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A mathematical study of the pantograph/catenary dynamic interaction on transition spans

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Abstract—**This paper presents the work carried out by CALPE team on the development of a mathematical model used in the study and simulation of the pantograph-catenarydynamic interaction in high speed railways considering two independent spans ofcatenary where the transition spans are overlapped. According to the developedmathematical model, a non lineal system of differential equations with variableconstraint depending on the pantograph position has been obtained. In order to verify the correctness of the model, a numerical integration algorithm based on an explicit method of the central differentials has been implemented. The procedure that has been designed allows us to study the more appropiate contact wire configuration, in order to make smoth transition of the pantograph between sets ofspans. This procedure can be generalized considering the case of several pantographs, obtaining very realistic simulations.**

Keywords: Railway catenary, pantograph-catenary interaction, high performance computing.

1 Introduction

In order to achieve appropiate performance in circulation of railway units, the pantograph-catenary contact force has to be kept as uniform as possible, avoiding losses of contact. Developing a mathematical model, that allows us to simulate the mechanical behavior system, can be helpful to specify optimal assembly conditions in the catenary or aereal contact line.

During the last few years, lot of studies about the pantograph-catenary dynamic interaction have appeared in science literature: Arnold and Simeon (2000), Collina and Burni (2002), Drugge et al. (1999), Jenssen and Trae (1997), Lesser et al. (1996) and Schaub and Simeon (2001). Nowadays, the common characteristics to all works presented is based on models where the pantograph interacts with only one contact wire, and these works suppose equal spans. But this assumption is not completely true, because the catenary structure is installed in series of 15 or 20 spans each one with 60m. which have not to be necessarily equals, and with the transition spans overlapped. In these transition spans, the pantograph can interact with the contact wires of two different spans at the same time and presents different configuration in the wires in order to obtain a smoth transition of the pantograph between sets of spans.

To develop a model where the pantograph can interact with two spans at the same time and with several contact wires can be helpful, overall when the system behaviour has to be evaluated, allowing the study of more adequate contact wires configuration in the transition spans, and more realistic numerical simulation with several pantograph, considering the complete traveled of each pantograph in the line.

This work has been structured in the following sections. In Section 2, the general model of the dynamic equations is introduced. In Section 3 and 4, the previous model has been particularized in order to explain the system model and the configuration for the transition spans. In Section 5, the numerical integration of differential equations is explained. Finally, conclusions about computational aspects and the possible uses of this method are outline in Section 6.

2 Dynamic equations of the system

The pantograph-catenary set can be considered compound of two subsystems that interact each other, under constraint conditions, because the pantograph-catenary contact force has to be kept as uniform as possible, avoiding contact losses. The system dynamic equation particularized for an instant of time t_n is given by 1, as follows:

$$\begin{pmatrix} M & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \ddot{q}_n \\ \ddot{\lambda}_n \end{pmatrix} + \begin{pmatrix} C_n & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \dot{q}_n \\ \dot{\lambda}_n \end{pmatrix} + \\ + \begin{pmatrix} K_n & \phi_n^t \\ \phi_n & 0 \end{pmatrix} \begin{pmatrix} Q_n \\ \lambda_n \end{pmatrix} = \begin{pmatrix} R_n \\ 0 \end{pmatrix} \quad (1)$$

Where M is the mass matrix of the system, C_n is the

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damping matrix, K_n is the stiffness matrix, ϕ_n is the constraint conditions matrix, R_n is the independent term vector, q_n is the generalized coordinates vector and λ_n is the multipliers vector of Lagrange which is equivalent to the constraint forces. In these equations, only the mass matrix is constant along the time, whereas the rest of terms can vary at each instant t_n.

If there is g generalized coordinates and r constraint, the system in 1 presents $g + r$ equations with $g + r$ unknown factors. The number of generalized coordinates depends on the meters of the catenary wires, as the discretization of the wires and the number and type of pantographs. The number of constraint conditions depends on the number and type of pantographs and the contact wires over which each pantograph interacts.

On the other hand, each terms and matrix of the equation 1 can be factorized in two terms, one of them concerns to the catenary and is represented with the subindex 1, and the other one concerns to the pantograph or pantographs, and is/are represented with the subindex 2, obtaining the expressions in 3.

$$M = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}, K_n = \begin{pmatrix} K_1n & 0 \\ 0 & K_2n \end{pmatrix}, \quad (2)$$

$$C_n = \begin{pmatrix} C_1n & 0 \\ 0 & C_2n \end{pmatrix}, R_n = \begin{pmatrix} R_1n \\ R_2n \end{pmatrix}, q = \begin{pmatrix} q_1n \\ q_2n \end{pmatrix}$$

3 Catenary model

The aerial contact line or catenary is built considering a range of spans, normally between 15 and 20, each of them about 60 m. When the catenary is modeled, the different types of elements can be consider: carrier wire, contact wire and droppers. The transition spans of each serie are overlapped, and present an special configuration in the contact wire, in order to make the transition of the pantograph smother (see Figure 1).

As the carrier wire as the contact wire are tightened by pulleys and independent counterbalances, located at the end of each series of spans. The catenary is a continuous system that can be modelized applying the techniques of anallysis of the Finite Element Method (FEM), according to Cook et al. (1998) or Bathe (1996).

With respect to the carrier and contact wires, the following Euler-Bernoulli differential equation considering a flexible pretensed wire in motion is:

$$(p/g) * \ddot{y} = -E * I * y^{IV} + T * y^{''} - p \quad (3)$$

Where p is the weight of the wire per unit of length, g the gravity aceleration, y the offset of the wire, T the mechanical tension, I the diametric moment of inertia and

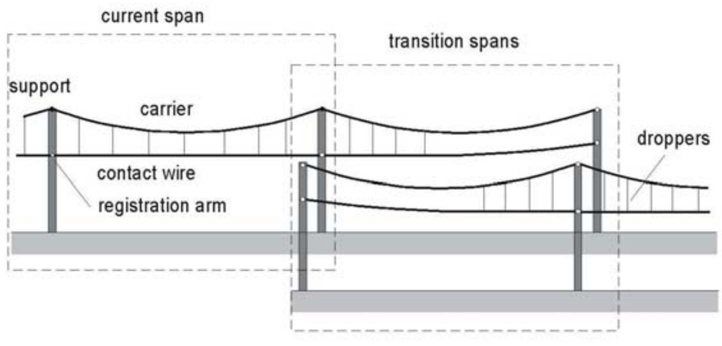

Figure 1: Catenary with the transition spans overlapped

E the elastic module of the material. The wires are modelized as pretensed beam elements, with two generalized coordinates by node: the offset and the turn angle.

Droppers behave as elastic bars of final length of assembly, which deform themselves from an initial length. In this case, each node presents only one generalized coordinate corresponding to the offset.

Notice that the droppers only work to traction so that its effect in the dynamic equations (and their inclusion in the stiffness matrix and independent term), it will be considered only when the effective length, measure as distance between the extreme nodes, will be greater or equal as the initial length l_0.

Also, the effect of the registration arm has been considered. This element is an articulated bar of length l joint at the end of the and whose functionality consists on fixing the contact wire so that it describes a zig-zag, to wear out uniformly the pantograph rubbing surface, behaving as a semi-rigid support.

The arm effect on the stiffness matrix and over the independet term can be approximate as a spring of stiffness k_b that exerts a dynamic force f_b over the contact wire, given by the equation:

$$f_b = f_m + (y_Am - y_A)\, k_b \quad (4)$$

where f_b represents the dynamic force exerted by the arm, f_m the static force of assembled that is a fixed value,y_{Am} is the assembly static height of the grip node (this value is also fixed), y_A is the generalized coordinate that is associate to the grip node of the arm and k_b is the stiffness, that is getting with the linearization of the statics equations and depends on the conditions of assembly.

For the mass matrix of the wires, we have supposed a digonal matrix. With all these considerations, it is possible to join the matrix of mass, stiffness and the independent term in the catenary.

For the damping matrix of the catenary, we have supposed a Rayleigh type damping, (Cook et al, 1989 or Bathe, 1996), where the damping matrix, is a lineal combination of the mass and stiffness matrix.

$$C_1 = \alpha M_1 + \beta K_1 \tag{5}$$

In this case a constant stiffness matrix for the catenary K_1 is supposed, with all droppers connected, and therefore the damping matrix C_1 is also constant. The numerical constants α and β are determined from the dampings supposed for two significant frequencies of oscillation of the system

4 Configuration of the transition of the span

The pantograph interacts with only one contact wire along its trajectory, but when the pantograph arrives to the last span, the transition span, the pantograph progressively loses the contact with the contact wire from the output serie and starts to get the contact with the contact wire from the input serie, and can interacts with two wires of different spans at the same time. In order to make the change of the pantograph from one serie to another smoother, the contact wires have to be configurated in a special manner (see Figures 1 and 2).

It is suppose that the trajectory of the pantograph goes from left to right, the sapan 1 of the figure goes to the output transition span and the span 2 to the input transition span, both of them have a symmetric configuration (the half left part of this configuration is represented in Figure 2). It can appreciate that the span 1 presents a zone of droppers, in the left part. From the last dropper, the contact wire lifts itself until the support in the right side of the span.

The contact wires from both spans intersect in the Q point that is situated in the middle of the span. In order to get a suitable configuration, it would be suitable to specify the droppers position and the height of the support point of the contact wire, given by the distance d, if E is the elasticity of the catenary in the middle of the span and F is an average estimation of the vertical force of the pantograph that is going to circulate around the line. This force will produce an elevation h at the wires given by:

$$h = E * F \tag{6}$$

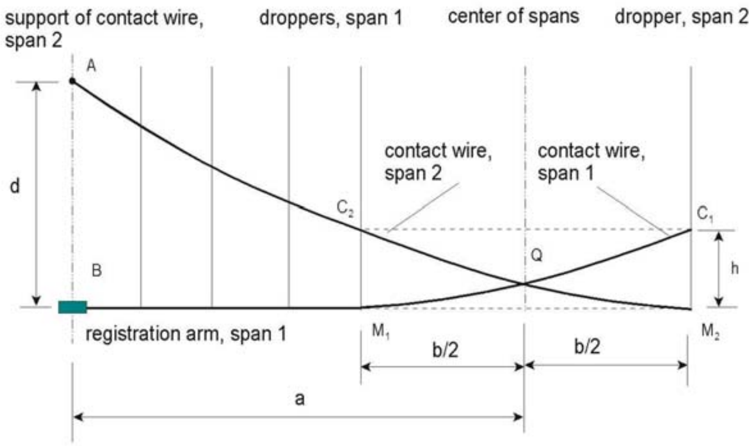

Figure 2: Details of the contact wires in a transition span.

There is a part of the contact wire that hangs free between the last dropper and the support, and presents a parabolic configuration where M_1 and M_2 are the minimum of the contact wires of the spans 1 and 2, respectively. The elevation h can be used to define a segment of the parabola between the points M_1 and C_1 from the span 1 or M_2 and C_2 from the span 2, in absence of dynamic effects, it guarantees the contact common of the pantograph with the two contact wires, for this, the intersect point Q, of the wires situated in the center of the span, has to be in the middle of the points M_1 and C_1 or M_2 and C_2.

Let T the mechanical tension of the contact wire, and p the weight of the wire by unit of length, the distance of C_1 respect to the minimum M_1 is given by the equation:

$$b = \sqrt{\frac{2 * T * h}{p}} \tag{7}$$

If a es the half of the length of the span, the height of the support A of the contact wire in the span 2 is:

$$d = \frac{p * \{a + b/2\}^2}{2 * T} \tag{8}$$

This same height is adopted for the contact wire in the span 1 assuring, in absence of dynamic effects, the contact of the pantograph with the wires of the spans 1 and 2 between the points $C_2 - M_2$ and $C_1 - M_1$, however this condition can be changed, varying the distance from the minimums M_1 and M_2 to the center of the span Q, and we can get different configurations, but in a standard

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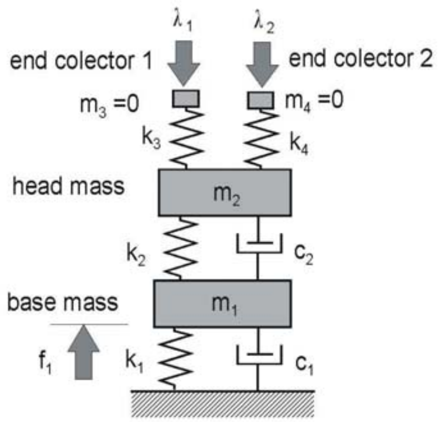

Figure 3: Pantograph on two contact wires.

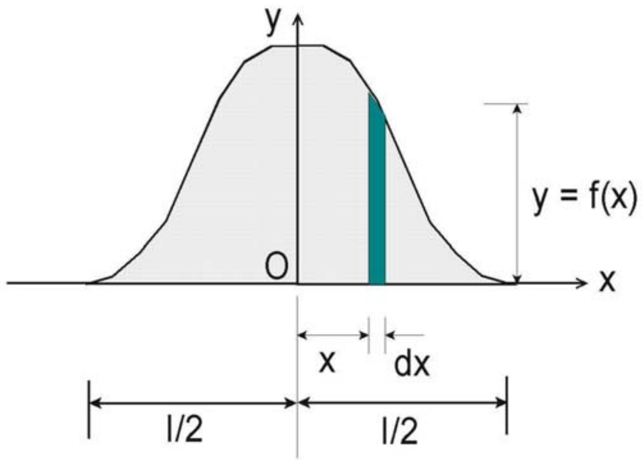

Figure 4: Distribution function of the load over the rub surface of the pantograph

assembly, it is supposed that the center of the span is corresponded with the center of the static safe contact zone C_2-M_2 and C_1-M_1.

4.1 Pantograph model

The pantograph is an articulated system that is modelized as a set of masses, springs and shock absorbers, although the values of these parametres can be obtained with tests in the laboratory. Generally, these values are usually specified by the manufacturer. Each mass has associated a generalized coordinate which corresponds to its vertical displacement.

In the Figure 3 a model of the pantograph with three masses is shown which is able to interact with the contact wires from the input and output transition spans. To make the differencial equation integration easier, two aditional elements without any mass have been added over the head mass, called terminal colectors, which get the force and contact from the wires and are linked to the head mass by a stiff spring, physically the stiffness of this spring represents the real stiffness of the contact wire-platen. λ_1 and λ_2 represent the constraint forces, or the pantograph-catenary contact force. There is the possibility to have several constraint conditions depending on how the pantograph interacts with the contact wire of one or other serie of spans, or with both of two contact wires of the two series at the same time. Finally, f_1 represents the vertical force of the pantograph.

The mass matrix from the pantograph model of the Figure 5, is a diagonal matrix with two null elements that corresponding to the mass of the terminal colectors m_3 and m_4, and the mass and stiffness matrix are as follow:

$$M_2 = \begin{pmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$K_2n = \begin{pmatrix} k_1+k_2 & -k_2 & 0 & 0 \\ -k_2 & k_2+k_3+k_4 & -k_3 & -k_4 \\ 0 & -k_3 & k_3 & 0 \\ 0 & -k_4 & 0 & k_4 \end{pmatrix} \tag{9}$$

4.2 Model of contact of the pantograph and the wire

According to previous experiences, to consider a puntual contact between the pantograph and the catenary could represent a problem because it involves to suppose a concentrated force during the motion, and there exist integration problems each time that the pantograph goes by a node obtained by the discretization of the contact wire.

To avoid this problem it would be suitable to suppose a distributed contact, according to a deteminate distribution function equivalent to express that the contact force is distributed over the rub zone of the pantograph situated in the terminal colector. By convenience, an Oxy (see Figure 4) axis system that moves itself with the pantograph is supposed. The x variable represents the position of the different points in the contact wire along the horizontal axi.

If l is the length of the rub surface, y_3 the generalized coordinate that corresponds to the vertical position of the terminal colector, according to the model of the Figure3,

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$y_n(x)$ the contact wire position around the rub surface of the pantograph, and $y = f(x)$, represents the contact distribution function, the following expression is given:

$$\int_{-\infty}^{\infty} f(x) * (y_h(x) - y_3) * dx = 0 \tag{10}$$

In addition the following expression is fulfilled:

$$\int_{-\infty}^{\infty} f(x) * dx = 1 \tag{11}$$

These equations allow us to get the terminal colector position of the pantograph, when the contact wire configuration at instant t_n is known, as a weighted measure of the nodes of the contact wire position that are situated over the rub zone, obtaining the following expression:

$$(y_3)_n = \int_{-\infty}^{\infty} f(x) * y_h(x) * dx \tag{12}$$

A similar equation is obtained for the position of the other terminal colector $(y_4)_n$. In general it is recommended that the weighted function $y = f(x)$ has a smooth form, in order to make the colector transition across the nodes easier. Several functions for $f(x)$ have been tested, and good results have been obtained by using a Hermite polynomial function given by

$$\begin{aligned} f(x) &= -\frac{32}{l^4} * x^3 - \frac{24}{l^3} + \frac{2}{l}, \quad -\frac{l}{2} \leq x \leq 0 \\ f(x) &= -\frac{32}{l^4} * x^3 - \frac{24}{l^3} + \frac{2}{l}, \quad 0 \leq x \leq \frac{l}{2} \\ f(x) &= 0, \quad x \leq -\frac{l}{2} \\ f(x) &= 0, \quad x \geq \frac{l}{2} \end{aligned} \tag{13}$$

The function $y_n(x)$ is the position of the different contact wire points in the pantograph environment. It can be expressed with the generalized coordinates associated to the nodes from the wire discretization, according to the FEM, so the terminal colector position of the pantograph is a weighted measure from the generalized coordinates of the contact wires nodes that are situated over the rub surface, so the Equation (10) in the t_n instant can be expressed as follow:

$$\phi_n * q_n = 0 \tag{14}$$

The previous expression have to be repeated for all constraint conditions of the system, so that the vector n is converted in the constraint conditions matrix.

The constraint conditions number depends on the contact wires number over the pantograph can interacts, of the type of pantograph and the number of the pantographs in the railway. Generally, there are catenaries with direct current with two contact wires, so, when the pantograph goes along the transition span, the mass of the head can interacts with four wires. In other hand, there are pantograph models with two mass in the head, and in this case there are eight constraint conditions pantograph-cantenary by pantograph, and at the same time, the railway units can have until four pantographs. To make a pantograph-catenary contact model, the constraint conditions vector is made in such a way that if there is r constraint conditions, and g generalized coordinates, the matrix dimension n is going to be rxg.

4.3 Constraint conditions model

In order to simplify the constraint condition model presented here, we are going to suppose only a pantograph with one mass of head that runs along the catenary which has only one contact wire. A general form of the constraint conditions matrix can be expressed in this case at intant t_n as follow:

$$\phi_n = \begin{pmatrix} \phi_{1n} \\ \phi_2 n \end{pmatrix} \tag{15}$$

where the vector ϕ_{1n} represents the constraint conditions vector of the collector 1 over the contact wire in the serie 1, and the vector ϕ_{2n} the the constraint conditions vector of the collector 2 over the contact wire in the serie 2. To the formulation of the equation (15) we are going to consider the follow cases:

1. The pantograph runs along the normal spans in the serie 1, it can interact only with one contact wire.
2. The pantograph runs along the normal spans in the serie 2, it can interact only with one contact wire.
3. The pantograph runs along the transition spans in both series, and in this case the pantograph can interact with two contact wire.

In the first case, the vector ϕ_{1n} is calculated according to the equation (10), while the terminal colector position 1, $(y_3)_n$ is calculated according to the equation (12). On the other hand, the collector 2 never can take contact with the wire in the serie 2, saying that it is disabled, and in this case, the constraint conditions vector ϕ_{2n}, is going to be created imposing the constraint that the collector position 2, $(y_4)_n$ and the head of the mass, $(y_2)_n$, will be coincident, that is:

$$(y_2)_n - (y_4)_n = 0 \tag{16}$$

The second case is similar to the previous case, but the collector 2 can contact the wire in serie 2 and the collector 1 will be deactivated. In the third case, the pantograph runs along the transition span and the collectors 1 and 2 can contact the wires in series 1 and 2, and both collectors are activated, the constraint conditions vectors,ϕ_{1n} and ϕ_{2n}, are calculated in the same way, using the Equation (10), and the position of the terminal collectors 1 and 2, $(y_3)_n$ and $(y_4)_n$, are calculated using Equation (12).

Finally, it is important to say that eventhough at the beginning the collector is interacting with the contact wire, the contact force can be canceled or have its sign changed. When this happens, the pantograph-cantenary contact does not exist and the constraint conditions have not sense. This circunstance has to be considered when we are going to solve Equation (1). When the collector lose the contact with its respective wire, it is equivalent to make null the stiffness collector.

5 Numerical integration of the dynamic equation

In the literature is possible to find different numerical integraion of the equaton (1). But, from the mechanical point of view the methods appeared in (Cook et al., 1989 and Bathe, 1996) to solve the equation system (1) have been considered. Good results have been obtained with the explicit integration method of the central differences, for that purpose, the speed vectors and acceleration of the generalized coordinates have been approximated according to the follow expressions:

$$\begin{aligned} \dot{q}_n &= \frac{1}{t}(q_{n+1} - q_{n-1}) \\ \ddot{q}_n &= \frac{1}{t^2}(q_{n+1} - 2q_n + q_{n-1}) \end{aligned} \tag{17}$$

If we replace these equations in Equation (1) the following expression is obtain

$$\begin{aligned} &\left(\frac{1}{t^2} * M + \frac{1}{2\Delta t} * C_n\right) * q_{n+1} = \\ &R_n - K_n * q_n - \lambda_n^t * \lambda_n + \frac{1}{t^2} * M * (2q_n - q_{n-1}) + \\ &+ \frac{1}{2\Delta t} C_n * q_{n-1} \end{aligned} \tag{18}$$

The previous expression allows us to get generalized coordinates q_{n+1} in the instant t_{n+1} but it does not allow us to know the generalized forces vector λ_{n+1} in that instant. Another important problem is that to determine q_{n+1} it is necessary to solve a lineal system of equations, because the damping matrix C_n is not diagonal.

One of the main advantages of the explicit methods is that it is possible to obtain the integration variables directly with simple matrix operations, without needing to solve a system with a high number of equations. In this cases it could be possible, if we would have supposed a diagonal damping matrix for the catenary, or if the damping had not been considered.

Although the considered options could be legitimate, we think that working with a Rayleigh damping technique is more realistic, according to Equation (5) and to modify in half step the speed vector in Equation (1). It allows us to get the integration variables in a direct form, obtaining the following system of equations:

$$\begin{pmatrix} M & 0 \\ 0 & 0 \end{pmatrix} * \begin{pmatrix} \ddot{q}_n \\ \ddot{\lambda}_n \end{pmatrix} + \begin{pmatrix} C_n & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \dot{q}_{n-\frac{1}{2}} \\ \dot{\lambda}_{n-\frac{1}{2}} \end{pmatrix} + \\ + \begin{pmatrix} K_n & \lambda_n^t \\ \lambda_n & 0 \end{pmatrix} * \begin{pmatrix} q_n \\ \lambda_n \end{pmatrix} = \begin{pmatrix} R_n \\ 0 \end{pmatrix} \tag{19}$$

According to Cook et al. (1989), this modification introduces a small error that can be ignored if a structural systems with a low damping is considered, as our case. The speed and acceleration vectors have been approached according to the expressions:

$$\begin{aligned} \dot{q}_{n-\frac{1}{2}} &= \frac{1}{2\Delta t} * (q_n - q_{n-1}) \\ \ddot{q}_n &= \frac{1}{2\Delta t} * (\dot{q}_n + \frac{1}{2} - \dot{q}_{n-\frac{1}{2}}) \\ \ddot{q}_n &= \frac{1}{\Delta t^2} * (q_{n+1} - 2q_n + q_{n-1}) \end{aligned} \tag{20}$$

If we replace these expressions in Equation (19) and solve it for the generalized coordinates vector q_{n-1}, considering that the mass matrix is diagonal, the resulting equation is as follows:

$$\begin{aligned} &\frac{1}{\Delta t^2} * M * q_{n+1} = \\ &R_n - K_n * q_n - \phi_n^t * \lambda_n + \\ &+ \frac{1}{\Delta t^2} * M * (q_n + \Delta t * \dot{q}_{n-\frac{1}{2}}) - C_n * \dot{q}_{n-\frac{1}{2}} \end{aligned} \tag{21}$$

The previous equation allows us to calculate the generalized coordinates vector q_{n+1}. However, there are still some aspect that are not solved: the generalized force vector λ_{n+1} can not be calculated and in addition Equation (21) can not determine the terminal collectors positions $(y_3)_{n+1}$, $(y_4)_{n+1}$ because its mass is null, and this makes the equation useless for its calculation. However, if the terminal collector is activated, it is possible to calculate its position using Equation (12) in t_{n+1}. If the collector is deactivated, its position will be the same than the head mass calculated according to Equation (16).

The pantograph-catenary constraint forces are the same than the forces of the springs, because the masses of the terminal collectors are null:

$$\begin{aligned}(\lambda_1)_{n+1} &= k_3 * [(y_3)_{n+1} - (y_2)_{n+1}], \\ (\lambda_2)_{n+1} &= k_4 * [(y_4)_{n+1} - (y_2)_{n+1}]\end{aligned} \tag{22}$$

The cycle of integration is completed in t_{n+1}. In order to initialize the algorithm the generalized coordinates values and constraint forces at the initial moment have been calculated, solving the folloing linear system:

$$\begin{pmatrix} K_0 & \phi_0^t \\ \phi_0 & 0 \end{pmatrix} * \begin{pmatrix} q_0 \\ \lambda_0 \end{pmatrix} = \begin{pmatrix} R_0 \\ 0 \end{pmatrix} \tag{23}$$

In addition, with regard to the generalized coordinates speeds, to the inital moment has been assumed that

$$\dot{q}_0 = 0 \tag{24}$$

The exposed integration procedure, based on the explicit method of the central differences, allows us to obtain the variables carrying out simple operations, without needing to solve systems of equations, treating nonlinearities as a direct form because the different variable terms can be updated: stiffness matrices, damping matrices, constraint conditions and independent terms at the end of each cycle of integration, and finally all of them are prepared for the following cycle.

5.1 Integration algorithm

According to the previous sections, the integration algorithm is organized as follow:

1. Introduction of the assembly data: characteristics of carrier and contact wires: weight, material, section, mechanical tension, number of contact wires, etc . Characteristics of droppers: lengths, weight, preload, etc. Also it is necessary to consider data about the pantograph or pantographs: number of pantographs, forces, masses, stiffness, dampings, etc.

2. To form the stiffness matrix K_0 and the independent terms R_0 at the initial moment, according to the model supposed for wires and pantograph, according to the Equations (3), (4) and (9). The Equations (6), (7) and (8) will be used to establish the boundary conditions fixing the contact wire.

3. To form the constraint conditions matrix at the initial moment ϕ_0, according to the Equations (10), (15) and (16). Initially, it is ssumed that the pantograph is located in a normal span of the first series of span.

4. To form the mass matrix M. A diagonal mass matrix will be assumed.

5. To form the initial damping matrix of the systema C_0, using the Equation (5) for damping matrix of the catenary.

6. To determine generalized coordinates and initial constraint forces, solving the linear system (23) and making null the speeds, according to (24).

7. Integration cycle of the differential equations in the time, $n = 0, 1, 2, \ldots$
   (a) Calculation of the generalized coordinates of the catenary and pantograph q_{n+1}, at the moment t_{n+1}, using the Equation (21).
   (b) Calculation of the terminal collectors position of the pantograph, $(y_3)_{n+1}$ and $(y_4)_{n+1}$, by means of the Equations (12) or (16), depending on which the pantograph runs along a normal span or a transition span.
   (c) Calculation of the constraint pantograph-catenary forces, $(\lambda_1)_{n+1}$ and $(\lambda_2)_{n+1}$, by means of the Equations (22).
   (d) To form the constraint conditions matrix ϕ_{n+1}, by means of the Equations (10), (15) and (16), considering if the pantograph is located on a normal span or a transition span.
   (e) To update of the different matrices and independent terms for the moment t_{n+1} : $K_{n+1}, C_{n+1}, R_{n+1}$, taking into account the disconnection of droppers and the lose contact between the pantograph and the catenary. In particular, the effect of the lose of contact will be considered of the following way: depending if the pantograph is on a normal span or a transition span, the discussed cases considered previously will be considered. If pantograph is in a transition span and the sign of the contact forces $(\lambda_1)_{n+1}$ and $(\lambda_2)_{n+1}$ calculated according to Equation (22) is zero or of negative sign, the contact exists and the stiffness at the terminal collectors do not varies, but if the sign is positive, the spring of the traction collector is a traction spring and it does not have sense and the contact disappears, this is equivalent to make zero the stiffness associated to the terminal collector. Thus, if K_0 represents the real stiffness of the contact

$$\begin{aligned}&if\ (\lambda_1)_{n+1} \leq 0,\ k_3 = k_0,\ else\ k_3 = 0 \\ &if\ (\lambda_2)_{n+1} \leq 0,\ k_4 = k_0,\ else\ k_4 = 0\end{aligned} \tag{25}$$

The stiffness matrix of the system will be updating for the following step.

(f) The speed in the coordinates generalized will be calculated too:

$$\dot{q}_{n+\frac{1}{2}} = \frac{1}{\Delta t} * (q_{n+1} - q_n) \qquad (26)$$

(g) Modification of the subscript of the variables and terms of the differential equation according to the cycle of integration: subscript $n+1$ corresponding to the terms calculated in the present cycle, it is transformed into n, returning to point 7.1 , repeating the process, until the pantograph has completed its trajectory.

5.2 Computational aspects

The procedures exposed in the previous sections have served as base for the development of a software application implemented in Visual C language that allows to simulate the dynamic behavior of the pantograph-catenary system considering two series of spans with transition span and several pantographs. At the time of developing this solfware, we have considered other important aspects relative to a high performance software, among others, (Nath Datta, 1995): robustness, portability, and efficiency in terms of reduction of memory requirements (Bytes of memory) and in terms of computacional cost of the algorithm (flops).

As much the stiffness matrix as the damping matrix present a high sparsity degree, they are stored in one of the storage formats that exist for sparse matrices and that we can find in scientific literature (Saad, 1996). To be precise, two formats have been chosen: The coordinate format (COO) by its simplicity and suitability for this algorithm, and the compressed row format (CSR) that allows to carry out operations between matrices (matrix-vector product, resolution of equation systems, etc) with a simple form. In this way, a reduction of memory requirements have been obtained to store the matrices that appear in the resolution of the dynamic problem, obtaining in addition a considerable reduction in the computation time.

On the other hand, the characteristics of robustness, portability and efficiency (in flops), have been obtained thanks to the use of standard linear algebra libraries: BLAS (Lawson et al., 1979) and SPARSKIT (Saad, 1994).

Thus in the study of a problem with two series of span of catenary with 1200 ms, using elements of 0.2 ms, a sparse stiffness matrix for the catenary is obtained K_{2n}, of 48000x48000, nevertheless the nonnull elements are approximately 4x48000. The memory requirements will be increassed when a 3-D model of the catenary will be considered in future works.

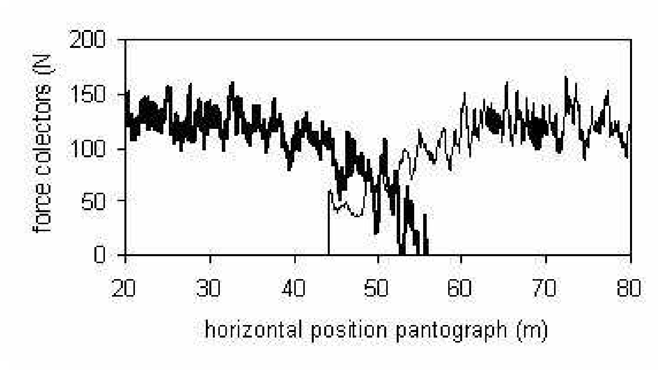

Figure 5: Contact force pantograph-catenary

The results of a simulation with a pantograph of three masses circulating to $120km/h$ along two series of three spans (being the central span a normal span and the both-ends spans a transition spans), the height between wire is $1.2m$, the material used is copper, the mechanical tension in the carrier and in the contact wire is $10KN$ with a diameter of $12mm$. The length of the spans is $20m$. Three droppers in the normal central span have been arranged, separated to a distance of $5m$, and two droppers for the transition spans, being in this case, the distance from the first dropper to the support of $3.83m$ and the distance between droppers of $2m$, with objective of fix the position of the minimum in the second dropper, getting a contact zone centered in center from the span. The droppers section is $25mm^2$.

A model of pantograph like the Figure 3 has been assumed, with: $m_1 = 15kg, m_2 = 7.2kg, k_1 = 50N/m, k_2 = 4200N/m, k_3 = k_4 = 50KN/m, c_1 = 90Ns/m, c_2 = 10Ns/m, c_3 = 0$, with a thrust force of $f_1 = 120N$. In Figure 5, the values of the pantograph-catenary force on the three central spans are shown, corresponding to a displacement of the pantograph between $20ms$ and $80ms$. The pantograph position between $40m$ and $60m$ corresponds to the transition span. The contact force on the first series is represented by the thick continuous line and the force on the second series by the line of fine outlines. It is possible to observe that the force on the first series is diminishing progressively from $40m$, being null completely for the position of $56m$ approximately, whereas the force on the second series appears from $44m$.

In the Figure 6 the position variation of the mass of head is shown. It is possible to observe that when the pantograph runs on normal spans, the mass of head presents a maximum displacements of $50mm$ and $47mm$ approximately for each span, whereas when the pantograph runs along the transition span, the maximum displacement does not get the $42mm$. The reason of this is because the pantograph must interact in a zone with two contact wires, presenting the line a greater stiffness. Varying the

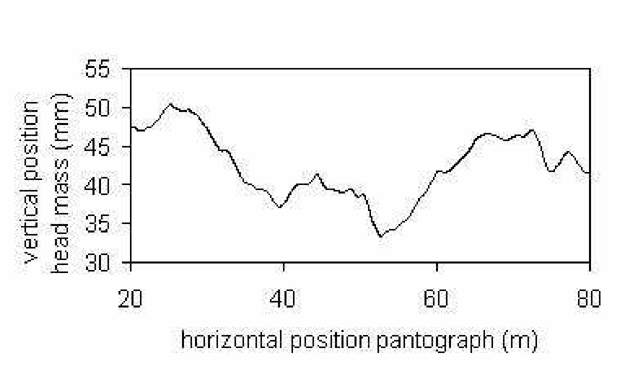

Figure 6: Upward displacement of the mass of head

configuration of the contact wire, it is possible to obtain a different form in the curves, in order to obtain a better behavior of the system.

6 Conclusion

In the present work, a procedure for the study and simulation of the dynamic pantograph-catenary system interaction in railway lines has been developed, considering two series of spans with the both-ends span overlapped. A differentials equation system with constraints has been obtained, where the catenary has been modeled using the FEM, whereas the pantograph has been considered like a system of masses, springs and shock-absorbings.

A numerical integration algorithm has been developed based on explicit method of the central differences, that solves the problem of a simple and direct form, like a differential ordinary equation system, where the pantograph can interact with only a contact wire, when it is runs in a normal span, or with two contact wires when it runs along the transition span. The procedure proposed allows to study the most suitable configuration of the contact wire, in order to facilitate the transition of the pantograph from a series of spans to another one, with a smooth form. This procedure also can be implemented for the case of several pantographs running, allowing to obtain a more realistic simulation because it considers the complete trajectory of the pantographs.

The results of this work have served as base to implemente a software program in Visual C language that allows to carry out simulations for different types of catenaries and pantographs, getting computational times very low, thanks to a spectacular reduction of the memory requirements.

The method presented can be extended for catenary with two contact wires, this type of catenary is used to impel traction units with direct current, and is very frequent in conventional lines of several European countries. Also it is possible to extende this procedure to the study of dynamic problems in three dimensions being able to obtain very realistic simulations with computational times really reasonable.

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