Economic Ordering Quantity Model for Optimal Buyer's Replenishment Policy Considering Expiring Goods

Ming-Feng Yang, Wu-Hsun Chung, Yen-Ting Chao and Hsiang-Ting Chien

***Abstract*—In this paper, based on the EOQ (Economic Ordering Quantity) model, we focus on the problem of wasting which became more and more serious. Thus, we provided some ideas to avoid wasting before the expiration date in this paper. The expiring goods are divided into two parts, the first is recyclable and contains residual value, and the second is unrecyclable which waiting to be discarded. Therefore, we combined the replenishment and recycling movement over multiple deliveries to reduce the cost, and the residual value of expiring item also could increase profits for buyers. This paper assumed that the ordering cycle is determined by the demand rate, delivery quantity per shipment, and the mathematical expectation of the expiring item rate in a lot. The loss of the expiring goods recycling and defective items processing are counted into the cost. As the result, this paper establish an EOQ inventory model for expiring items with equal-size shipment policy to derive the optimal ordering cycle, ordering quantity, and number of deliveries. In the end, through the theoretical proving procedure and numerical example, then analyze the results.**

***Index Terms*—Replenishment Policy, Expiring Goods, Defective Item, Economic Ordering Quantity Model.**

I. Introduction

This paper studied the problem of wasting that is very important for modern country. We found the channel would abandon the goods which are available but nearly the expiry date. The retailer offer items at a discounted price when they are close to the expiration date or look upon as sub-quality to avoid food wasting Aschemann-Witzel J et al. [1]. Nowadays in the retailer market, Prayoga and Shi-Woei [2] found a unique phenomenon that some of the retailers resell nearly expired grocery products which are bought from other retailers. In this way, the better market segmentation and the utilities of consumer and retailers will increase. Furthermore, Younes and Nicola [3] took an investigation into the shortfalls reasons of promotional on-shelf-availability in retailing. The research indicated the amount of data for retailers is great higher than for manufacturers, reflecting the importance of retailers in ensuring on-shelf-availability.

As a result, we are based on replenishment policy to establish a model to solve the expiring goods. F.W.Harris [4] introduced the economic order quantity (EOQ) model firstly, and settle two fundamental questions: "when should an order be placed?" and "how much should it be ordered?" . Maddah and Jaber [5] extend the original EOQ model by allowing for several batches of imperfect quality items which could be consolidated and shipped in one lot, and rectified a defect in original model related to the approach of assessing the expected profit. Goyal and Cárdenas-Barrón [6] based on the simple approach with the optimal method, and compared the results for determining the economic production quantity and get the optimal results to fit reality. M.F. Yang et al. [7] thought it is important factor that impact the inventory policy by order quantity and transport lot. To established equal-size shipment policy Lu Yueli et al. [8] considering the possibility of residue or stockout in the end of a cycle and assuming that the ordering cycle is determined by the demand rate, delivery quantity per shipment, and the mathematical expectation of the defective rate. Then Maddah and Jaber [5] analyzed the effect of screening speed and variability of the supply process on the order quantity. When the low variability of the yield rate, the result shows the order quantity in their model is larger than the classical EOQ model. In addition, for Marco Bijvanka et al. [9]'s new replenishment policy, they compared the (s, S) policy and (s, nQ) policy, then inferred the steady-state inventory distribution (s, nQ) policy could be used for inventory systems with different objectives or service level constraints.

For example, convenience store is a normal business circumstance. Thus, these stores are important to efficiently control their stock replenishment to reduce operating costs. Po-Chien and Jia-Yen [10] obtained the basic reorder quantity by consideration of the probability forecast of demand, hypothesis testing and the newsboy method. In order to better understand the impact of consumer behavior and shelf-life, Geoffrey et al.[11] purposed three key decisions (a) whether to discount older items, (b) how much discount to offer, and (c) what should be the replenishment policy. In terms of perishable products, Leandro and Gilbert [12] compared two suboptimal selling priority policies with an optimized policy: increasing revenue by sell the fresher items first, and avoiding spoilage by selling the oldest

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available items first.

Integrate the above model, regarding the setting of 'T', most of them pertain to the cycle time T is random variable which is relative to the demand rate D, the order quantity Q_d ,and the defective rate p, that is, $T = (1 - p)\ Q_d/D$. However, another one is the cycle time is a deterministic variable; that is, $T = (1 - (p))/D$. The optimal cycle time is obtained when the optimal ordering quantity is found by Lu Yueli et al. [8]. This paper established an economic ordering quantity model for expiring items with equal-size shipment policy to derive the optimal ordering cycle, order quantity, and number of deliveries which expands the model proposed by Maddah and Jaber [5] and Lu Yueli et al. [8].

II. ASSUMPTIONS AND NOTATIONS

A. Assumptions

1) Infinite time horizon.
2) Rate of demand,D, is constant and known.
3) Shortages are not allowed.
4) According to first-in-first-out(FIFO) inventory method.
5) Lead time is ignored, and instantaneously delivery.
6) The item provided by supplier contain a percentage p of expiring items, which p is a random variable between interval [0, 1] with the probability density function (PDF) f(p) and the mathematical expectation μ, under exponential distribution.
7) Due to the μ is expected random probability between 0 and 1, therefore, $\lambda > 1$ under the exponential distribution.
8) Expiring goods are divided into two parts, the first is recyclable and contains residual value, and the second is unrecyclable.
9) Expiring goods which cannot be recycled are stored and consolidated at a lot until the end of the final period in nth time interval t, that is, $[(n - 1t), nt]$.
10) The expected ordering cycle of buyer is $T = (1 - \mu)Q/D$. The supplier shipped them in equal size Q_d in n shipments with the time interval $t = (1 - \mu)Q_d/D$. As a result, $Q = nQ_d$ and $T = nt$.
11) Replenishment interval isn't longer than keeping interval of items.
12) The expiring items of (n-1)th time are recycled by nth replenishment. The residual value of expiring items would be sold at the discounted price v which could make buyer gain the profit.

B. Notations

T: ordering cycle, decision variable.
Q: ordering quantity each cycle, decision variable.
n: number of deliveries, decision variable.
Q_d: replenishment quantity of each shipment, decision variable.

D: annual demand rate.
C: purchasing price per unit item.
C_o: ordering cost per order.
C_h: holding cost for the buyer per unit per year.
C_r: recycling cost per unit per unit.
C_p: processing cost of scarps per cycle.
R: replenishment cost per unit time.
p: percentage of the item nearly expiry in a lot, random variable.
μ: expectation of random variable p.
p_i: percentage of expiring goods which cannot be recycled.
S: selling price of available items per unit.
v: residual value of the item nearly expiry per unit.

III. MODEL FORMULATION

A. Revenue

- The revenue of available items is

$$Sn\min\{(1-p)Q_d,\ Dt\} = SnQ_d\min\{(1-p),(1-\mu)\} \quad (1)$$

- The residual value of expiring item is $vp(1-p_i)nQ_d$.

In a cycle, the total revenue of the buyer is

$$TR(Q_d,n) = SnQ_d\min\{(1-p),(1-\mu)\} + vpnQ_d = nQ_d\{S\min\{(1-p),(1-\mu)\} + v(p - p_i * p)\} \quad (2)$$

B. Cost

- The purchasing cost is $(C_o + nCQ_d)$.

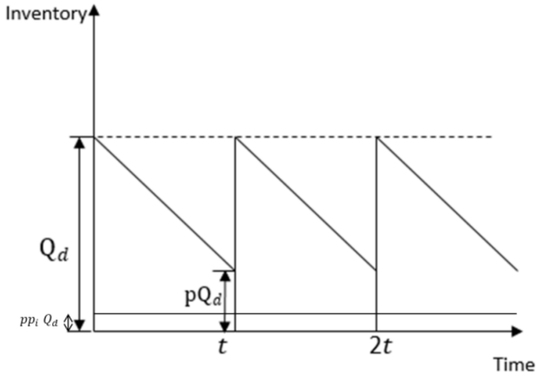

Fig. 1. Inventory level of available goods and recyclable expiring goods.

- The holding cost of available goods and recyclable expiring goods is

$$\left[\frac{(pQ_d+Q_d)tn}{2} - pp_iQ_dtn\right] * C_h = \frac{[p(1-2p_i)+1]Q_dtnC_h}{2} \quad (3)$$

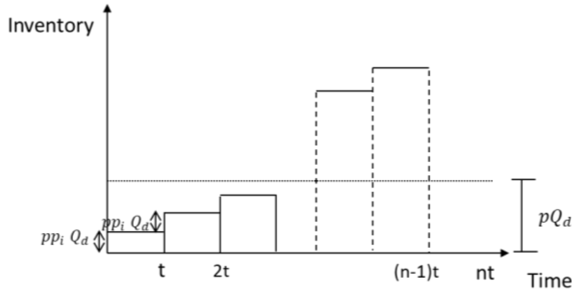

Fig. 2. Inventory level of expiring goods which cannot be recycled.

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- The holding cost of expiring goods which cannot be recycled

$$\sum_{i=1}^{n-1} i * pp_i\, Q_d tn C_h = \frac{n*(n-1)}{2} pp_i Q_d t C_h \tag{4}$$

- The cost of replenishment is nR.
- The recycling cost is $p(1-p_i)Q_d(n-1)tC_r$.
- The processing cost of scarps is C_p.

In a cycle, the total cost of the buyer is

$$TC(Q_d,n) = C_o + nCQ_d + \frac{[p(1-2p_i)+1]Q_d tnC_h}{2} + \frac{n(n-1)}{2} pp_i Q_d tC_h + nR + p(1-p_i)(n-1)Q_d tC_r + C_p \tag{5}$$

C. Total Profit

Integrated above, the total profit of the buyer in a cycle is

$$\begin{aligned} TP(Q_d,n) &= TR(Q_d,n) - TC(Q_d,n) \\ &= nQ_d\{S\min\{(1-p),(1-\mu)\} + vp(1-p_i) - C\} \\ &\quad - \left\{C_o + \frac{[p(1-2p_i)+1]Q_d ntC_h}{2} + \frac{n(n-1)}{2D} pp_i Q_d tC_h + nR + pQ_d(1-p_i)(n-1)tC_r + C_p\right\} \\ &= nQ_d\{Smin\{1-p,1-\mu\} + vp - C\} - \frac{Q_d tnC_h}{2}[(p+1) - pp_i(3-n)] - C_o \\ &\quad - nR - pQ_d tC_r(1-p_i)(n-1) - C_p \end{aligned} \tag{6}$$

The expected ordering cycle of buyer is $T = (1-\mu)Q/D$. The supplier shipped them in equal size Q_d in n shipments with the time interval $t = (1-\mu)Q_d/D$. Buyer's expected total profit in a cycle is

$$\begin{aligned} ETP(Q_d,n) &= nQ_d\{SE[min\{1-p,1-\mu\}] + v\mu(1-p_i) - C\} - C_o \\ &\quad - \frac{{Q_d}^2 nC_h}{2D}[(\mu+1) - \mu p_i(3-n)](1-\mu) - nR \\ &\quad - \frac{C_r\mu {Q_d}^2(1-p_i)(n-1)(1-\mu)}{D} - C_p \end{aligned} \tag{7}$$

Buyer's expected total profit per unit time in a cycle is

$$\begin{aligned} ETPU(Q_d,n) &= \frac{E[TP(Q_d,n)]}{E[T]} \\ &= \frac{D}{1-\mu}\{SE[min\{1-p,1-\mu\}] + v\mu(1-p_i) - C\} \\ &\quad - \frac{(C_o + nR + C_p) * D}{(1-\mu)nQ_d} - \frac{Q_d C_h(1-\mu)}{2}[(\mu+1) - \mu p_i(3-n)] \\ &\quad - \frac{C_r\mu Q_d(1-p_i)(n-1)}{n} \end{aligned} \tag{8}$$

Under exponential distribution, the probability destiny function between interval [1,0] is

$$f(p) = \lambda e^{-\lambda p} \tag{9}$$

Where,

$$p \sim Exp(\lambda)$$
$$\lambda = \frac{1}{\mu}$$

The model we set above is to maximize the $ETPU(Q_d, n)$, and the Q_d and n are integer.

IV. Theoretical Proving Procedure Numerical

A. Theoretical 1

For a given n, in order to maximizes $ETPU(Q_d, n)$, we seek for the optimal value of Q_d at the first. The first derivative of $ETPU\ (Q_d, n)$ with respect to Q_d is obtained

$$\frac{\partial ETPU(Q_d,n)}{\partial Q_d} = \frac{D(C_o + nR + C_p)}{(1-\mu)n{Q_d}^2} - \frac{C_h(1-\mu)}{2}[(\mu+1) - \mu p_i(3-n)] - \frac{C_r\mu(1-p_i)(n-1)}{n} \tag{10}$$

Then, substitute with $k(n), a(n), b(n)$ in to (10).

$$\frac{\partial ETPU(Q_d,n)}{\partial Q_d} = \frac{Dnk(n)}{(1-\mu){Q_d}^2} - \frac{C_h a(n)}{2} - \frac{C_r b(n)}{n} \tag{11}$$

Where

$$a(n) = (1-\mu)[(\mu+1) - \mu p_i(3-n)]$$
$$b(n) = \mu(1-p_i)(n-1)$$
$$k(n) = \frac{(C_o + nR + C_p)}{n^2}$$

To be maximized, the first derivative with regard to Q_d is equal to zero.

$$\frac{\partial ETPU(Q_d,n)}{\partial Q_d} = 0 \tag{12}$$

Equating the first derivative with regard to Q_d to zero and solving the equation, the economic replenishment lot size is

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$$Q_d{}^* = \sqrt{\frac{2Dn^2k(n)}{(1-\mu)[nC_h a(n)+2C_r b(n)]}} \tag{13}$$

Since

$$\frac{\partial^2 \mathrm{ETPU}(Q_d, n)}{\partial^2 Q_d} \le 0$$
$$\frac{-2D(C_o + nR + C_p)}{(1-\mu)nQ_d{}^3} \le 0 \tag{14}$$

Therefore, at $Q_d > 0$, $\mathrm{ETPU}(Q_d, n)$ is a concave function. $Q_d{}^*$ is the optimal value that maximizes $\mathrm{ETPU}(Q_d, n)$ on $Q_d \in [0, +\infty)$. We substitute $Q_d{}^*$ for Q_d into the function ETPU. Then, presenting the $\mathrm{ETPU}^*(n) = \mathrm{ETPU}(Q_d{}^*, n)$.

$$\mathrm{ETPU}^*(n) = \frac{D\{SE[\min\{1-p, 1-\mu\}] + v\mu(1-p_i) - C\}}{1-\mu} - \frac{\sqrt{2Dk(n)[nC_h a(n) + 2C_r b(n) + 1](1-\mu)}}{1-\mu} \tag{15}$$

According the situations, the expectation of $E[\min\{1-p, 1-\mu\}]$ may be divided into two outcomes:

$$E[\min\{1-p, 1-\mu\}] = \begin{cases} (1-\mu)e^{-\frac{1}{\mu}} + 1 - \mu & ,\text{When } 1-p > 1-\mu \\ \mu e^{\frac{-1}{\mu}} + 1 - \mu \ , & \text{When } 1-\mu > 1-p \end{cases} \tag{16}$$

B. Theoretical 2

Then, we do the first derivative of ETPU (Q_d, n) with respect to n is obtained

$$\frac{\partial ETPU(Q_d, n)}{\partial n} = \frac{(C_o + C_p)D + C_r\mu Q_d{}^2(p_i - 1)(1-\mu)}{Q_d(1-\mu)n^2} - \frac{2C_r\mu(1-p_i)Q_d}{2} \tag{17}$$

$$\frac{\partial^2 ETPU(Q_d, n)}{\partial^2 n} = \frac{-2\{(C_o + C_p)D + C_r\mu Q_d{}^2(p_i - 1)(1-\mu)\}}{Q_d(1-\mu)n^3} < 0 \tag{18}$$

Therefore, $ETPU(Q_d, n)$ is strongly concave on $n > 0$. Thus, in each replenishment, determining the optimal number of deliveries n*, is simplified to obtain a global optimum.

Algorithm

Step1: n=1
Step2: Determine the Q_d by taking first-order partial derivative of with respect to Q_d.
Step3: Take Q_d into ETPU.
Step4: Set n=n+1, repeat steps 2-3 to get $\mathrm{ETPU}(Q_d, n)$.
Step5: If $\mathrm{EJTP}(Q_d, n) \geqq \mathrm{EJTP}(Q_d, n-1)$, goes to step 4. if not, go to step 6 and stop.
Step6: After obtained each optimal solution in cases, choose the best ETPU. The highest value is the most optimal solution in the end.

V. Numerical Example

To examine the model we set, we consider a situation with the following parametric values partially adopted in Maddah and Jaber's model [5] and M.F. Yang et al. [7].

(i) Annual demand rate D= 5000.
(ii) Purchasing price per unit item C=$5/unit.
(iii) Ordering cost per order C_o=$100/cycle.
(iv) Holding cost for the buyer per unit per year C_h=$5/unit.
(v) Recycling cost per unit per unit C_r=$2/unit.
(vi) Selling price of available items per unit S=$50/unit.
(vii) Replenishment cost per unit time R=$50/unit time.
(viii) Residual value of the item nearly expiry per unit v=$10/unit.
(ix) Percentage of expiring goods which cannot be recycled p_i=10%
(x) Processing cost of scarps per cycle C_p=$80/cycle.
(xi) Percentage of the item nearly expiry in a lot, random variable, p~Exp(λ) , μ=1/λ.

The relevant results are with different λ, which are shown in Table1. and Fig3.

We calculated n=1 to 10 with λ=2~10 in computer, we got 100 results with different numerical data, then chose the best expected total profit with every λ=2~10. As seen from Table 1, λ and μ are the reciprocal relationship. When λ increase, then μ decrease, the optimal of delivery, ordering quantity, ordering cycle, and expected total profit per unit time all increase.

In Fig. 3, we can see the relationship between ETPU and n. with the optimal replenishment policy for different λ (μ=1/λ). When expected random probability μ decrease, the quantity of expiring goods are also reduced. As a result, the expected total profit would increase and the buyer would like to increase the optimal of delivery, in order to get more profit. However, when λ>4, the increase of ETPU* is getting smaller. It means the marginal utility of ETPU is significantly decreased while λ>4 and n>14.

TABLE I
THE EFFECTS OF Λ AND M ON N*, Q*, T*, AND ETPU*

λ	*μ*	*n**	*Q**	*T**	*ETPU**
2	0.5	12	951681.511	9.5168	107027.00
3	0.333	13	1017373.112	13.565	721514.92
4	0.25	14	1144962.649	17.174	994870.44
5	0.2	16	1808028.318	28.928	1050663.03
6	0.1667	17	2042816.846	34.047	1087813.55
7	0.14286	18	2306154.071	39.534	1113711.08
8	0.125	19	2596447.759	45.438	1132568.49
9	0.111	20	2913106.16	51.789	1146829.04
10	0.1	21	3256016.817	58.608	1157960.38

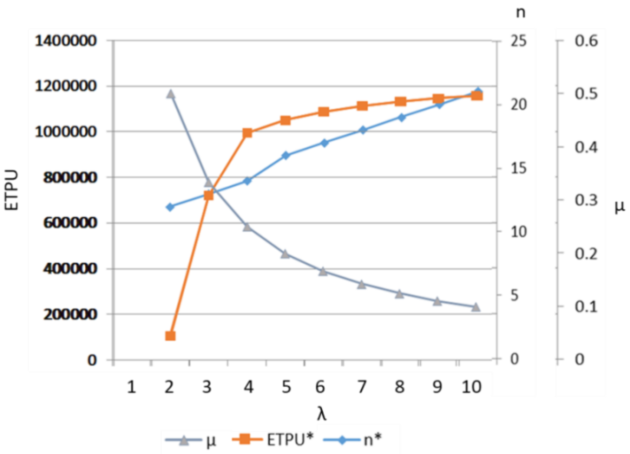

Fig. 3. The relationship of μ , ETPU* and n* with the optimal replenishment policy for different λ (μ =1/λ).

VI. Conclusion

Nowadays, the wasting problem became more and more serious. Most research didn't consider the processing of expiring goods in replenishment policy; however, it's important to reality. In the EOQ model which extended the concept of Maddah and Jaber's [5] model, we provided not only the quantity of delivery per shipment, but also derived the optimal cycle time to help the buyer making the replenishment policy. The recycling of available expiring goods residual value we considered would increase the revenue, and the goods get more utilization, then the wasting will also reduce. In this benign cycle can make buyers more willing to do and make more people benefit. In the future extensions, the research of the replenishment policy could integrate the vendor and buyer inventory based on this model or consider more factors, for example: carbon tax, inflation...improving the model to adapt to reality.

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