

On Detectability and Recoverability of Input/State Asynchronous Sequential Machines with Transient Faults in Non-fundamental Mode

Jung–Min Yang

Abstract—Corrective control has been successfully applied to compensating for faulty behaviors of asynchronous sequential machines. In this paper, a corrective control scheme is studied for dealing with transient faults that may happen in non-fundamental mode operations. Since a fault can occur to a machine in transient transitions, the procedure of fault diagnosis and fault tolerant control should be more complicated compared with the case of fundamental mode. We show that certain properties, called detectability and recoverability, are requisites for the existence of a fault tolerant corrective controller that makes the closed-loop system immune against any fault occurrence in non-fundamental mode. A simple example is provided to illustrate the proposed notions and the controller existence.

Index Terms—asynchronous sequential machines, corrective control, fault tolerant control, non-fundamental mode.

I. INTRODUCTION

Corrective control, an automatic control theory aiming at compensating for the stable-state behavior of asynchronous sequential machines, has been showing good performance when applied to diagnosing and tolerating various faulty behavior of the machines. This is mainly due to the property of corrective control that it can materialize immediate fault recovery against state transition faults, which otherwise could lead to malfunctions of the machine when the external input further changes. Notable among the related results are elimination of critical races [1], [2] and infinite cycles [3], [4]. Furthermore, the author presented general schemes of fault tolerant corrective control for overcoming transient faults [5], [6], permanent faults [7], and intermittent faults [8]. Recently, corrective control is being studied based on a matrix framework called semi-tensor product (STP); see, e.g., [9]–[11].

The main concern of this paper is to study fault tolerant corrective control of input/state asynchronous sequential machines with transient faults that occur in non-fundamental mode operations. Here, transient faults are referred to as infiltration of outer disturbance entities or an outbreak of internal failures such that the asynchronous sequential machine undergoes an unauthorized state transition. In all the prior work, it is assumed that occurrences of transient faults comply with the principle of fundamental mode operations

This research was supported in part by the Bio & Medical Technology Development Program of the National Research Foundation (NRF) funded by the Korean government (MSIP) (No. 2015M3A9A7067220) and in part by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (No. NRF-2018R1D1A1A09082016).

J.–M. Yang is with the School of Electronics Engineering, Kyungpook National University, Daegu, 41566, Republic of Korea. E-mail: jmyang@knu.ac.kr.

[12] whereby only one variable may change at a time in the dynamics of asynchronous sequential machines. This means that a transient fault has to occur only when the machine stays at a stable state. Admittedly, the speed of transient transitions is very fast in the operation of asynchronous sequential machines so that the possibility of fault occurrences during the transitions is low. But it is not impossible that a transient fault occurs during transient transitions, especially if the transition speed becomes slower due to time-delay. Hence it would be more general to consider fault occurrences in both fundamental and non-fundamental mode.

To determine the possibility of fault diagnosis and fault tolerant control against transient faults occurring in non-fundamental mode, relevant properties of the controlled machine must be investigated. In this paper, we introduce detectability and recoverability of the controlled machine and show that they are both needed to describe the conditions on fault diagnosis and fault recovery against transient faults in non-fundamental mode. In particular, detectability is necessary for fault diagnosis, and recoverability for fault tolerant control. Our main consideration is devoted to addressing the existence condition for fault tolerant corrective controllers in terms of these properties. A detailed process of controller synthesis is omitted and will be only sketched conceptually.

The rest of this work is structured as follows. Section II provides a modeling formalism of input/state asynchronous sequential machines and the characteristics of transient faults occurring in non-fundamental mode operations. In Section III, the notions of detectability and recoverability are addressed and the existence condition for a fault tolerant corrective controller is presented based on the proposed properties. A simple example is provided in Section IV to demonstrate the proposed notions and scheme. Finally, Section V concludes the paper.

II. PRELIMINARIES

A. Modeling

The considered asynchronous sequential machine is input/state type in which the output is equal to the present state of the machine. An input/state asynchronous sequential machine Σ is modeled as a deterministic finite state machine

$$\Sigma := (A, X, x_0, f),$$

where A is the input set, X is the state set, $x_0 \in X$ is the initial state, and

$$f : X \times A \rightarrow X$$

is the state transition function partially defined on $X \times A$. A is further divided into

$$A := A_n \cup A_d$$

where A_n and A_d are the sets of normal and adversarial inputs, respectively.

A state-input combination $(x, v) \in X \times A$ is termed valid if $f(x, v)$ is defined in Σ . A valid combination (x, v) is divided into stable and transient combinations. If $f(x, v) = x$, (x, v) is a stable combination with x a stable state. On the other hand, if $f(x, v) \neq x$, it is a transient combination with x a transient state. x can be either stable or transient depending on the present input value. To address this, let

$$U(x) := \{v \in A_n | f(x, v) = x\}$$

$$T(x) := \{v \in A_n | f(x, v) \neq x\}$$

be the subsets of A_n of which members make stable and transient combinations with x , respectively.

Since Σ has no global synchronizing clock, it responds only with the change of the external input. Σ stays at a stable combination (x, v') ($v' \in U(x)$) indefinitely unless the input value changes. If the external input changes to $v \in T(x)$ that makes a transient combination with x , Σ engages in a chain of transient transitions, e.g.,

$$f(x, v) = x_1$$

$$f(x_1, v) = x_2$$

$$\vdots$$

during which the input v remains fixed. Supposing that no infinite cycles exist in Σ , Σ reaches a stable state x_k such that

$$x_k = f(x_{k-1}, v) = f(x_k, v), \exists k < \infty,$$

i.e., $v \in U(x_k)$. x_k is called the *next stable state* of (x, v) [12]. Owing to the absence of a synchronizing clock, the transient transitions are passed through instantaneously. Hence, from outer users's viewpoint, only stable states are observed in the operation of Σ . To epitomize this feature, we define the stable recursion function s [1] by

$$s : X \times A \rightarrow X$$

$$s(x, v) := x_k$$

where x_k is the next stable state of (x, v) . A chain of transitions from one stable combination to another, as described by s , is termed a *stable transition*. The domain of s is expanded from $X \times A$ to $X \times A^+$ in a natural way as follows, where A^+ is the set of non-empty strings made of characters in A . For $x \in X$ and $v_1 v_2 \dots v_k \in A^+$,

$$s(x, v_1 v_2 \dots v_k) := s(s(x, v_1), v_2 \dots v_k).$$

We also define

$$\tau(x, v) := \{x, x_1, \dots, x_k\}$$

as the set of transient states traversed by Σ when it takes the stable transition $(x, v) \rightarrow (s(x, v), v)$. If $f(x, v) = x$, $\tau(x, v) := \emptyset$.

A transient combination (x, v) with $v \in T(x)$ can be either the first or a middle transient combination of the stable transition characterized by (x, v) . If x has been the stable state of Σ when the external input changes to v , x is the first

transient state of the stable transition $(x, v) \rightarrow (s(x, v), v)$. On the other hand, if $\bar{x} \in X$ exists such that $x \in \tau(\bar{x}, v)$, x can be an intermediate transient state of the stable transition $(\bar{x}, v) \rightarrow (s(\bar{x}, v), v) (= (s(x, v), v))$. For later usage, we specify as follows the states preceding a given transient state and the *root state*, which is the state that precedes any other transient states of a stable transition characterized by a given transient combination.

Definition 1. Given a transient combination $(x, v) \in X \times A_n$, $\bar{x} \in X$ is a preceding state of (x, v) if $\bar{x} \neq x$ and $x \in \tau(\bar{x}, v)$; $P(x, v) \subset X$ is the set of all preceding states of (x, v) and

$$P(x) := \cup_{v \in T(x)} P(x, v);$$

(x, v) is a proper transient combination if $P(x, v) \neq \emptyset$; and $r(x, v) \in P(x, v)$, the root state of a proper transient combination (x, v) , is the state such that $P(r(x, v), v) = \emptyset$. $(r(x, v), v)$ is called a root transient combination.

For $\bar{x} \in P(x, v)$, it is straightforward that

$$\tau(x, v) \subset \tau(\bar{x}, v) \subset \tau(r(x, v), v)$$

and

$$s(x, v) = s(\bar{x}, v) = s(r(x, v), v).$$

For example, assume that a stable transition $(x, v) \rightarrow (s(x, v), v)$ has k transient states $\tau(x, v) = \{x, x_1, \dots, x_k\}$ except for x as described in Fig. 1 and there exists no \bar{x} such that $x \in \tau(\bar{x}, v)$. Then, (x_i, v) , $i = 1, \dots, k$, is a proper transient combination, $P(x_i, v) = \{x, x_1, \dots, x_{i-1}\}$, and $r(x_i, v) = x$.

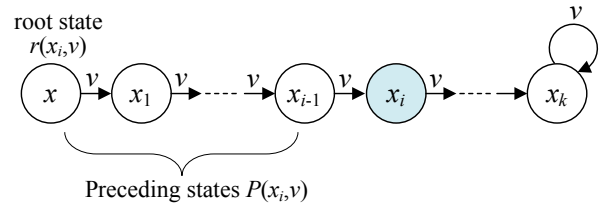


Fig. 1. Preceding states and root state.

B. Transient Faults in Non-fundamental Mode

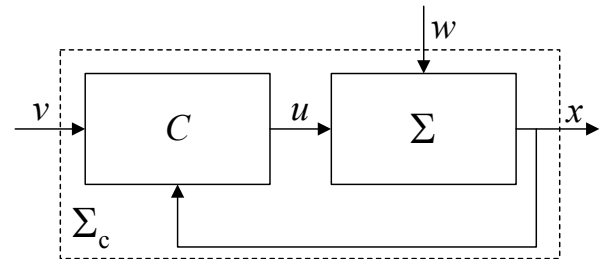


Fig. 2. Corrective control system for tolerating transient faults in non-fundamental mode.

Fig. 2 shows the corrective control system for an input/state asynchronous sequential machine Σ that is exposed to transient faults occurring in non-fundamental mode. C is the corrective controller, which has the form of an input/output asynchronous sequential machine, and Σ_c is the closed-loop system consisting of C and Σ . Receiving the

external input $v \in A_n$ and state feedback $x \in X$, C generates the control input $u \in A_n$. Hence C is modeled as

$$C = (A_n \times X, A_n, \Xi, \xi_0, \phi, \eta)$$

where $A_n \times X$ and A_n are the input and output sets, respectively, Ξ is the state set, $\xi_0 \in \Xi$ is the initial state,

$$\phi : \Xi \times A_n \times X \rightarrow \Xi$$

is the state transition function, and

$$\eta : \Xi \rightarrow A_n$$

is the output function. The design of C equals the assignment of the function values of ϕ and η . As stated before, in this paper we focus concern on elucidating the existence condition for a fault tolerant controller and will not present the controller synthesis in detail.

Note that C is activated only when a transient fault is diagnosed. In the normal behavior, C just delivers the external input v as the control input $u = v$. If a transient fault is detected, on the other hand, C provides a sequence of control inputs so that Σ can recover the normal input/state behavior immediately.

In Fig. 2, $w \in A_d$ denotes the adversarial input. If w infiltrates into Σ , it overrides the current control input u and causes Σ to undergo an unauthorized transition. For $x \in X$, let

$$W(x) := \{w \in A_d | (x, w) \text{ is valid and } s(x, w) \neq x\}$$

be the set of adversarial inputs that are defined at x , i.e., when Σ has the state x , any $w \in W(x)$ may occur. If $w \in W(x)$ occurs to Σ that has been staying at the stable state x , Σ undergoes the unauthorized transition from x to $s(x, w)$ regardless of the current input u . In this case, C must be designed so that it takes Σ toward the original state x before further change of the external input. The latter topic is presented in the author's previous studies [5], [6].

By contrast, supposed that w happens when Σ has the transient state x . This means that Σ has been engaging in a stable transition from, e.g., (\bar{x}, v) such that $v \in T(x) \cap T(\bar{v})$ and $\bar{x} \in P(x, v)$ and that w occurs at the instant that Σ is exactly passing through x . As the result of the fault, Σ would reach $s(x, w)$ instead of the nominal next stable state $s(\bar{x}, v) = s(x, v)$. To regain the normal behavior, Σ must be controlled to transfer from $s(x, w)$ to $s(x, v)$. Unlike the case of fault occurrences at stable states, the desired state varies depending on the current external input v . Hence the existence condition for a fault tolerant corrective controller will be stricter than the case of those occurring in fundamental mode.

Remark: Among transient combinations having non-empty $W(x)$, we only have to consider proper transient combinations, i.e., those having preceding states (see Definition 1). Since a state in a non-proper transient combination will be the root state of the corresponding stable transition, the unauthorized transition can be directly diagnosed by observing the change of state feedback.

III. MAIN RESULT

A. Detectability

We first discuss how to diagnose an occurrence of a transient fault occurring in non-fundamental mode. Compared with transient faults in fundamental mode, diagnosis on transient faults occurring at a transient state is confounding since an unauthorized transition happens in the middle of transient transitions. Suppose that Σ has been experiencing a stable transition from (\bar{x}, v) where $\bar{x} \in P(x, v)$ is a preceding state of (x, v) , when an adversarial input $w \in W(x)$ occurs to Σ at the instant that Σ is passing through the transient state x . Σ is then enforced to reach $s(x, w)$, not the nominal next stable state $s(\bar{x}, v) = s(x, v)$. Referring to Fig. 2, C may perceive an occurrence of such a transient fault by observing that the state feedback x changes to $s(x, w)$ after transmitting the control input $u = v$. To validate this speculation, let us assume further that $s(x, w) \in \tau(\bar{x}, v)$. Then, upon receiving the state feedback $s(x, w)$, C is faced with the following two ambiguous situations:

- (i) Σ may have undergone an unauthorized transition from x to $s(x, w)$ by an occurrence of $w \in W(x)$;
- (ii) Σ may have undertaking a normal stable transition from (\bar{x}, v) , traversing $s(x, w)$ on the way to the next stable state $s(\bar{x}, v)$ (see Fig. 3).

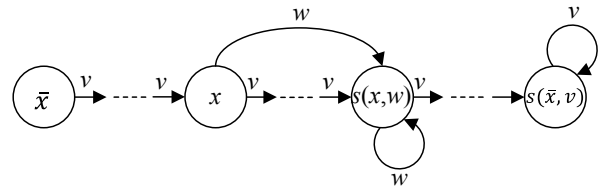


Fig. 3. Ambiguous situation of a transient fault occurring in non-fundamental mode.

Hence C cannot determine with certainty whether or not a transient fault really happens, so the subsequent fault tolerant control action cannot be initiated. To eliminate the underlying ambiguity, the deviated state $s(x, w)$ must not be contained in the transient state trajectory of the corresponding stable transition. But note that (x, v) may be a transient combination of a stable transition starting at any preceding state of (x, v) , and that

$$\tau(\bar{x}, v) \subset \tau(r(x, v), v)$$

for every preceding state $\bar{x} \in P(x, v)$, where $r(x, v)$ is the root state of (x, v) ; see Definition 1. Hence the latter condition is valid if and only if

$$s(x, w) \notin \tau(r(x, v), v).$$

If a transient combination satisfies this condition, it is called *fault detectable* in this paper. Let us formalize the detectability of transient combinations as follows.

Definition 2. Given $\Sigma = (A, X, x_0, f)$, a proper transient combination $(x, v) \in X \times A_n$ with $W(x) \neq \emptyset$ is fault detectable if

$$\forall w \in W(x), s(x, w) \notin \tau(r(x, v), v)$$

where $r(x, v)$ is the root state of (x, v) addressed in Definition 1.

If a transient combination (x, v) is fault detectable, C can diagnose an occurrence of any $w \in W(x)$ by observing the change of the state feedback to $s(x, w)$.

B. Recoverability

Suppose that an adversarial input $w \in W(x)$ infiltrates into Σ when Σ is passing through a proper transient combination (x, v) that is fault detectable. As stated in the previous section, the desired state that must be reached by fault tolerant control is not the original state x but the next stable state $s(x, v)$ Σ is supposed to reach in the normal behavior. In the framework of corrective control, the necessary and sufficient condition for realizing a corrective controller achieving this desired transition is that $s(x, v)$ is stably reachable from the deviated state $s(x, w)$, namely there must exist a sequence of input characters that take Σ from $s(x, w)$ toward $s(x, v)$ [1]. In this paper, the latter property is called *recoverability* of a transient combination with transient faults.

Definition 3. Given $\Sigma = (A, X, x_0, f)$, a proper transient combination $(x, v) \in X \times A_n$ with $W(x) \neq \emptyset$ is recoverable if

$$\forall w \in W(x), \exists t_w \in A_n^+ \text{ such that } s(s(x, w), t_w) = s(x, v).$$

Fig. 4 illustrates the notion of recoverability of a transient combination. Here, it is assumed that an input string $t \in A_n^+$ with $|t| = k$ and

$$t := u_1 u_2 \cdots u_k$$

exists that takes Σ from $s(x, w)$ toward $s(x, v)$ in k stable transitions, i.e., $s(s(x, w), t) = s(x, v)$. $x^1, x^2, \dots, x^{k-1} \in X$ denote the intermediate stable states traversed by Σ in the stable transition from $s(x, w)$ to $s(x, v)$. In terms of the stable recursion function s , we have

$$\begin{aligned} s(s(x, w), u_1) &= x_1 \\ s(x_1, u_2) &= x_2 \\ &\vdots \\ s(x_{k-2}, u_{k-1}) &= x_{k-1} \\ s(x_{k-1}, u_k) &= s(x, v). \end{aligned}$$

t will be utilized by the corrective controller in realizing the desired recovery path from $s(x, w)$ to $s(x, v)$.

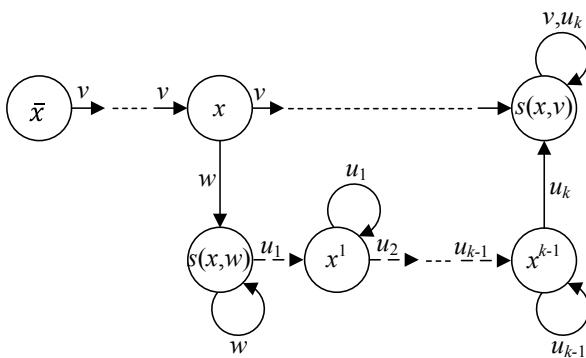


Fig. 4. Notion of recoverability of a transient combination (x, v) .

Before addressing the existence condition for a fault tolerant corrective controller, we note that x with $W(x) \neq \emptyset$ may make a proper transient combination in more than one

stable transition depending on the input. For instance, assume that

$$x \in \tau(x_1, v_1) \cap \tau(x_2, v_2)$$

in which $x_1, x_2 \in P(x)$ and $s(x_1, v_1) \neq s(x_2, v_2)$, in other words $s(x, v_1) \neq s(x, v_2)$. This means that x may be traversed as a transient state in two stable transitions beginning from the preceding states x_1 and x_2 and that those stable transitions converge to different next stable states $s(x, v_1)$ and $s(x, v_2)$, respectively. To guarantee fault recovery against every transient fault occurring at x in non-fundamental mode, both (x, v_1) and (x, v_2) must be fault detectable and recoverable, that is, both $s(x, v_1)$ and $s(x, v_2)$ must be stably reachable from $s(x, w)$ for all $w \in W(x)$. If $|W(x)| \gg 1$, the existence condition would be very tight as Σ must have stable reachability with respect to every possible recovery trajectory between the deviated state $s(x, w)$ and the desired next stable state. But an alternative scheme of fault tolerance can be utilized as follows to resolve the possible absence of recoverability.

For the sake of simplicity, we assume that (x, v) with $v \in T(x)$ and $W(x) \neq \emptyset$ is not recoverable and $x \in \tau(\bar{x}, v)$ for some $\bar{x} \in P(x, v)$. Assume further that there exists an input string $t' := u'_1 \cdots u'_l \in A_n^+$ such that

$$\begin{aligned} s(\bar{x}, v) &= s(\bar{x}, u'_1 \cdots u'_l), \\ x'_i &:= s(x'_{i-1}, u'_i), \\ i &= 1, \dots, l, \end{aligned}$$

where $x'_0 = \bar{x}$ and $x'_l = s(\bar{x}, v)$. Furthermore, suppose that for all (x'_{i-1}, u'_i) and $x' \in \tau(x'_{i-1}, u'_i)$, $W(x') = \emptyset$, that is, no transient fault may occur in this state trajectory. Then, we can prevent every unrecoverable transient fault of $W(x)$ from occurring by providing t' , instead of v , if the external input changes to v when Σ has been staying at the stable state x . This scheme is not so much fault tolerant control; rather, it is a kind of fault avoidance. In this paper, we do not include this alternative scheme into the existence condition. This topic will be tackled as a future study.

C. Existence of Controllers

We now address in formal terms the existence condition for a fault tolerant corrective controller that can invalidate transient faults occurring in non-fundamental mode operations in an input/state asynchronous sequential machine Σ . Predictably, detectability and recoverability serve as key ingredients for the controller existence.

Proposition 1. Given an input/state asynchronous sequential machine $\Sigma = (A, X, x^0, f)$ with $A = A_n \cup A_d$, suppose that $W(z) \neq \emptyset$ for some state $z \in X$ and that all transient faults by $W(x)$ occur in non-fundamental mode operations. Then, there exists a corrective controller C in the structure of Fig. 2 that achieves fault recovery against any transient fault by $W(z)$ if and only if for all $a \in T(z)$ such that (z, a) is a proper transient combination,

- (i) (z, a) is fault detectable, and
- (ii) (z, a) is recoverable.

A formal proof of the above proposition is omitted. In general, a transient fault may occur in both fundamental and non-fundamental mode. Since fault diagnosis and fault

tolerant control against those faults occurring in fundamental mode are fully addressed in the prior work [5], [6], they can be easily combined with the presented result so as to specify the existence condition for a corresponding corrective controller.

IV. EXAMPLE

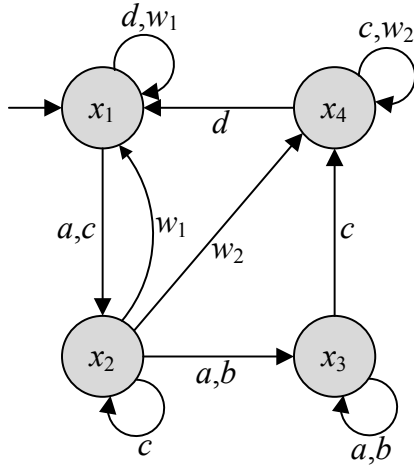


Fig. 5. Example machine Σ .

Consider a simple example input/state machine $\Sigma = (A, X, x_0, f)$ whose state-flow diagram is shown in Fig. 5, where

$$\begin{aligned} A_n &= \{a, b, c, d\} \\ A_d &= \{w_1, w_2\} \\ X &= \{x_1, x_2, x_3, x_4\} \\ x_0 &:= x_1. \end{aligned}$$

Here, we have

$$\begin{aligned} W(x_2) &= \{w_1, x_2\} \\ W(x_i) &= \emptyset, \quad \forall i = 1, 3, 4. \end{aligned}$$

Since only $W(x_2) \neq \emptyset$, we solely have to consider transient faults occurring at the transient state x_2 . Referring to Fig. 5, we can induce that (x_2, a) is a proper transient combination of the stable transition from (x_1, a) to (x_3, a) , i.e.,

$$\begin{aligned} P(x_2, a) &= \{x_1\}, \\ r(x_2, a) &= x_1 \\ \tau(x_1, a) &= \{x_1, x_2\} \\ s(x_1, a) &= s(x_2, a) = x_3. \end{aligned}$$

Note that the other transient combination (x_2, b) does not need to be considered since it is not proper.

We first investigate detectability of (x_2, a) . From Fig. 5, it is derived that

$$\begin{aligned} s(x_2, w_1) &= x_1 \in \tau(x_1, a) \\ s(x_2, w_2) &= x_4 \notin \tau(x_1, a). \end{aligned}$$

Thus (x_2, a) is not fault detectable by Definition 2. In particular, we cannot recognize an occurrence of w_1 since the deviated state x_1 is an intermediate transient state of the corresponding stable transition. Next, let us determine

recoverability of (x_2, a) . Referring to Fig. 5, we easily know that every state is stably reachable from another one, as Σ has a closed loop

$$x_1 \rightarrow \dots \rightarrow x_4 \rightarrow x_1.$$

Hence (x_2, a) is recoverable. In summary, a fault tolerant corrective controller can be designed that achieves fault recovery against w_2 ; fault recovery against w_1 is not possible.

V. CONCLUSION

In this paper, we have studied a fault tolerant control problem of asynchronous sequential machines subjected to transient faults occurring in non-fundamental mode. It has been found that the existence condition for a fault tolerant corrective controller is tighter than the case of fault recovery against transient faults occurring in fundamental mode. To describe the existence condition, detectability and recoverability of transient combinations have been defined and relevant properties have been studied. Based on the presented properties, the necessary and sufficient condition for the existence of an appropriate corrective controller has been addressed. We have also validated the proposed notions and schemes through the example. Following the preliminary result of the current study, we will continue our future research on presenting a design procedure of a fault tolerant corrective controller.

REFERENCES

- [1] T. E. Murphy, X. Geng, and J. Hammer, "On the control of asynchronous machines with races," *IEEE Trans. Automat. Control*, vol. 48, no. 6, pp. 1073–1081, 2003.
- [2] J. Peng and J. Hammer, "Bursts and output feedback control of non-deterministic asynchronous sequential machines," *Eur. J. Control*, vol. 18, no. 3, pp. 286–300, 2012.
- [3] N. Venkatraman and J. Hammer, "On the control of asynchronous sequential machines with infinite cycles," *Int. J. Control*, vol. 79, no. 7, pp. 764–785, 2006.
- [4] N. Venkatraman and J. Hammer, "Stable realizations of asynchronous sequential machines with infinite cycles," in *Proc. 2006 Asian Control Conf.*, Bali, Indonesia, pp. 45–51, 2006.
- [5] J.-M. Yang and J. Hammer, "State feedback control of asynchronous sequential machines with adversarial inputs," *Int. J. Control*, vol. 81, no. 12, pp. 1910–1929, 2008.
- [6] J.-M. Yang, "Corrective control of input/output asynchronous sequential machines with adversarial inputs," *IEEE Trans. Automat. Control*, vol. 55, no. 3, pp. 755–761, 2010.
- [7] J.-M. Yang, "Fault tolerance in asynchronous sequential machines using output feedback control," *IEEE Trans. Automat. Control*, vol. 57, no. 6, pp. 1604–1609, 2012.
- [8] J.-M. Yang and S. W. Kwak, "Fault tolerance in switched ASMs with intermittent faults," *IET Control Theory Appl.*, vol. 11, no. 9, pp. 1443–1449, 2017.
- [9] X. Xu and Y. Hong, "Matrix approach and model matching of asynchronous sequential machines," *IEEE Trans. Automat. Control*, vol. 58, no. 11, pp. 2974–2979, 2013.
- [10] B. Wang, J. E. Feng, and M. Meng, "Matrix approach to model matching of composite asynchronous sequential machines," *IET Control Theory Appl.*, vol. 11, no. 13, pp. 2122–2130, 2017.
- [11] J. Wang, X. Han, Z. Chen, and Q. Zhang, "Calculating skeleton matrix of asynchronous sequential machines based on the semi-tensor product of matrices," *IET Control Theory Appl.*, vol. 11, no. 13, pp. 2131–2139, 2017.
- [12] Z. Kohavi and N. K. Jha, *Switching and Finite Automata Theory*, 3rd ed., Cambridge, U.K.: Cambridge Univ. Press, 2010.