Computer Applications to Structural Analysis of Pavements

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Abstract—**Surface wave method can be used to determine the properties of the component materials of a pavement structure. The measurements are made of the differences in the phase of a wave of the Rayleigh type, received at two points at known radial spacings from the source of the wave. Pavements for highways or airport runways are usually constructed of cement concrete. Such pavements are treated as a single layer of a constant thickness overlying a semi-infinite medium. The system is dispersive and inverse methods can be employed to determine the properties of the materials of the component layers of the structure. Algebraic inverse method has been developed and employed to conduct the inversion process of the experimental dispersion curve in order to back calculate the unknown parameters of the system.**

Index Terms— **Computer application, Pavement, Surface wave, Inversion.**

I. INTRODUCTION

A pavement system is a layered half space with the deepest layer extending to infinite depth. The determination of in-situ elastic properties of pavement layers materials is of great importance in pavement engineering and pavement management system. The stiffness moduli of pavement layers are required in characterizing materials, designing constructed layers, assessing structural capabilities of existing pavements and evaluating pavement conditions and rehabilitation needs.

Pavements respond to dynamic loads by transmitting waves through the pavement structure. It is logical to use the results of measurements of wave propagation for pavement testing and analysis. The wave propagation method, as a nondestructive testing procedure, is an effective, fast and economic technique to deduce the necessary information for pavement analysis. Surface waves of the Rayleigh type are used for this purpose. The waves are generated artificially, by means of a hammer blow applied to the surface of the pavement.

In an infinite, homogeneous, linear elastic material two types of wave may occur, compressional waves and shear waves. The velocities of these waves will depend on the elastic parameters and on the density of the material. In a layered system, the phase velocity is not constant, but varies with the frequency.

The manner of the variation is used to determine the required parameters of the system (the velocity of wave propagation in each medium, and the thickness of each layer). Wave velocity is a direct indication of the elastic stiffness of the material, i.e. higher wave velocities are associated with higher stiffnesses and vice versa. Thus the full range of the materials used in the pavement layers can be assessed by this method.

The results of measurements are interpreted with the aid of inverse methods. Solutions of the relevant wave equation are used to express the surface phase velocities in terms of the frequency. The resulting expressions are inversed in order to determine the elastic constants of the components of the pavement structure. The solution of these problems is facilitated by the use of computer-based algebra.

II. WAVE METHOD

Waves of the Rayleigh type are generated by applying an impulsive load, a hammer blow, at right angles to the surface of the pavement. The measurements are made of the differences in the phase of the Rayleigh wave, received at two points at known radial spacings from the source of the wave, from which the elastic properties and thicknesses of the component materials of the pavement layers are derived. Two vertical sensors (usually accelerometers) are placed on the surface of the system at known horizontal distances to monitor the vertical ground motions induced by the hammer. The analog motions captured by sensors are amplified and converted into a digitized form by the Analog-to-Digital (A/D) converter. A wide range of PC-based data acquisition and control systems provide facilities for these kinds of readings and conversions and permit analysis of data with the help of software. On completion of the A/D conversion, the digitized signals are transformed from the time domain to the frequency domain by the Fast Fourier Transform (FFT).

The velocity of the generated wave is determined as a function of the frequency, enabling a frequency-dispersion curve to be plotted. This curve shows the phase velocity of a wave at the surface of the structure. The frequency-dispersion data (experimental dispersion curve) can be used to determine the properties of the media of which the structure is composed. The properties can be determined by the application of inverse methods to the results obtained through the establishment of the experimental dispersion curve.

III. INVERSION PROCESS

The problem is to determine the properties of the component materials of the pavement structure. The inversion process is used to back calculate the unknown parameters of the system from the data measured in the field and presented in the form of the experimental dispersion curve.

Most of the inverse techniques developed thus far are based on the numerical methods. With the numerical inverse method, an initial model is assumed for the system, and the theoretical dispersion curve is obtained by determining the phase velocity at different frequencies using the elastic theory of wave propagation in layered media. In most applications, values of the unknown parameters of the system are guessed by trial and error, and the forward calculation is repeated until the agreement between the calculated and the measured results is sufficiently close. Once an adequate match between the experimental dispersion curve and the theoretical one for the trial model is established, the last modified profile is accepted as a solution. However this process is time-consuming, greatly dependent on the experience of the user and requires engineering judgment. Ideally, a true mathematical inverse should be found.

Graphical inverse methods sometimes suffice for obtaining approximate results (Akhlaghi, 2004), although most systems must be analyzed with the aid of relevant solutions of the wave equation of the particular system studied. The simplest assumption is that the structure consists of a free plate, which is far from reality. A free plate is an idealized, rather than a truly physically-realizable, model but it is a good approximation to a number of practical configurations (Akhlaghi & Cogill, 1996). By employing the free plate solutions and applying to a single-layered system, it is possible to determine some unknown parameters of the medium from the characteristic equation of the system.

A layered system is a more accurate model of a pavement than a free plate. The system described by Lee (1932) is composed of a single layer overlying a semi-infinite medium. This system is discussed in the following section. The use of computerized algebra offers the means of obtaining a true inverse. The result is obtained in the form of a polynomial. The application of such a method is proposed to obviate the need for trial and error and for numerical iterative solutions.

IV. SYSTEM EQUATION

Pavements contain a surface layer having a stiffness which is higher than that of the underlying medium. As the difference in the densities is not significant, the compressional wave velocity (α) and shear wave velocity (β) are higher in the surface layer. Then:

$$k^2 > k_{\beta 1}^2 > k_{\alpha 1}^2 \tag{1}$$

where:

$$k = \frac{\omega}{c} \quad ; \quad k_\alpha = \frac{\omega}{\alpha} \quad ; \quad k_\beta = \frac{\omega}{\beta} \tag{2}$$

ω = angular frequency; and c = phase velocity.

Let:

$$r_1^2 = k_{\alpha 1}^2 - k^2 \quad ; \quad r_2^2 = k^2 - k_{\alpha 2}^2 \tag{3}$$

$$s_1^2 = k_{\beta 1}^2 - k^2 \quad ; \quad s_2^2 = k^2 - k_{\beta 2}^2 \tag{4}$$

Suffixes 1 and 2 denote the surface layer and the underlying medium respectively.

The values of r_1 and s_1 are imaginary, and it is convenient, following Lee (1932), to change the notation. We substitute:

$$R^2 = k^2 - k_{\alpha 1}^2 \quad ; \quad S^2 = k^2 - k_{\beta 1}^2 \tag{5}$$

The determinant of the system can be written (Ewing *et al.* 1957) as:

$$\xi_1 \eta_2 - \xi_2 \eta_1 = 0 \tag{6}$$

where:

$$\xi_1 = (2 - \frac{k_{\beta 1}^2}{k^2})[X \cosh Rh + \frac{r_2}{R} Y \sinh Rh] \\ - 2\frac{S}{k}[\frac{r_2}{k} W \sinh Sh + \frac{k}{S} Z \cosh Sh] \tag{7}$$

$$\xi_2 = (2 - \frac{k_{\beta 1}^2}{k^2})[\frac{s_2}{k} W \cosh Rh + \frac{k}{R} Z \sinh Rh] \\ - 2\frac{S}{k}[X \sinh Sh + \frac{s_2}{S} Y \cosh Sh] \tag{8}$$

$$\eta_1 = (2 - \frac{k_{\beta 1}^2}{k^2})[\frac{r_2}{k} W \cosh Sh + \frac{k}{S} Z \sinh Sh] \\ - 2\frac{R}{k}[X \sinh Rh + \frac{r_2}{R} Y \cosh Rh] \tag{9}$$

$$\eta_2 = (2 - \frac{k_{\beta 1}^2}{k^2})[X \cosh Sh + \frac{s_2}{S} Y \sinh Sh] \\ - 2\frac{R}{k}[\frac{s_2}{k} \sinh Rh + \frac{k}{R} Z \cosh Rh] \tag{10}$$

where *X, Y, Z, W* are the terms introduced by Love (1911) as follows:

$$X = \frac{\mu_2}{\mu_1}\frac{k_{\beta 2}^2}{k^2} - (\frac{\mu_2}{\mu_1} - 1) \quad ; \quad Y = \frac{k_{\beta 1}^2}{k^2} + 2(\frac{\mu_2}{\mu_1} - 1) \tag{11}$$

$$Z = \frac{\mu_2}{\mu_1}\frac{k_{\beta 2}^2}{k^2} - \frac{k_{\beta 1}^2}{k^2} - (\frac{\mu_2}{\mu_1} - 1) \quad ; \quad W = 2(\frac{\mu_2}{\mu_1} - 1)$$

If it is assumed that $\lambda_1 = \mu_1$, $\lambda_2 = \mu_2$ and that h is small, then

$$R^2 = k^2 - \frac{1}{3}k_{\beta 1}^2 \quad ; \quad S^2 = k^2 - k_{\beta 1}^2 \tag{12}$$

and (7) to (10) are as follows:

$$\xi_1 = (\frac{k_{\beta 1}}{k^2})[2hr_2(1+W)+(2-X)] \quad (13)$$

$$\xi_2 = (\frac{k_{\beta 1}^2}{k^2})[hk(2X-2-Z)-\frac{s_2}{k}(2+W)] \quad (14)$$

$$\eta_1 = (\frac{k_{\beta 1}^2}{k^2})[hk(\frac{2}{3}Z-2-Z)-\frac{s^2}{k}(2+W)] \quad (15)$$

$$\eta_2 = (\frac{k_{\beta 1}^2}{k^2})[hs_2(\frac{2}{3}W+2-Y)+(2-X)] \quad (16)$$

By applying (6), (13) to (16) lead to a cubic in k^2 which can be used to solve for k in terms of the frequency ω. The frequency ω is defined implicitly in the remaining independent variables in (7) to (10). The relevant solution must be selected.

V. COMPUTER APPLICATIONS

The following segment of a forward program in Fortran is used to obtain values of the wavelength for given values of the surface phase velocity.

```
   250 CONTINUE
   R1H = (2./V)*R1K
   KOUNT = KOUNT + 1
   R2H = (2./V)*R2K
   S1H = (2./V)*S1K
   S2H = (2./V)*S2K
C*********************************************************************
   XI1 = (2.-CB1*CB1)*(X*COS(R1H)+R2R1*Y*SIN(R1H)
  1     +2.*S1K*(R2K*W*SIN(S1H)-XKS1*Z*COS(S1H)
   XI2 = (2.-CB1*CB1)*(S2K*W*COS(R1H)+XKR1*Z*
  1   SIN(R1H) +2.*S1K*(X*SIN(S1H)-S2S1*Y*COS(S1H)
C*********************************************************************
   ETA1 = (2.-CB1*CB1)*(R2K*W*COS(S1H)+XKS1*Z*
  1             SIN(S1H)   +2. *R1K*(X*SIN(R1H)-
R2R1*Y*COS(R1H)
   ETA2 = (2.-CB1*CB1)*(X*COS(S1H)+S2S1*Y*SIN(S1H)
  1     +2.*R1K*(S2K*W*SIN(R1H)-XKR1*Z*COS(R1H)
C*********************************************************************
   XITEM1 = XI1*ETA2
   XITEM2 = XI2*ETA1
C*********************************************************************
   DEL = XITEM1 – XITEM2
```

Typical results for the fundamental mode are shown in Fig. 1. This figure shows the reciprocal of wavelength plotted against the frequency. The slope of the curve yields the reciprocal of the limiting phase velocities at short and at long wavelengths. As the expression contains trigonometric functions, higher modes exist but are likely to be suppressed by material damping.

The inverse of the results shown in Fig. 1 is needed for analyzing field measurements. The following code is developed for use with Mathematica (Wolfram, 1992). It has been used to calculate a thickness 'h' corresponding with successive pairs of values of phase velocity and frequency as data. It yields an inverse to the Fortran Program above.
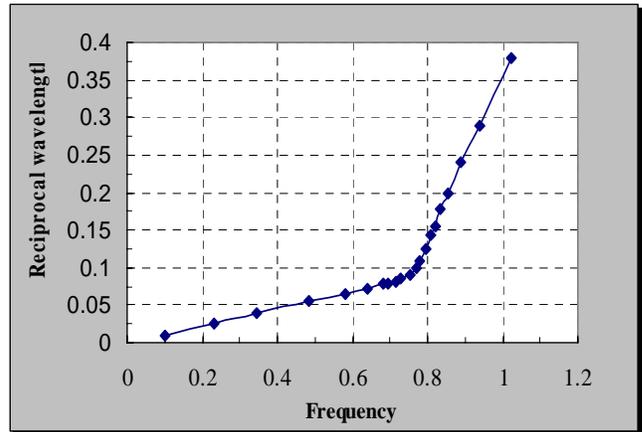

Figure 1: Fundamental mode dispersion data for single layer overlying a semi-infinite medium

(* The following is a definition of the period equation and leads to a polynomial 'y' in h to the power of nine *).

k = w / c ;   kb1 = w / b1 ;   kb2 = w / b2 ;

r1 = (- 1 / c ^ 2 + 1 / a1 ^ 2) ^(1/2) ;
s1 = (- 1 / c ^ 2 + 1 / b1 ^ 2) ^(1/2) ;
r2 = (+ 1 / c ^ 2 - 1 / a2 ^ 2) ^(1/2) ;
s2 = (+ 1 / c ^ 2 - 1 / b2 ^ 2) ^(1/2) ;

(* The definition of the ancillary variables follows *)

ww=2(mu2/mu1-1) ;
xx=mu2/mu1 kb2^2/k^2 – ww ;
yy=kb1^2/k^2 + ww ;
zz=mu2/mu1 kb^2/k^2 – kb1^2 / k^2 – ww ;

pow := 9 ; (* Set the degree of polynomial *)

x1= ((2-kb1^2/k^2) (xx Cos [w r1 h] + r2/r1 yy Sin [w r1 h])
     + 2 s1/k (r2/k ww Sin [w s1 h] – k/s1 zz Cos [w s1 h]
))
x2=((2-kb1^2/k^2) (s2/k ww Cos[w r1 h] + k/r1 zz Sin[w r1 h])
     + 2 s1/k (xx Sin [w s1 h] – s2/s1 yy Cos [w s1 h]))
e1=((2-kb1^2/k^2)(r2/k ww Cos[w s1 h] + k/s1 zz Sin[w s1 h])
     + 2 r1/k (xx Sin [w r1 h] – r2/r1 yy Cos [w r1 h]))
e2= ((2-kb1^2/k^2) (xx Cos [w s1 h] + s2/s1 yy Sin [w s1 h])
     + 2 r1/k (s2/k ww Sin [w r1 h] – k/r1 zz Cos [w r1 h]))

(* Expand each of the elements in a series, and simplify *)

xi1 = Factor [Normal [Series [x1, {h, 0, pow}]]] ;
xi2 = Factor [Normal [Series [x2, {h, 0, pow}]]] ;
eta1 = Factor [Normal [Series [e1, {h, 0, pow}]]] ;

eta2 = Factor [Normal [Series [e2, {h, 0, pow}]]] ;

(* Now write the determinant, to be solved by reversion for the thickness, h *)

f = Normal [Series [{ xi1 eta2 – xi2 eta1 } , {h, 0, pow}]] ;
y = Simplify [Factor [f]] ;

(* 'y' is a polynomial, and the coefficients of hˆn can be extracted by a Coefficient [y, h, n] command *)

z = -Coefficient [y, h, 0]/Coefficient [y, h, 1] ;

(* Independent variable of the reversion *)

y1 = y - Coefficient [y, h, 0] ;
y2 = y1/ Coefficient [y1, h, 1] ;

(* Subtract the constant term from the series, and arrange that hˆ1 has coefficient of unity *)

The series y2 is reversed in order to obtain a series for the layer thickness h. The result is obtained in terms of the velocities of compressional and shear waves in the component media (a1, a2, b1, b2), the measured values of frequency and the corresponding surface phase velocity (the data pairs w and c). The reciprocal slopes of the plot in Fig. 1 are the limiting velocities of waves of the Rayleigh type in two component materials. If a value of Poisson's ratio is assumed, the parameters a1, a2, b1 and b2 can be estimated from the field results. The series represents the behavior of a physical system, and it is expected that it must be convergent. However, as in the case of forward program, an infinite number of modes exists and the relevant one must be selected. The solution shown can be adapted to the case of a layer of material having a higher stiffness overlying a semi-infinite medium of material having a lower stiffness.

VI. SUMMARY AND CONCLUSIONS

Surface wave method can be used to determine the mechanical and physical properties of the materials of a layered structure such as highway pavements and airport runways. The dispersion curve of Rayleigh waves can be determined in the field with the aid of measurements of ground motions induced by a hand hammer. The phase velocity is not constant, but varies with the frequency. The manner of the variation is used to determine the required parameters of the system using one of the inverse methods.

Inverse problems in relation to the study of layered systems such as highway pavements have been discussed, and numerical results have been demonstrated. The inverse can be obtained numerically, by trial and error. Values of the unknown parameters of the system are guessed, and the forward calculation is repeated until the agreement between the calculated and the measured results is sufficiently close. This method is time-consuming and requires experienced operator and engineering judgment. Also graphical inverse method is available; however it is approximate procedure and is applicable only to simple cases. Ideally, the inverse should be obtained by means of a true mathematical inverse. This inverse method has been developed for the case of single layer pavement system during this research work. The frequency equation for the system is expanded as a polynomial, and is solved for the unknown parameters of the system. The unknowns are usually the layers thicknesses and the elastic moduli of the materials within each layer. The unknowns can be calculated directly and no iteration is required.

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