Phase-Modulations Analyses in Coherent Homodyne Optical CDMA Network Using a Novel Prime Code Family

M. M. Karbassian, *Member, IAENG* and H. Ghafouri-Shiraz

Abstract— **This paper examines the bit-error rate (BER) for an optical direct-detection code-division multiple-access (OCDMA) system employing a novel prime code family, hereby referred to as 'Double-Padded Modified Prime Code (DPMPC)' for the spreading and de-spreading operations in addition to coherent homodyne dual-balanced detection. As a coherent modulation, binary phase-shift keying (BPSK) format is deployed where the phase is modulated by either an external phase-modulator or injection-locking methods. The phase limitation and the performance for both phase-modulations including multiple access interferences (MAI) in a shot-noise limited regime are studied and moderate penalty associated with the limited phase excursion is revealed.**

Index Terms—**Coherent balanced-detection, injection-locking, limited phase excursion, multiple access interference (MAI), optical code-division multiple-access (OCDMA).**

I. INTRODUCTION

Coherent time spreading optical code division multiple access (OCDMA) as a multiple access protocol that takes advantage of the excess bandwidth in single-mode optical fiber has attracted a lot of attention because of the superior performance over incoherent schemes [1]-[3]. It maps low-rate electrical or optical signals into high-rate optical sequences to achieve random multiple access. In OCDMA, many nodes transmit simultaneously on a common frequency band.

Accordingly, signals must be designed to reduce mutual interferences. In order to achieve this result, each data bit is subdivided into a number of binary "chips". The effect of this further modulation is to spread the signal's frequency spectrum. The chip sequence constitutes a code that permits a bit stream broadcast on a network to be selected by means of a correlation process at the receiver destination. A large number of chip codes can be assigned to different users, in the process of which the set of optical sequences essentially becomes a set of address codes or signature sequences for the network. Several architectures have been considered for the use of CDMA within an optical fiber, the most common systems use direct-detection with bipolar codes such as Gold sequences; whereas, unipolar codes like prime code families, particularly the novel proposed one (section II), have more flexible code-length and are almost orthogonal. To retain the advantage of 0/1 codes as a power saving option, we consider in this study unipolar signaling along with the employing of binary phase shift keying (BPSK) as a coherent modulation. The capacity of the system using prime codes is limited by the maximum achievable bit-rate of the electronic circuitry generating the pseudo-noise (PN) sequences. We settle on a maximum attainable chip-rate of 10Gchip/s and a desired bit-rate of hundreds of Mbps, leading to a limit in the length of the spreading sequences on the order of hundreds of chips per bit.

In this study, the analyses as the signal-to-noise ratios of optical CDMA systems using a novel prime code family and two phase modulation methods with coherent detection are investigated. The performance penalty imposed on the OCDMA system as a result of the limited phase excursion of ±0.42π introduced in [4] is also evaluated.

II. 'DOUBLE-PADDED MODIFIED PRIME CODE (DPMPC)' STRUCTURE

According to the various prime code families introduced in [5]-[7], the proposed optical signature sequence (DPMPC) is generated through repeating (padding) firstly, the final sequence-stream of the Modified Prime Code (MPC) [5] sequence and secondly the final sequence-stream of the previous modified prime sequence (rotating) in the same group. In fact, the padding order can also be applied vice versa, whereas if the padding order changes during the code generation, the cross-correlation value also changes into undesirable values (increases); therefore, the padding order has to be followed for the whole code sequences. It is now clear where the expression 'Double-Padded' comes from. Finally, these two sequences are padded into each MPC sequence and consequently the code enlarges by *2P* as compared to MPC and by *P* as compared to both Padded Modified Prime Code (PMPC) [6] and new-Modified Prime Code (nMPC) [7]. This implies an increase in the chip-rate (processing factor in spreading) which makes the proposed new code more secure (i.e. less or no interception) and also permits the OCDMA system to operate at higher bit-rate. It is necessary to note that

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the padded sequences cannot only be the final sequence-stream of MPC but also they can be any stream of MPC sequences. This is due to the uniqueness of each MPC sequence-stream that makes each code matchless against each other. The new code-family has P groups, each of which has P sequence codes. The length of each code is (P^2+2P) and the weight is ($P+2$). The total number of available sequences is P^2. Table I shows an example of the DPMPC for $P=3$. Referring to Table I each code consists of two parts, MPC and group sequence-stream (GSS) parts. For example, in sequence C_{10} the MPC part is '100 010 001' and its GSS part is '001 010' which comprises the last sequence-stream of C_{10} and C_{12},

i.e.: $C_{10} \rightarrow 100\ 010\ 001\ (MPC)\ +\ 001\ 010\quad (GSS)$

Similarly, by adding the '001' (the last part of C_{10}, MPC sequence-stream) to the last sequence-stream of C_{11} we can generate C_{11} of DPMPC. Since synchronous OCDMA is considered, the auto- and cross-correlation function C_{mn} for any pair of codes m and n at synchronous time (T) is given by,

$$C_{mn} = \begin{cases} P+2, & \text{if } m=n \\ 0, & \text{if } m \neq n, m \text{ and } n \text{ share the same group} \\ 1, & \text{if } m \neq n, m \text{ and } n \text{ are from different groups} \end{cases} \quad (1)$$

where $m,n \in \{1,2,...P^2\}$. Figures (1) to (3) illustrate the correlation function C_{mn} for codes of $P=5$.

In Figure (1), the auto-correlation values of C_{41} are displayed at each chip synchronization positions. As clearly shown in Figures (2) and (3), the cross-correlation values of different codes at exactly synchronized time (T) is '0' for the codes in a same group and '1' for those are in different groups. As an example, data stream '11010' is followed by the code sequences to show how it works. In Figure (1) at each synchronous time T, the system follows the data at its maximum value (auto-correlation), while in Figures (2) and (3) at minimum (cross-correlation). If either correlation value or data is '0' the output is also '0' at each synchronized time of T; since the code and data are multiplied.

III. Coherent OCDMA System Architecture

In this optical direct-detection CDMA system, the outgoing data is first encoded using proposed prime code sequences. The encoded data is then modulated using an external phase modulator; this corresponds to multiplication of the optical carrier and enables the system to employ codes based on in-phase correlations. The use of an external Mach-Zehnder modulator for this application was verified experimentally in [8]. At the intended receiver, another phase modulator is used to demodulate the received signal using the same prime code sequence used for encoding at the transmitter. Based on the correlation properties of the selected family of prime codes, the signal that has been modulated twice by the same code will de-spread, while signals modulated by different codes will be further spread in the frequency domain and hence removed. The

TABLE I
Double-Padded Modified Prime Codes for P=3

Codes	MPC			GSS Part	
C_{00}	100	100	100	100	010
C_{01}	001	001	001	001	100
C_{02}	010	010	010	010	001
C_{10}	100	010	001	001	010
C_{11}	010	001	100	100	001
C_{12}	001	100	010	010	100
C_{20}	100	001	010	010	001
C_{21}	001	010	100	100	010
C_{22}	010	100	001	001	100

optical phase modulator could be followed by a Fabry-Perot optical filter that rejects unwanted crosstalk originating form the multiuser interference outside the filter bandwidth. The optical output of the filter consists of the desired signal and residual multiple access interference.

In coherent detection OCDMA system, a high quality reference laser is centrally located and individual users use this reference for injection-locking of inexpensive distributed feed-back (DFB) lasers to act as a source for transmission and also serve as the local oscillator for coherent homodyne detection. The phase is modulated by controlling the injection current to the DFB laser. In this case, data is phase modulated by a prime sequence and transmitted over the network. At the receiver a DFB laser is slaved to the reference laser, phase modulated by the original sequence and then combined coherently with the received CDMA signal. Since the same source is used for transmission and reception we have homodyne detection. The coherently mixed optical signals are incident on a dual-balanced detector whose electrical output conserves phase information. The possible bipolar electrical signal is integrated over a bit interval and the result compared to a threshold to form the final bit estimate. When phase modulation is achieved by an external modulator, chip-rate can range to the maximum of 10Gchip/s; in contrast, the injection current adjusted to achieve phase modulation has the maximum chip-rate of the order of 1Gchip/s due to the limited phase excursion [4]. This reduction in chip-rate may be justified to avoid the expense of an external modulator.

On the other hand, as mentioned, if we pose the limit of 10Gchip/s to the chip-rate and want to keep the bit-rate sufficiently high (hundreds of Mbps), we are limited to spreading sequences having lengths in the order of hundreds. As Gold sequences are $N = 2^n - 1$ chips long, with n being an odd integer, we have limited the results to $n = 9$, i.e. a length of 511, whereas with DPMPC there are two steps further where P equals 23 and 29 with achievable code-lengths ($N=P^2+2P$) of 575 and 899 respectively.

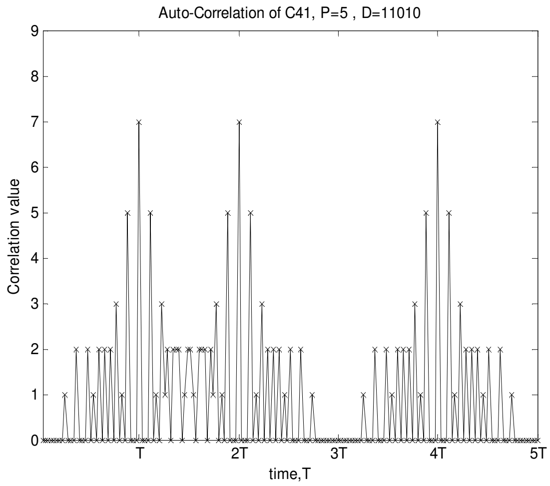

Fig. 1. Auto-correlation values of the C_{41} DPMPC for data stream of '11010'. (*T* is synchronization time)

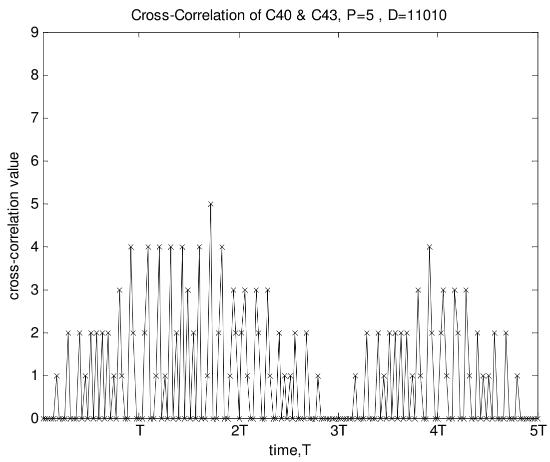

Fig. 2. Cross-correlation values of C_{40} and C_{43} (Same Group) DPMPC for data stream of '11010'.(*T* is synchronization time)

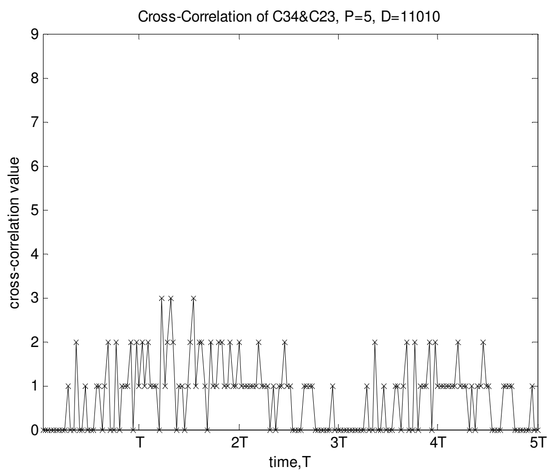

Fig. 3. Cross-correlation values of C_{34} and C_{23} (Different Groups) DPMPC for data stream '11010'. (*T* is synchronization time)

IV. External Phase Modulation with Dual-Balanced Detection

To conserve information contained in the phase of the optical carrier, coherent detection is used whereby a local optical source is coherently combined with the received information-bearing signal. In homodyne detection the local oscillator is at the carrier wavelength and the output electrical signal is at baseband. To eliminate the direct current (DC) component in this baseband signal a dual-balanced detector is used. Obviously, a local oscillator with much greater power than the total received signal results in a shot-noise limited regime where dark current and receiver thermal noise are negligible.

The first step is to examine the equations that govern the electrical output of a dual-balanced detector in a CDMA system. Let *K* be the number of active users and $S_i(t-\tau_i)$ be the signature sequence of user *i*. Let τ_i be the relative time-delay between users *i* and the desired user (e.g. user #1). The use of a unipolar spreading sequence is assumed; so that the sequence takes values from the set $\{0,1\}$ and also all users have roughly the same average power given by $\hat{S}^2$ (they travel nearly the same distance in the fiber and suffer the same losses due to dispersions) plus all users are received with the same polarization. At any instant of time *t*, $C_i(t)$ denotes the piecewise-constant function which is the product of the bit value and the sequence value of user *i* at time *t*, thus also taking values from the set $\{0,1\}$. The initial phase offset of user *i* is θ_i, a random variable uniformly distributed over the interval $(0,2\pi)$. The received signal is given by:

$$s(t) = \sqrt{2}\hat{S}\sum_{i=1}^{K} C_i(t)\cos(\omega_c t + \theta_i) \qquad (2)$$

or in a phasor format as:

$$S = \hat{S}\sum_{i=1}^{K} e^{j(\omega_c t + C_i(t).\pi/2 + \pi/2 + \theta_i)} \qquad (3)$$

The local oscillator will be modulated by the spreading sequence of the desired user, $S_1(t)$. The phasor form of the local oscillator signal is then given by:

$$L = \hat{L}\sum_{i=1}^{K} e^{j(\omega_c t + S_1(t).\pi/2 + \pi/2 + \theta_{LO})} \qquad (4)$$

The output of the dual-balance detector is hence:

$$\begin{aligned}\frac{2\eta e}{h\nu}\mathrm{Re}[LS] &= \frac{2\eta e}{h\nu}\hat{L}\hat{S}\sum_{i=1}^{K}\cos\left[(S_1(t) - C_i(t)).\pi/2 + \theta_{LO} - \theta_i\right] \\ &= \frac{2\eta e}{h\nu}\hat{L}\hat{S}\sum_{i=1}^{K} S_1(t).C_i(t).\cos(\theta_{LO} - \theta_i)\end{aligned} \qquad (5)$$

where η is the (common) quantum efficiency of the two detectors, *h* is Planck's constant, *e* is the fundamental charge of an electron, and ν is the employed optical frequency. Note that we assume the local oscillator tracks the phase of the desired user, so without loss of generality we use $\theta_{LO} = \theta_1 = 0$. As mentioned earlier, when a strong local oscillator is employed, the noise at the output is shot-noise limited and has one-sided power spectral density of:

$$N_0 = \frac{2\eta \mathrm{e}}{h\nu}\hat{L}^2 \qquad (6)$$

The output of the detector (equ.(5)) is integrated over a bit interval, leading to the following statistic [4]:

$$\begin{aligned} S_{out} &= \frac{2\eta \mathrm{e}}{h\nu}\hat{L}\hat{S}\sum_{i=1}^{K}\int_{0}^{T} S_1(t).C_i(t)\,dt + \sqrt{N_0}\int_{0}^{T} n(t)\,dt \\ &= \frac{2\eta \mathrm{e}}{h\nu}\hat{L}\hat{S}.b_0^1.T + \sqrt{\frac{\eta \mathrm{e}^2 T}{h\nu}}\hat{L}.n \\ &+ \frac{2\eta \mathrm{e}}{h\nu}\hat{L}\hat{S}\sum_{i=2}^{K}\left[b_{-1}^{i}.R_{i,1}(\tau_i) + b_0^{i}.\hat{R}_{i,1}(\tau_i)\right]\cos\theta_i \end{aligned} \quad (7)$$

where b_0^1 represents the information bit being detected and b_{-1}^i is the overlapping previous bit of user *i*, and b_0^i is the overlapping following bit of user *i*. The continuous-time partial cross-correlation functions are defined as:

$$\begin{aligned} R_{i,j}(\tau) &= \int_{o}^{\tau} s_i(t-\tau).s_j(t)\,dt \\ \hat{R}_{i,j}(\tau) &= \int_{\tau}^{T} s_i(t-\tau).s_j(t)\,dt \end{aligned} \quad (8)$$

where the argument is $\tau = \tau_i - \tau_j$, with τ_1 taken to be zero. The noise *n(t)* in equ.(7) is assumed a Gaussian random variable with zero mean and unit variance; all data bits are independent, equiprobable, and the delays are also independent and uniformly distributed over a bit interval. The first term in equ.(7) is due to the desired user, while the second term is additive white Gaussian noise, and the third is the multiple access interference.

For further analysis, the multiple access interference has been assumed a Gaussian distributed. Simulations in [9] show the validity of this approximation for a reasonable number of active users. The variance of each term in the multiple access interference sums has been shown in [4] to be approximately equal to $T^2/3N$, where *N* is the length of the signature sequence ($2T^2/3N$ corresponds to the correlation and *1/2* is due to the phase offset). The signal-to-noise ratio (SNR) is given by:

$$SNR = \frac{\left(\frac{2\eta e}{h\nu}\right)^2 \hat{L}^2.\hat{S}^2.T^2}{\left(\frac{2\eta e}{h\nu}\right)^2 \hat{L}^2.\hat{S}^2.T^2.\frac{K-1}{3N} + \frac{\eta e^2 T}{h\nu}.\hat{L}^2} = \frac{1}{\frac{K-1}{3N} + \frac{h\nu}{4\eta\hat{S}^2 T}} \quad (9)$$

Note that $4\eta\hat{S}^2 T/h\nu$ is the single-user SNR (E_b/N_0), that shows SNR equals E_b/N_0 if the multiple access interference were zero (*K=1*). Since we have BPSK signaling in Gaussian noise, the bit-error rate (BER) can be calculated directly from the accomplished SNR.

Figure (4) clearly explains the system behavior versus the number of simultaneous active users (*K*) and comparisons between DPMPC of *P=29* based on above discussions and Gold-sequences of 511 code-length discussed in [4]. It is shown that MAI is the limiting factor in system performance. The BER threshold of 10^{-9} is shown in the figure to assist the eye. As represented by Figure (4), obviously the higher the single-user SNR the lower the BER achieved, and the different performances with single-user SNR of 13dB, 16dB and 19dB

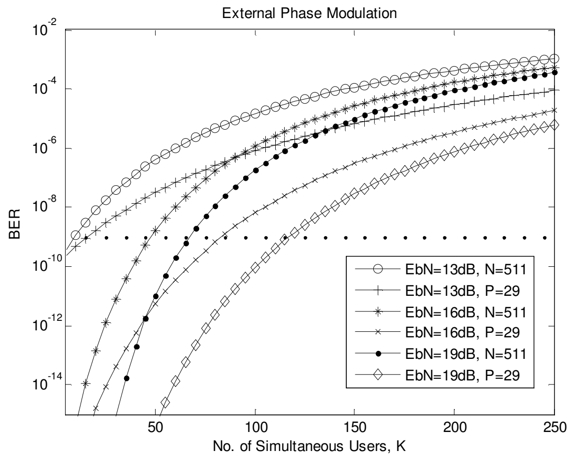

Fig. 4. Performance of BPSK-OCDMA system using an external modulator versus number of simultaneous active users (*K*)

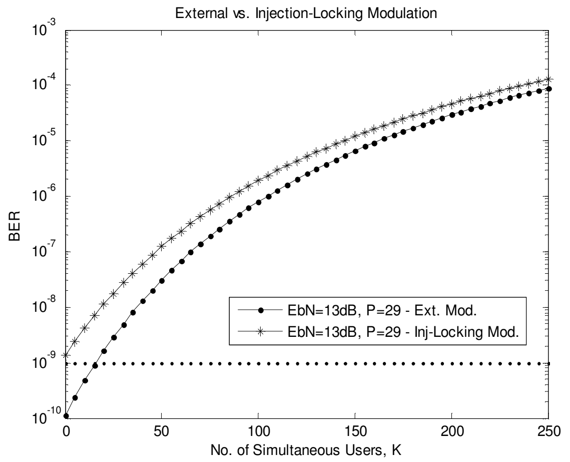

Fig. 5. Performance comparisons of externally and injection-locking modulated BPSK-OCDMA system versus number of simultaneous active users (*K*)

are illustrated. By comparing these two codes, it is noticeable that DPMPC increases the system capacity in terms of accommodating more simultaneous active users up to two times more. For example, following the single-user SNR of 16dB in Figure (4), the system employed DPMPC tolerates at the maximum of 82 (10% of full-load in case of *P=29*) simultaneous users having BER=10^{-9} while the system employed Gold-sequences tolerates 42 simultaneous users.

V. Injection-Locking phase modulation with Phase Limitation

Now examining the effect of a phase limitation on the modulation process is studied. In this case, the injection current of DFB laser diode is modulated to accomplish PSK signaling at the transmitter, with phase excursion limited to ±0.42π and modulation speed no greater than 1Gchip/s. At the receiver we demodulate via injection-locking of the local oscillator (as it is no longer possible to track the desired user's initial phase offset by injection-locking, it is supposed to track by another method in that $\theta_1 = \theta_{LO}$ is still zero). By writing $\pm 0.42\pi = \pm \pi/2 \mp 0.08\pi$, the information and local oscillator signals are therefore given as the following form [4]:

$$
\begin{aligned}
S &= \hat{S}\sum_{i=1}^{K} e^{\,j(\omega_c t + C_i(t).(\pi/2-0.08\pi)+\pi/2+\theta_i)} \\
&= \hat{S}\sum_{i=1}^{K} e^{\,j(\omega_c t + \pi/2.C_i(t)+\pi/2+\theta_i)}.e^{\,j0.08\pi C_i(t)}
\end{aligned}
\tag{10}
$$

$$
L = \hat{L}\sum_{i=1}^{K} e^{\,j(\omega_c t + S_1(t).\pi/2+\pi/2)}.e^{\,j0.08\pi.S_1(t)}
\tag{11}
$$

The output of the dual-balanced detector under this condition contains two new terms, due to the phase limitation, as compared to equ.(5). The first term is going to be constant in time, and for the purposes of our analysis, this can be estimated and subtracted with zero error. The last term involves the weighted-sum of the difference of two codes (essentially a new "pseudo"-code). The weighted-sum is bounded by the sum over the pseudo-codes, which in turn is negligible compared with the main signal term in the output as the code-length becomes arbitrarily larger than the number of active users ($N \gg K$). Therefore, we approximate the output by the main term alone plus noise by integrating over a bit interval, thus the output signal is obtained as:

$$
\begin{aligned}
S_{out} &= \frac{2\eta e}{h\nu}\hat{L}\hat{S}\cos^2(0.08\pi)\sum_{i=1}^{K}\int_0^T S_1(t).C_i(t).\cos\theta_i dt + \sqrt{N_0}\int_0^T n(t)dt \\
&= \frac{2\eta e}{h\nu}\hat{L}\hat{S}.b_0^1.T.\cos^2 0.08\pi + \sqrt{\frac{\eta e^2 T}{h\nu}}\hat{L}.n \\
&\quad + \frac{2\eta e}{h\nu}\hat{L}\hat{S}\cos^2 0.08\pi.\sum_{i=2}^{K}\left[b_{-1}^i.R_{i,1}(\tau_i) + b_0^i.\hat{R}_{i,1}(\tau_i)\right]\cos\theta_i
\end{aligned}
\tag{12}
$$

Using the same Gaussian approximation for the multiple access interference, the SNR is achieved as follows:

$$
SNR = \frac{\left(\frac{2\eta e}{h\nu}\cos^2 0.08\pi\right)^2 \hat{L}^2.\hat{S}^2.T^2}{\left(\frac{2\eta e}{h\nu}\cos^2 0.08\pi\right)^2 \hat{L}^2.\hat{S}^2.T^2.\frac{K-1}{3N} + \frac{\eta e^2 T}{h\nu}.\hat{L}^2} = \frac{1}{\frac{K-1}{3N} + \frac{h\nu}{4\eta\hat{S}^2 T\cos^4 0.08\pi}}
\tag{13}
$$

By comparing equations (9) and (13), it is noticed that the only difference is the cosine term which causes 0.6dB degradation, while this is analogous to the single-user where it is expected a power-loss of twice since the limitation applies to both modulator and demodulator, then the degradation is dually equal to 1.2dB ($20\log\theta$, θ is the phase limitation).

The behavior of these two systems is compared and shown in Figure (5) in terms of the number of simultaneous active users (K) where single-user SNR and P are set to 13dB and 29 respectively. The external modulation indicates remarkably enhanced performance, particularly at the lower number of simultaneous active users.

VI. Conclusion

The equations were introduced for the BER of optical CDMA using novel prime code family (DPMPC) and coherent detection with both external phase modulation and injection-locking under the assumption of a Gaussian distribution for the multiple access interferences. Accordingly, employing DPMPC outperforms the conventional bipolar sequences in terms of flexible code-lengths and more accommodating active users. The limited phase excursion causes several complications i.e. firstly separate phase tracking is required as it can no longer be accomplished simultaneously with phase modulation; secondly from a practical point of view, there is a DC-bias level in the dual-balanced detector output requiring estimation and removal; finally there is the degradation in BER equivalent to a signal loss of 0.6dB; however this loss is small in comparison to the MAI penalty for multiple simultaneous users.

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