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Robot Motion Planning using Hyperboloid Potential Functions

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***Abstract*—A new approach to robot path planning using hyperboloid potential functions is presented in this paper. Unlike parabolic potential functions, where the control force increases with distance from the goal and is unbound, and conic potential functions where a singularity occurs at the goal, hyperboloid potential functions avoid both these drawbacks. However, they do combine the advantages of both parabolic and conic potentials as the asymptotic property of the hyperbolic function ensures bounded control forces, while stability and smooth contact are guaranteed at the goal point.**

***Index Terms*— Obstacle avoidance, potential functions, robot motion planning.**

I. Introduction

Potential field methods provide an elegant, simple and computationally efficient approach for single and multiple robot motion planning. The method was introduced first as a motion planning algorithm for manipulator arms [1], and then generalized for robot motion planning in dynamic environments [2]. Space applications of the potential field method have been developed for various tasks such as formation-flying [3], proximity manoeuvring [4], and on-orbit assembly [5], [6].

The potential field method constructs an attractive potential field that is responsible for directing a manoeuvring object toward its goal configuration. Collisions with other objects or obstacles in the workspace are avoided through constructing high potential fields surrounding them. Various functional forms of repulsive potential fields have been investigated, including: FIRAS [1], superquadric functions [7], navigation functions [8], Gaussian distributions and power laws [9], and the Laplace equation [10].

This paper introduces a new representation for the attractive potential field using hyperbolic functions. They provide key advantages over paraboloid or conic functions. In addition, superquadric obstacle potentials will be used where the orientation of the superquadric is defined using quaternions. As will be seen, rotational and translation motion then becomes strongly coupled, allow efficient manoeuvring [11].

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II. Translational Attractive Potential

A manoeuvring object is stimulated to move toward its goal configuration through an attractive potential field. Any function could be utilized providing it satisfies Lyapunov's stability conditions and its global minimum is placed at the goal configuration. Previously, two main types of attractive potential have been used: parabolic and conical [12]. The new hyperbolic function provides bounded control action while also providing smooth motion in the neighbourhood of the goal configuration.

A. Parabolic Attractive Potential

The parabolic function is commonly used in motion planning problems, as shown in Fig. 1. A manoeuvring object is defined with the position, $\mathbf{r}$, and velocity, $\dot{\mathbf{r}}$, and is required to move to a goal point at position, $\mathbf{r}_G$, with goal velocity, $\dot{\mathbf{r}}_G$. Defining the potential function as [13]:

$$V_{att,trans} = \frac{\lambda_p}{2}\left|\mathbf{r}-\mathbf{r}_G\right|^2 + \frac{\lambda_v}{2}\left|\dot{\mathbf{r}}-\dot{\mathbf{r}}_G\right|^2 \tag{1}$$

where λ_p and λ_v are constant gain factors, the time derivative of the parabolic potential function is expressed as:

$$\dot{V}_{att,trans} = \left(\dot{\mathbf{r}}-\dot{\mathbf{r}}_G\right).\left(\lambda_p\left(\mathbf{r}-\mathbf{r}_G\right)+\lambda_v\left(\ddot{\mathbf{r}}-\ddot{\mathbf{r}}_G\right)\right) \tag{2}$$

To ensure stability using Lyapunov's second theorem the time derivative should be non-positive everywhere in the workspace since the potential function is positive definite. A suitable control law is then expressed as:

$$\ddot{\mathbf{r}} = \ddot{\mathbf{r}}_G - \frac{\lambda_p}{\lambda_v}\left(\mathbf{r}-\mathbf{r}_G\right) - \lambda\left(\dot{\mathbf{r}}-\dot{\mathbf{r}}_G\right) \tag{3}$$

for some control gain λ. As the distance between the manoeuvring object and its goal increases, the required control force to ensure stability increases and is unbound. Hence, for real systems actuator saturation may occur and the stability of the problem is not guaranteed. To circumvent this problem a conical potential function can be used.

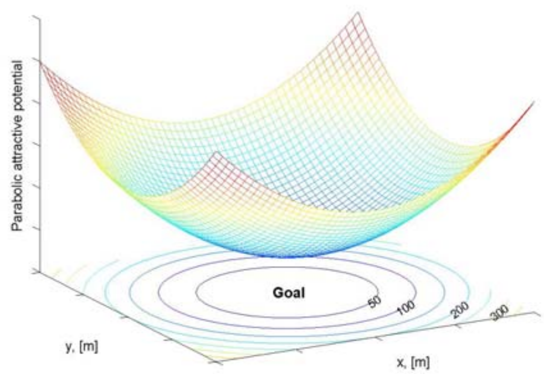

Fig. 1 Parabolic-well attractive potential

B. Conical Attractive Potential

The conical attractive potential does not have the difficulties discussed above, as shown in Fig. 2. It can be expressed as [13]:

$$V_{att,trans} = \lambda_p |\mathbf{r} - \mathbf{r}_G| + \frac{\lambda_v}{2} |\dot{\mathbf{r}} - \dot{\mathbf{r}}_G|^2 \tag{4}$$

The time derivative of the potential is then expressed as:

$$\dot{V}_{att,trans} = (\dot{\mathbf{r}} - \dot{\mathbf{r}}_G) \cdot \left(\lambda_p \frac{\mathbf{r} - \mathbf{r}_G}{|\mathbf{r} - \mathbf{r}_G|} + \lambda_v (\ddot{\mathbf{r}} - \ddot{\mathbf{r}}_G) \right) \tag{5}$$

Therefore, a suitable control law is then given by:

$$\ddot{\mathbf{r}} = \ddot{\mathbf{r}}_G - \frac{\lambda_p}{\lambda_v} \frac{\mathbf{r} - \mathbf{r}_G}{|\mathbf{r} - \mathbf{r}_G|} - \lambda (\dot{\mathbf{r}} - \dot{\mathbf{r}}_G) \tag{6}$$

The control force remains bound regardless of the location of the manoeuvring object. However, the goal point is singular.

C. Hyperbolic Attractive Potential

Continuous control of a mobile robot is carried out either by merging the parabolic and conical potentials each over a certain range, or now by using a hyperbolic function, as shown in Fig. 3. Near its global minimum, the hyperbolic function has a smooth shape like the parabola, while away from the minimum it becomes asymptotic with constant gradient like the conical field. The hyperbolic potential is described as:

$$V_{att,trans} = \lambda_p \left(\sqrt{1 + |\mathbf{r} - \mathbf{r}_G|^2} - 1 \right) + \frac{\lambda_v}{2} |\dot{\mathbf{r}} - \dot{\mathbf{r}}_G|^2 \tag{7}$$

The time derivative is then expressed as:

$$\dot{V}_{att,trans} = (\dot{\mathbf{r}} - \dot{\mathbf{r}}_G) \cdot \left(\frac{\lambda_p}{\sqrt{1 + |\mathbf{r} - \mathbf{r}_G|^2}} + \lambda_v (\ddot{\mathbf{r}} - \ddot{\mathbf{r}}_G) \right) \tag{8}$$

Finally, a suitable bounded, smooth and singularity-free control law is expressed as:

$$\ddot{\mathbf{r}} = \ddot{\mathbf{r}}_G - \frac{\lambda_p}{\lambda_v \sqrt{1 + |\mathbf{r} - \mathbf{r}_G|^2}} - \lambda (\dot{\mathbf{r}} - \dot{\mathbf{r}}_G) \tag{9}$$

The control law defined in Eq. (9) is significantly better than those defined in Eqs. (3) and (6) as the control force remains bounded whatever the distance to goal is, and the singularity at the goal is removed. Hence, global convergence is achieved.

The single hyperbolic function is also more computationally efficient to implement compared to the use of a combined conical and parabolic potential with a switching function in the neighbourhood of the goal. Again, the single function contains the key features of both the parabolic and conical fields.

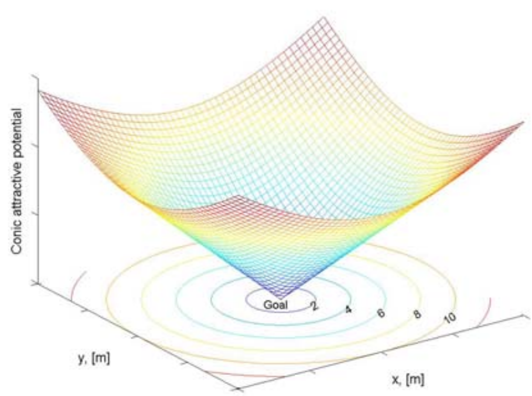

Fig. 2 Conic-well attractive potential

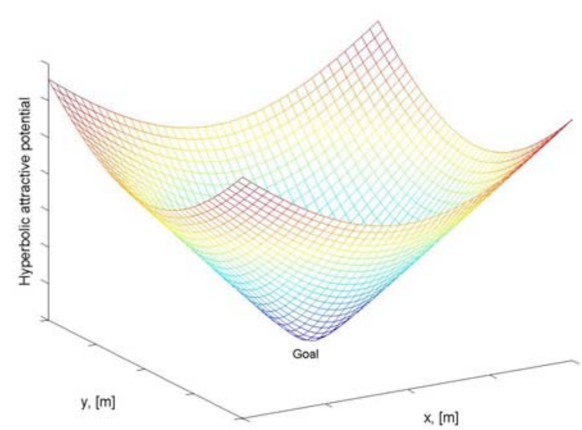

Fig. 3 Hyperbolic-well attractive potential

III. Global Potential Function

Translational motion of the manoeuvring object to the goal is not the sole objective of the potential function. Orientation is also of importance for extended rigid bodies manoeuvring to some goal orientation. In addition, collision avoidance with other manoeuvring objects and obstacles is a key requirement of the global potential function which is expressed as:

$$V = \lambda_p \left(\sqrt{1+\left|\mathbf{r}-\mathbf{r}_G\right|^2} - 1 \right) + \frac{\lambda_v}{2}\left|\dot{\mathbf{r}}-\dot{\mathbf{r}}_G\right|^2 + \lambda_q \overline{\mathbf{q}}.\overline{\mathbf{q}} + \frac{\lambda_\omega}{2}\boldsymbol{\omega}.\boldsymbol{\omega} + V_{obs} \tag{10}$$

where λ_{q} and λ_{ω} are constant gains, $\overline{\mathbf{q}} = \left[q_1 \;\; q_2 \;\; q_3 \right]^T$ is the quaternion vector representing orientation, omitting the forth term q_4, $\boldsymbol{\omega}$ is the angular velocity vector, and V_{obs} is the obstacle potential field.

The time derivative of the global hyperbolic potential is then expressed as:

$$\dot{V} = \left(\dot{\mathbf{r}}-\dot{\mathbf{r}}_G\right). \left(\frac{\lambda_p}{\sqrt{1+\left|\mathbf{r}-\mathbf{r}_G\right|^2}} + \lambda_v\left(\ddot{\mathbf{r}}-\ddot{\mathbf{r}}_G\right) + \nabla V_{obs} \right) + 2\lambda_q \dot{\overline{\mathbf{q}}}.\overline{\mathbf{q}} + \lambda_\omega \dot{\boldsymbol{\omega}}.\boldsymbol{\omega} + \dot{\overline{\mathbf{q}}}.\nabla^q V_{obs} \tag{11}$$

with $\nabla = \left[\partial/\partial x \;\; \partial/\partial y \;\; \partial/\partial z \right]^T$ and $\nabla^q = \left[\partial/\partial q_1 \;\; \partial/\partial q_2 \;\; \partial/\partial q_3 \right]^T$. Equation (11) can be simplified using the following relations:

$$\dot{\overline{\mathbf{q}}} = \frac{1}{2}\mathbf{Q}\boldsymbol{\omega} \tag{12}$$

while

$$\mathbf{Q} = \begin{bmatrix} q_4 & q_3 & -q_2 \\ -q_3 & q_4 & q_1 \\ q_2 & -q_1 & q_4 \end{bmatrix} \tag{13}$$

Finally, the time derivative can be expressed as

$$\dot{V} = \left(\dot{\mathbf{r}}-\dot{\mathbf{r}}_G\right). \left(\frac{\lambda_p}{\sqrt{1+\left|\mathbf{r}-\mathbf{r}_G\right|^2}} + \lambda_v\left(\ddot{\mathbf{r}}-\ddot{\mathbf{r}}_G\right) + \nabla V_{obs} \right) + \boldsymbol{\omega}. \left(\lambda_q \mathbf{Q}\overline{\mathbf{q}} + \lambda_\omega \dot{\boldsymbol{\omega}} + \frac{1}{2}\mathbf{Q}\nabla^q V_{obs} \right) \tag{14}$$

Hence, the control laws are expressed as:

$$\ddot{\mathbf{r}} = \ddot{\mathbf{r}}_G - \frac{\lambda_p}{\lambda_v\sqrt{1+\left|\mathbf{r}-\mathbf{r}_G\right|^2}} - \lambda\left(\dot{\mathbf{r}}-\dot{\mathbf{r}}_G\right) - \frac{1}{\lambda_v}\nabla V_{obs} \tag{15-a}$$

and

$$\dot{\boldsymbol{\omega}} = -\left(\frac{\lambda_q}{\lambda_\omega} q_4 \overline{\mathbf{q}} + \lambda_\omega \boldsymbol{\omega} + \frac{1}{2\lambda_\omega}\mathbf{Q}\nabla^q V_{obs} \right) \tag{15-b}$$

The control laws defined by Eq. (15) are of a general form for any repulsive potential. Superquadric obstacle potentials are chosen for the remainder of the paper [11]. These functions provide a means of capturing the geometric shape of extended rigid bodies and allow coupled translational and rotational motion planning.

IV. Numerical Results

The hyperbolic potential field is used with continuous control to assemble seven beam elements to form a truss structure. Objects are initially placed parallel to the *z*-axis, Fig. 4. The assembly of the objects is demonstrated in Fig. 5, where Fig. 6 shows the evolution of the object dynamics. Noting the log-scale, it can be seen that a smooth acceleration- coast-braking profile is generated by the control law.

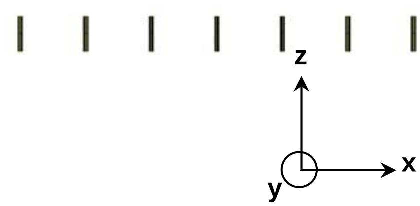

Fig. 4 Initial object configuration

Fig. 5-a) Object configuration (t = 37 sec)

Fig. 5-b) Object configuration (t = 120 sec)

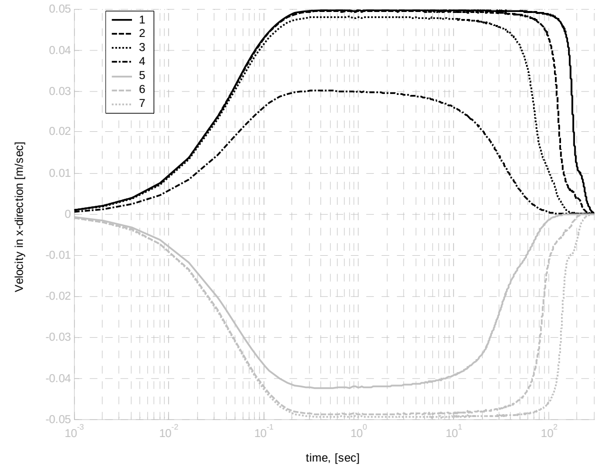

Fig. 6-a) Object velocities in *x*-direction

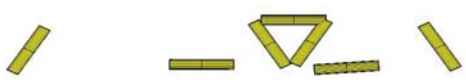

Fig. 5-c) Object configuration (t = 180 sec)

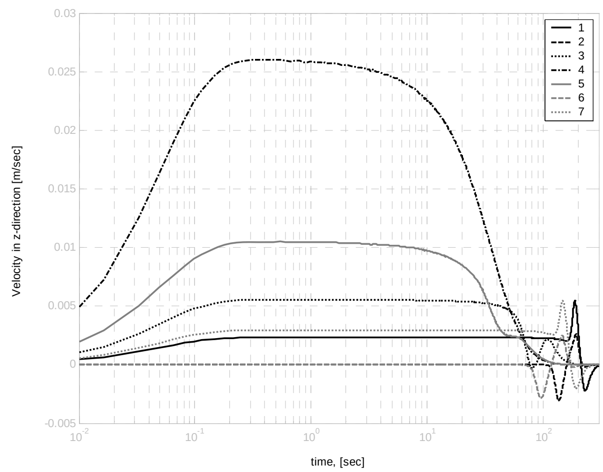

Fig. 6-b) Object velocities in *z*-direction

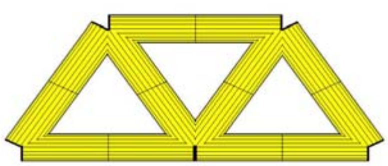

Fig. 5-d) Final object configuration (t = 300 sec)

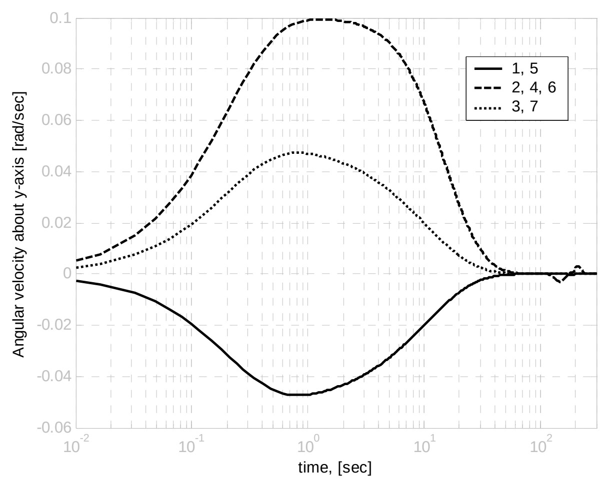

Fig. 6-c) Object angular velocities about *y*-axis

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V. CONCLUSION

Adding a velocity term to a hyperbolic potential function provides successful continuous control with bounded control action. The resulting controlled velocities are nearly constant over the entire workspace, except in the neighbourhood of obstacles. Global stability and convergence of the system is proven and tested for a dense workspace. Proximity motion of the manoeuvring objects shows the coupling between translational and rotational motion in the presence of obstacles.

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