, 2$, $E_2 = 3$, $E_3 = 4$, and $E_4 = 5$. We solve the single operating room scheduling problem by minimizing the CVaR with $\alpha = 0.8$.

TABLE I
Parameters of an example problem

Surgery j	1	2	3	4	5
$\mathbb{E}[p_j]$ (min.)	334	215	207	309	107
Var$[p_j]$ (min.)	7046	4700	2586	4845	1850
b_j (min.)	1	50	100	150	1
d_j (min.)	335	265	307	459	108

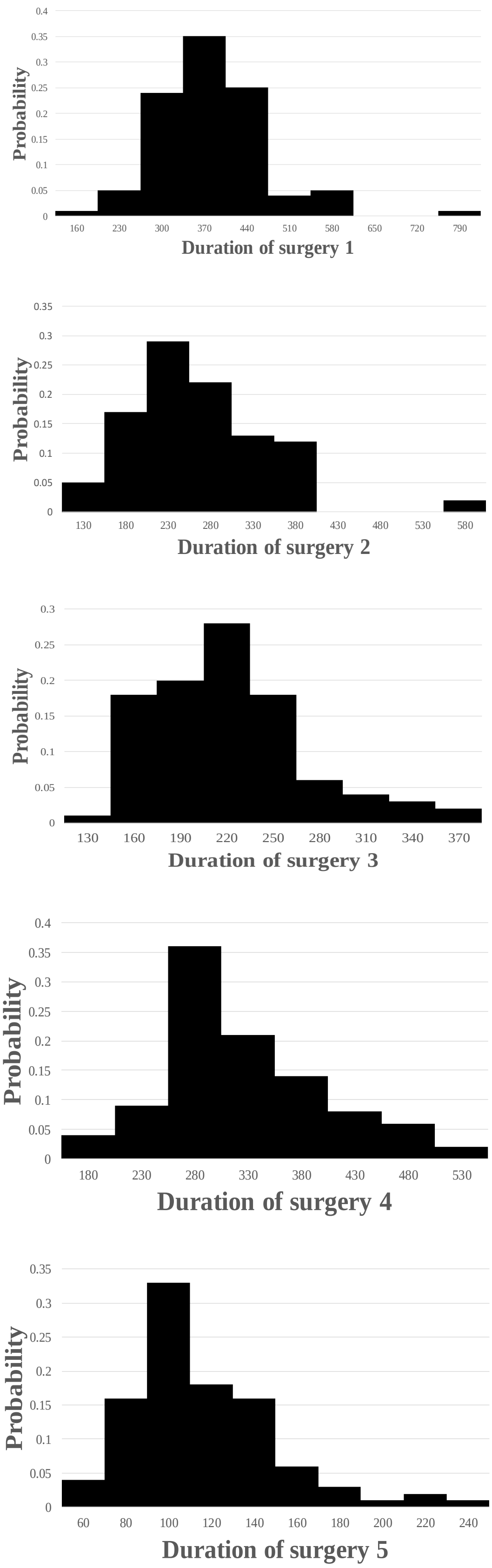

Fig. 4. Histograms of the scenario-wise values of the duration of surgery j (p_j).

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TABLE II
RESULTS UNDER DIFFERENT β

β	Optimal sequence δ_β	Objective function value (min.)	Expected value of total weighted delay (min.)	CVaR
0	5, 3, 2, 1, 4	181119	1811.19	3018
0.1	5, 3, 2, 1, 4	163309	1811.19	3018
0.2	5, 3, 2, 1, 4	145498.8	1811.19	3018
0.3	5, 3, 2, 1, 4	127688.7	1811.19	3018
0.4	5, 3, 4, 2, 1	111924.8	1845.70	2957
0.5	5, 3, 4, 2, 1	93763.5	1845.70	2957
0.6	5, 3, 4, 2, 1	75597.8	1845.59	2957
0.7	5, 3, 4, 2, 1	57440.9	1845.70	2957
0.8	5, 3, 4, 2, 1	39279.6	1845.70	2957
0.9	5, 3, 4, 2, 1	21118.3	1845.70	2957
1	5, 3, 4, 2, 1	2957	2955.83	2957

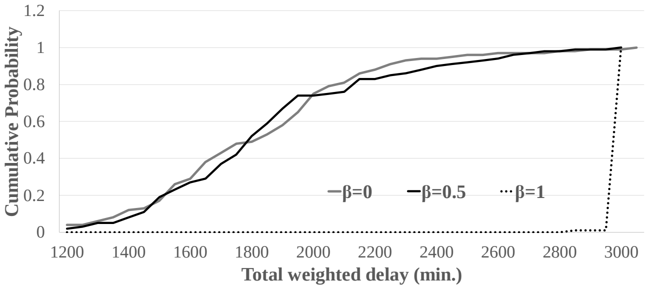

Fig. 5. Cumulative distribution function of the total weighted delay of surgeries for $\beta = 0$, $\beta = 0.5$ and $\beta = 1$.

B. Results

The computer used to generate the schedule was equipped with an Intel Core 2.30GHz processor (i5-6200U) and 8 GB RAM. The CPU time required to solve the formulated stochastic programming problem was 3 s using the IBM ILOG CPLEX 12.6.3 solver. In the generated test instances, there were 2729 constraints and 1130 variables. We provide some numerical examples for various weighting factors of β that range from 0 to 1 to assess the effects of the weighting factor β on the optimal solution (Table 2). The case of $\beta = 0$ minimizes the expected value of the total weighted delay without considering the risk measure, i.e., risk neutral behavior. Conversely, the case of $\beta = 1$ minimizes the risk measure, i.e., risk-averse decision making. Minimizing the expected value of the total weighted delay ($\beta = 0$) results in the optimal sequence $\delta_0 = 5$, 3, 2, 1, and 4. The expected value of the total weighted delay becomes a small value of 1811.19; however, the value of the CVaR is a large value of 3018. In this case, there is a risk that a large delay might occur. The criterion of minimizing the CVaR, i.e., $\beta = 1$, resulted in the optimal sequence $\delta_1 = 5$, 3, 4, 2, and 1. Even though the CVaR becomes a small value of 2957, the expected value of the total weighted delay is a large value of 2955.83. The objective function value of $\beta = 1$ is 61.25 times less than that of $\beta = 0$ and 31.70 times less than that of $\beta = 0.5$. The expected value of the total weighted delay of $\beta = 0$ is 1.63 times less than that of $\beta = 1$ and 1.02 times less than that of $\beta = 0.5$. The CVaR of $\beta = 1$ is 1.02 times less than that of $\beta = 0$.

The cumulative distribution functions of the total weighted delay under both sequences are shown in Fig. 5. Therefore, the cumulative distribution function of $\beta = 1$ and $\beta = 0.5$ reach a probability of 1 at a total weighted delay value of 3000, while $\beta = 0$ has a substantial associated probability of exceeding this value. This example represents the risk-averse nature of the CVaR and its effectiveness in reducing the variability. Figure 6 shows the histograms of the total weighted delays of surgeries for $\beta = 0$, $\beta = 0.5$, and $\beta = 1$.

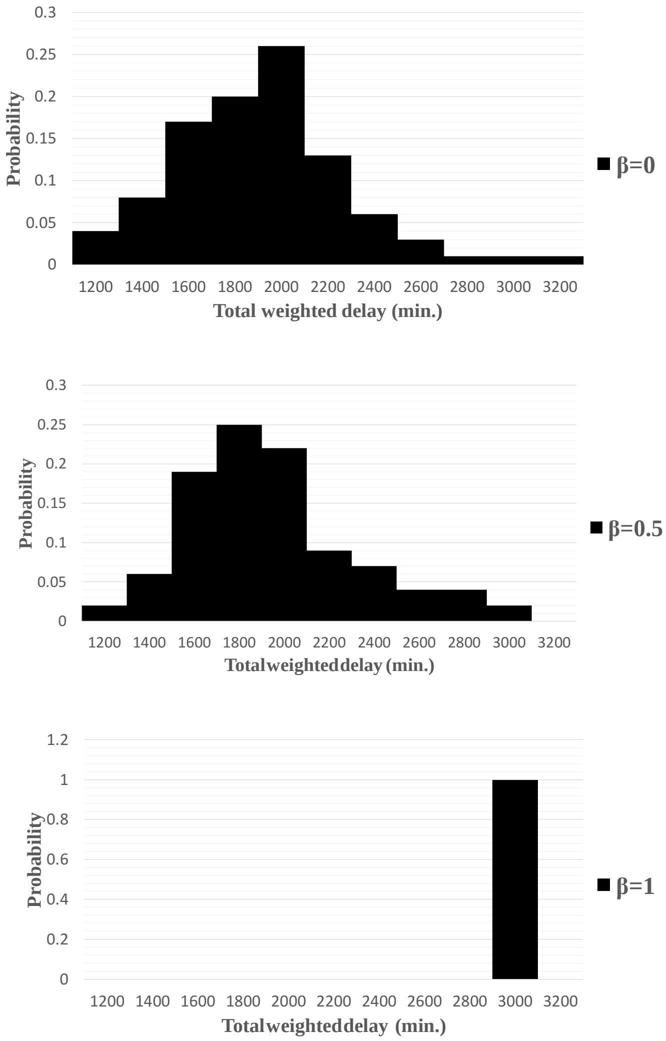

Fig. 6. Histograms of the total weighted delay of surgeries for $\beta = 0$, $\beta = 0.5$, and $\beta = 1$.

The objective function value of $\beta = 1$ is smaller than those of the other β values. The expected value of the total weighted delay of $\beta = 0$ is the smallest of the β values. The CVaR of $\beta = 0.5$ and $\beta = 1$ is smaller than that of $\beta = 0$. The maximum values of the total weighted delays of $\beta = 0.5$ and $\beta = 1$ are smaller than that of $\beta = 0$. It is important to consider both the expected value of the total weighted delay and the CVaR to create a single operating room schedule.

Reducing the expected value as well as the CVaR can avoid the collapse of the entire schedule. A small delay in a surgery only results in a small delay for the other surgeries in the same operating room; however, a large delay in a surgery forces the entire schedule to change. If the surgery is a little late at an operating room, the other scheduled surgeries will need to wait until the surgery is completed with a small delay. If the surgery is very late in an operating room, the scheduler will make a new schedule to conduct the other surgeries in other operating rooms. Moved surgeries also change the expected start times of surgeries scheduled in the other operating rooms.

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V. CONCLUDING REMARKS

We develop a stochastic programming model for scheduling a single operating room. We derive an optimal sequence for surgeries that minimizes the expected value of the total weighted delay and the CVaR. We analyze the effects of the weighting factor on the expected value of the total weighted delay and the CVaR. Thus, we find that it is important to consider the expected value of the total weighted delay and the CVaR to create a single operating room schedule.

In the future, we will compare the results obtained by the objective function including the CVaR to other approaches with the VaR or variance as the risk measures. In addition, the proposed model should be expanded to a multi-operating room scheduling model.

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