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On-Line Fast Algebraic Parameter and State Estimation for a DC Motor Applied to Adaptive Control

G. Mamani, J. Becedas and V. Feliu-Batlle *†

Abstract—**This paper presents an adaptive position control scheme for DC motors based on an on-line closed loop continuous-time identification method. A fast, non-asymptotic, algebraic identification method is used to estimate simultaneously the unknown system parameters and the unmeasured states to update an adaptive position control scheme.**

Keywords: Algebraic Identification, System Identification, Adaptive Control, States Estimation

1 INTRODUCTION

In recent years, the importance of the continuous-time model identification problem has been recognized in the areas of identification and self tuning adaptive control. A survey on identification of continuous-time systems is [1]. Parameter estimation has been an important topic in system identification literature. The traditional theory is well developed in [2].The advantages of direct continuous time estimation in relation to its discrete-time alternative are defined in [3]. It is well known that for an observable system, represented in state space, the state estimation problem is intimately related to the computation of time derivatives of the output signals, in a sufficient number.

Adaptive control covers a set of techniques which attain the control performance when the plant dynamics is unknown or changes in time. There are several survey papers [4], [5] and books [6] and [7] among others. The adaptive control scheme proposed in [8] was used to control a DC servomotor based on speed control method, this approach is only used to control first order systems. In that paper the method is implemented in discrete time. In [9], the multiple model adaptive system of the DC motor was proposed. This is based upon the indirect adaptive control and more than one identifier (to estimates the unknown parameters of the plant) are incorporated.

The main contribution of our article is that we use an on-line closed loop algebraic method of continuous-time nature for the estimation of the unknown parameters and unmeasured states of a DC servomotor model in order to implement an adaptive position control scheme. Our approach uses the model of the system which is well known. The advantages of the method are that it does not need any statistical knowledge of the noises corrupting the data; the estimation does not require initial conditions or dependence between the system input and output; and the algorithm is computed on-line in a closed loop and in real time. The motor parameters and states are simultaneously estimated. The only measured variables are the motor position, as given by an encoder, and the input voltage to the armature circuit of the motor. After the estimates of the motor inertia, viscous friction, velocity and acceleration are obtained, the Coulomb's friction coefficient is instantaneously estimated. The importance of this coefficient estimation is explained in [10] in order to appropriately control the system by compensating this non-linearity. The identification method is based on elementary algebraic manipulations of the following mathematical tools: module theory, differential algebra and operational calculus, see [11]. Recently, the algebraic method has been applied in [12]. In this work the algebraic method independence to the input signal design is also demonstrated. Finally, we mention that the algebraic method has also been applied in [13] in the area of signal processing applications, and [14] in flexible robots.

This paper is structured as follows: In section 2 the DC servo motor model and the algebraic identification method are presented. The identification of Coulomb's friction coefficient is attempted. In Section 3 the closed loop adaptive PD controller design is explained. These results are verified via simulation in Section 4. Finally, Section 5 is devoted to concluding remarks.

2 MOTOR MODEL AND IDENTIFICATION PROCEDURE

This section is devoted to explain the linear model of the DC motor and the algebraic identification method.

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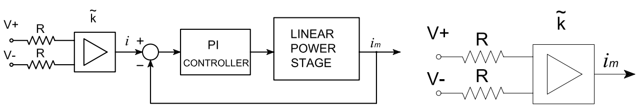
2.1 DC Motor model

A common electromechanical actuator in many control systems is constituted by the DC motor. The DC motor used is fed by a servo-amplifier with a current inner loop control. We can write the dynamic equation of the system as:

$$kV = J\ddot{\hat{\theta}}_m + \nu\dot{\hat{\theta}}_m + \hat{\Gamma}_c(\dot{\hat{\theta}}_m) \qquad (1)$$

where J is the unknown inertia of the motor $\left[kg \cdot m^2\right]$, ν is the unknown viscous friction coefficient $[N \cdot m \cdot s]$, $\hat{\Gamma}_c$ is the unknown Coulomb friction torque which affects the motor dynamics $[N \cdot m]$. This nonlinear friction term is considered as a perturbation, depending only on the sign of the angular velocity of the motor of the form $\mu sign(\dot{\hat{\theta}}_m)$ with μ constant. The parameter k is the electromechanical constant of the motor servo-amplifier system $[Nm/V]$. $\ddot{\hat{\theta}}_m$ and $\dot{\hat{\theta}}_m$ are the angular acceleration $\left[rad/s^2\right]$ and the angular velocity of the motor $[rad/s]$ respectively. The constant factor n is the reduction ratio of the motor gear; thus $\theta_m = \hat{\theta}_m/n$, where θ_m stands for the position of the motor gear and $\hat{\theta}_m$ for the position of the motor shaft. $\Gamma_c = \hat{\Gamma}_c n$, where Γ_c is the Coulomb friction torque in the motor gear. V is the motor input voltage $[V]$ acting as the control variable for the system. This is the input to a servo-amplifier which controls the input current to the motor by means of an internally PI current controller (see Fig.1(a)). The electrical dynamics can be neglected because it is much faster than the mechanical dynamics of the motor. Thus, the servo-amplifier can be considered as a constant relation, k_e, between the voltage and the current to the motor: $i_m = k_e V$ (see Fig.1(b)), where i_m is the armature circuit current and k_e includes the gain of the amplifier, $\tilde{k}$, and R is the input resistance of the amplifier circuit. The total torque delivered to the motor



(a) (b)

Figure 1: (a) Complete amplifier scheme.(b) Equivalent amplifier scheme.

Γ_T is directly proportional to the armature circuit in the form $\Gamma_T = k_m i_m$, where k_m is the electromechanical constant of the motor. Thus, the electromechanical constant of the motor servo-amplifier system is $k = k_m k_e$.

In order to obtain the transfer function of the system the following perturbation-free system is considered:

$$KV = J\ddot{\theta}_m + \nu\dot{\theta}_m \qquad (2)$$

where $K = k/n$. To simplify the developments, let $A = K/J$ $[N/(V \cdot kg \cdot m)]$, $B = \nu/J$ $[N \cdot /(kg \cdot m)]$. The DC motor transfer function is then written as:

$$G(s) = \theta_m(s)/V(s) = A/(s(s+B)) \qquad (3)$$

In our parameter identification scheme we will compute, in an algebraic manner A and B from linear identifiability.

2.2 The Procedure of parameter and state estimation

Consider the second order perturbed system given in (1). J and ν are unknown parameters and they are not linearly identifiable. Nevertheless, the parameter $\frac{K}{J}$ denoted by A and the parameter $\frac{\nu}{J}$ denoted by B are linearly identifiable. Taking this into account and also from the fact that $K = k/n$, after some rearrangements, we have:

$$\ddot{\theta}_m + B\dot{\theta}_m + \Gamma^* = AV \qquad (4)$$

where $\Gamma^* = \frac{\hat{\Gamma}_c}{nJ}$. We consider this last parameter as a constant perturbation input and proceed to compute the unknown system parameters A and B as follows:

Taking Laplace transforms, of (4) yields,

$$(s^2\theta_m(s) - s\theta_m(0) - \dot{\theta}_m(0)) + \qquad (5)$$
$$+B(s\theta_m(s) - \theta_m(0)) + \frac{\Gamma^*}{s} = AV(s)$$

we obtain multiplying out by s,

$$(s^3\theta_m(s) - s^2\theta_m(0) - s\dot{\theta}_m(0)) + \qquad (6)$$
$$+B(s^2\theta_m(s) - s\theta_m(0)) + \Gamma^* = AsV(s)$$

Taking the third derivative with respect to the complex variable s, we obtain independence of initial conditions. Then (6) results in an expression free of the initial conditions $\dot{\theta}_m(0)$, $\theta_m(0)$ and the Coulomb's friction coefficient Γ^*.

$$\frac{d^3}{ds^3}\left[s^3\theta_m(s)\right] + B\frac{d^3}{ds^3}\left[s^2\theta_m(s)\right] = A\frac{d^3}{ds^3}[sV(s)] \qquad (7)$$

The terms of (7) are developed as:

$$\frac{d^3}{ds^3}\left[s^3\theta_m(s)\right] = s^3\frac{d^3\theta_m(s)}{ds^3} + 9s^2\frac{d^2\theta_m(s)}{ds^2} + \qquad (8)$$
$$+ 18s\frac{d\theta_m(s)}{ds} + 6\theta_m(s)$$

$$\frac{d^3}{ds^3}\left[s^2\theta_m(s)\right] = s^2\frac{d^3\theta_m(s)}{ds^3} + 6s\frac{d^2\theta_m(s)}{ds^2} + 6\frac{d\theta_m(s)}{ds} \qquad (9)$$

$$\frac{d^3}{ds^3}[sV(s)] = s\frac{d^3V(s)}{ds^3} + 3\frac{d^2V(s)}{ds^2} \qquad (10)$$

Recall that multiplication by s in the operational domain corresponds to derivation in the time domain. To avoid derivation, after replacing the expressions (8, 9, 10) in equation (7), we multiply both sides of the resulting expression by s^{-3}. We obtain the first equation for the unknown parameters A, B, $\dot{\theta}_m$ and $\ddot{\theta}_m$.

$$B(s^{-1}\frac{d^3\theta_m(s)}{ds^3} + 6s^{-2}\frac{d^2\theta_m(s)}{ds^2} + 6s^{-3}\frac{d\theta_m(s)}{ds}) -$$

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$$A(s^{-2}\frac{d^3V(s)}{ds^3}+3s^{-3}\frac{d^2V(s)}{ds^2})=-[\frac{d^3\theta_m(s)}{ds^3}+$$
$$+9s^{-1}\frac{d^2\theta_m(s)}{ds^2}+18s^{-2}\frac{d\theta_m(s)}{ds}+6s^{-3}\theta_m(s)] \qquad (11)$$

The expression (11) is multiplied both sides by s^{-1} once more. This leads to a second linear equation for the estimates A, B, $\dot{\theta}_m$ and $\ddot{\theta}_m$. After replacing the expressions (8, 9, 10) in equation (7), we multiply both sides of the resulting expression by s^{-2}. We obtain the third equation for the unknown parameters A, B, $\dot{\theta}_m$ and $\ddot{\theta}_m$.

$$s\frac{d^3\theta_m(s)}{ds^3}+9\frac{d^2\theta_m(s)}{ds^2}+18s^{-1}\frac{d\theta_m(s)}{ds}+6s^{-2}\theta_m(s)$$
$$+B(\frac{d^3\theta_m(s)}{ds^3}+6s^{-1}\frac{d^2\theta_m(s)}{ds^2}+6s^{-2}\frac{d\theta_m(s)}{ds})=$$
$$A(s^{-1}\frac{d^3V(s)}{ds^3}+3s^{-2}\frac{d^2V(s)}{ds^2}) \qquad (12)$$

In the time domain, we have:

$$-\frac{d}{dt}(t^3\theta_m)+9t^2\theta_m-18\int t\theta_m+6\int^{(2)}\theta_m+B((-t^3\theta_m)$$
$$+6\int t^2\theta_m-6\int^{(2)}t\theta_m)=A(-\int t^3V+3\int^{(2)}t^2V) \qquad (13)$$

The quantity $\dot{\theta}_m$ may be computed, once A and B has been determined

$$\dot{\theta}_m=\frac{d\theta_m}{dt}=\frac{1}{t^3}(6t^2\theta_m-18\int t\theta_m$$
$$+6\int^{(2)}\theta_m)+\frac{B}{t^3}(-t^3\theta_m+6\int t^2\theta_m$$
$$-6\int^{(2)}t\theta_m)+\frac{A}{t^3}(\int t^3V-3\int^{(2)}t^2V) \qquad (14)$$

After replacing the expressions (8, 9, 10) in equation (7), we multiply both sides of the resulting expression by s^{-1}. We obtain the fourth equation for the unknown parameters A, B, $\dot{\theta}_m$ and $\ddot{\theta}_m$.

$$(s^2\frac{d^3\theta_m(s)}{ds^3}+9s\frac{d^2\theta_m(s)}{ds^2}+18\frac{d\theta_m(s)}{ds}+6s^{-1}\theta_m(s))$$
$$+B(s\frac{d^3\theta_m(s)}{ds^3}+6\frac{d^2\theta(s)}{ds^2}+6s^{-1}\frac{d\theta_m(s)}{ds})$$
$$=A(\frac{d^3V(s)}{ds^3}+3s^{-1}\frac{d^2V(s)}{ds^2}) \qquad (15)$$

which may be written in the time domain as:

$$-\frac{d^2}{dt^2}(t^3\theta_m)+9\frac{d}{dt}(t^2\theta)-18t\theta_m+6\int\theta_m+B(\frac{d}{dt}(-t^3\theta_m)$$
$$+6t^2\theta_m-6\int t\theta_m)=A(-t^3V+3\int t^2V) \qquad (16)$$

We obtain the following expression for the motor acceleration, $\frac{d^2\theta_m}{dt^2}$. The quantity $\ddot{\theta}_m$ may be computed, once A, B, and $\dot{\theta}_m$ has been computed.

$$\ddot{\theta}_m=\frac{d^2\theta_m}{dt^2}=\frac{1}{t^3}(3t^2\frac{d\theta_m}{dt}-6t\theta_m+6\int\theta_m)+\frac{B}{t^3}(3t^2\theta_m-t^3\frac{d\theta_m}{dt}$$
$$-6\int t\theta_m)+\frac{A}{t^3}(t^3V-3\int t^2V) \qquad (17)$$

The unknown parameters A, B, $\dot{\theta}_m$ and $\ddot{\theta}_m$ are clearly linearly identifiable since they can be computed from the linear equation, in the time domain we have

$$\begin{bmatrix} p_{11}(t) & p_{12}(t) & p_{13}(t) & p_{14}(t) \\ p_{21}(t) & p_{22}(t) & p_{23}(t) & p_{24}(t) \\ p_{31}(t) & p_{32}(t) & p_{33}(t) & p_{34}(t) \\ p_{41}(t) & p_{42}(t) & p_{43}(t) & p_{44}(t) \end{bmatrix}\begin{bmatrix} B \\ A \\ \dot{\theta}_m \\ \ddot{\theta}_m \end{bmatrix}=\begin{bmatrix} q_1(t) \\ q_2(t) \\ q_3(t) \\ q_4(t) \end{bmatrix} \qquad (18)$$

where p_{11}, p_{12}, p_{13}, p_{14} and q_1 are

$$p_{11}(t)=-\int t^3\theta_m+6\int^{(2)}t^2\theta_m-6\int^{(3)}t\theta_m \qquad (19)$$
$$p_{12}(t)=\int^{(2)}t^3V-3\int^{(3)}t^2V \qquad (20)$$
$$p_{13}(t)=0 \qquad (21)$$
$$p_{14}(t)=0 \qquad (22)$$
$$q_1(t)=-t^3\theta_m-9\int t^2\theta_m+18\int^{(2)}t\theta_m-6\int^{(3)}\theta_m \qquad (23)$$

and $p_{21}(t)=\int p_{11}(t)$, $p_{22}(t)=\int p_{12}(t)$, $p_{23}(t)=0$, $p_{24}(t)=0$, $q_2(t)=\int q_1(t)$, being

$$p_{21}(t)=-\int^{(2)}t^3\theta_m+6\int^{(3)}t^2\theta_m-6\int^{(4)}t\theta_m \qquad (24)$$
$$p_{22}(t)=\int^{(3)}t^3V-3\int^{(4)}t^2V \qquad (25)$$
$$p_{23}(t)=0 \qquad (26)$$
$$p_{24}(t)=0 \qquad (27)$$
$$q_2(t)=\int t^3\theta_m-9\int^{(2)}t^2\theta_m+18\int^{(3)}t\theta_m-6\int^{(4)}\theta_m \qquad (28)$$

where $p_{31}(t)$, $p_{32}(t)$, $p_{33}(t)$, $p_{34}(t)$, $q_3(t)$ are

$$p_{31}(t)=t^3\theta_m-6\int t^2\theta_m+6\int^{(2)}t\theta_m \qquad (29)$$
$$p_{32}(t)=-t^3V+3\int^{(2)}t^2V \qquad (30)$$
$$p_{33}(t)=t^3 \qquad (31)$$
$$p_{34}(t)=0 \qquad (32)$$
$$q_3(t)=6t^2\theta_m-18\int t\theta_m+6\int^{(2)}\theta_m \qquad (33)$$

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and

$$p_{41}(t) = -t^2\theta_m + t^3\dot{\theta}_m + 6\int t\theta_m \tag{34}$$

$$p_{42}(t) = -t^3V + 3\int^{2} t\ V \tag{35}$$

$$p_{43}(t) = -3t^2 \tag{36}$$

$$p_{44}(t) = t^3 \tag{37}$$

$$q_4(t) = -6t\theta_m + 6\int \theta_m \tag{38}$$

Notation[1]. Let us clarify that the estimation of the states $\dot{\theta}_m$ and $\ddot{\theta}_m$ is dependant of the system parameters A and B. Therefore, the estimation of the states is initially carried out with initial arbitrary values of the motor parameters A_0 and B_0 which the designer can chose. When the real values A and B are estimated the identifier is updated with this new values. Thus, the real signals of the states are estimated.

2.3 Identification of Coulomb's friction coefficient

It is well established that for a system operating at relatively high speed, the Coulomb's friction torque is a function of the angular velocity. For those systems, the Coulomb's friction is often expressed as a signum function dependent on the rotational speed [?]. Consider system (4) with $\Gamma^* = \mu sign(\dot{\theta}_m)$. From this equation, and due to the fact that A, B, $\dot{\theta}_m$ and $\ddot{\theta}_m$ are estimated with the on-line algebraic method, we have

$$\mu sign(\dot{\theta}_m) = AV - \ddot{\theta}_m - B\dot{\theta}_m \tag{39}$$

Notation[2]. With the motor spinning only in one direction, Coulomb's friction coefficient will not change its sign, and can be considered as a constant. When the motor angular velocity is close to zero, the Coulomb's friction effect is that of a chattering high frequency signal:

$$\Gamma^* = \mu sign(V) \tag{40}$$

Then, if the motor spins always in the same direction, in the identification time interval we have that $\Gamma^* = \mu$ and

$$\mu = AV - \ddot{\theta}_m - B\dot{\theta}_m \tag{41}$$

After the values of A, B, $\dot{\theta}_m$ and $\ddot{\theta}_m$ are obtained by the previous method, the scaled Coulomb's friction coefficient, μ, is directly obtained from equation (41).

[1] $\int^{(n)} \phi(t)$ representing the iterated integral $\int_0^t \int_0^{\sigma_1} ... \int_0^{\sigma_{n-1}} \phi(\sigma_n) d\sigma_n ... d\sigma_1$ with $(\int \phi_{(t)}) = (\int^{(1)} \phi_{(t)}) = \int_0^t \phi(\sigma) d\sigma$

[2] The term: $\mu sign(\dot{\theta}_m)$ is a perturbation produced by the Coulomb's friction torque, where μ is the scaled Coulomb's friction amplitude, or coefficient. Note that $\Gamma^* = \frac{\hat{\Gamma}_c}{nJ} = \mu sign(\dot{\theta}_m)$ then, the Coulomb's friction coefficient is $\xi = Jn\mu$. The model $sign(\dot{\theta}_m)$ is defined as: $sign(\dot{\theta}_m) = \left\{ 1 \ (\dot{\theta}_m > 0), -1 \ (\dot{\theta}_m < 0) \right\}$

3 Closed loop adaptive PD controller

This section is devoted to explain the design of a closed loop PD adaptive controller based on the algebraic identification method (See Fig.2). The PD controller is designed by locating all the closed loop poles in a reasonable place of the negative real axis. The controller works with initial arbitrary motor parameters A_0 and B_0. The estimator previously proposed estimates in closed loop and in real time the true values of the motor A, B, $\dot{\theta}_m$ and $\ddot{\theta}_m$. After this estimations, the controller is updated with these new parameters providing an accurate tracking of the desired trajectory.

A PD controller is proposed, $C_{pd}(s) = k_p + k_v s$, whose gains are $\{k_p, k_v\}$. Suppose that the motor system has initial values A_0 and B_0 which we can arbitrarily choose. The initial transfer function of the motor is:

$$G_{m_0} = A_0/(s(s + B_0)) \tag{42}$$

The stability condition on the closed loop expression $(1 + G_{m_0}(s)C_{pd}(s))$ leads to the following characteristic polynomial,

$$s^2 + (kvA_0 + B_0)s + k_pA_0 = 0 \tag{43}$$

We can equate the corresponding coefficients of the closed loop characteristic polynomial (43) with those of a desired second order Hurwitz polynomial. Thus, we can choose to place all the closed loop poles at some value of the negative real axis, $-a$ with $a > 0$, using the following desired polynomial expression,

$$p(s) = (s + a)^2 = s^2 + 2as + a^2 \tag{44}$$

where the parameter a represents the common location of all the closed loop poles. Identifying the corresponding terms of the equations (42) and (43), the parameters k_p and k_v may be uniquely obtained by computing the following equations,

$$k_{p0} = a^2/A_0, \ k_{v0} = (2a - B_0)/A_0 \tag{45}$$

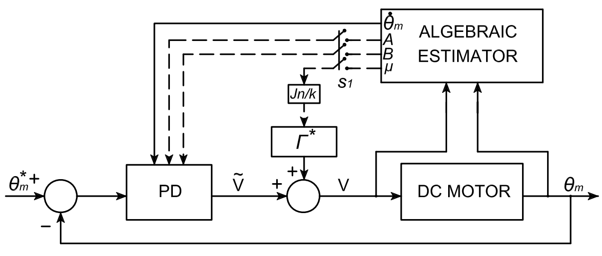

Figure 2: closed loop adaptive system

The system begins to work, in a very short period of time, t_1, the algebraic estimator estimates the true values of A, B, $\dot{\theta}_m$, $\ddot{\theta}_m$ and, instantaneously after, μ. The PD controller is instantaneously updated, by switching ON

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Table 1: Parameters of the DC motor used in the simulations

A	B	k	n	μ
61.13	15.15	0.21	50	34.74

s_1, and recalculated with the new parameters A and B. Note that in (45) we design the PD controller parameters dependable of the parameters of the motor. The new updated controller is now of the form:

$$k_p = a^2/A, k_v = (2a - B)/A \quad (46)$$

With the estimation of the Coulomb‘s friction torque, Γ^*, a compensation term is introduced in the system in order to eliminate the effect of this perturbation (See [10]). The compensation term is included in the control input voltage to the motor, $\tilde{V}$, and this is of the form:

$$\tilde{V} = \hat{\Gamma}_c/k(-sign(\dot{\theta}_m)) = \mu \cdot J \cdot n/k(-sign(\dot{\theta}_m)) \quad (47)$$

when $\dot{\theta}_m \neq 0$. When $\dot{\theta}_m \approx 0$, the compensation term is included as:

$$\tilde{V} = \hat{\Gamma}_c/k(-sign(V)) = \mu \cdot J \cdot n/k(-sign(V)) \quad (48)$$

4 Results

This section is devoted to show the good performance of the proposed method. The values of the motor parameters used in simulations are depicted in Table 1. We consider that there exists a servo amplifier used to supply voltage to the DC motor; this amplifier accepts control inputs from the computer in the range of $[-10, 10]$ $[V]$. The input reference signal to the system is a Bezier's eighth order polynomial with an offset of 0.2 (rad). Thus, we have considered the following initial conditions for the motor to show the robustness of the method to such initial conditions: $\theta_m(0) = 0, \dot{\theta}_m(0) = 0$.

In Fig.3(a)-3(d) the estimations of the motor parameters and states are represented. Fig.3(a) depicts the estimation of the parameter A, whose estimated value is 61.13 $[N/(V \cdot kg \cdot m)]$. In Fig.3(b) is shown the estimation of the parameter B; the estimated value is 15.15 $[N \cdot s/(kg \cdot m)]$. Both estimates are obtained in 0.02 $[s]$. The observers initially work with initial arbitrary values of the motor parameters which we have chosen to be $A_0 = 1$ and $B_0 = 1$. When the estimated parameters of A and B are obtained, the observers are immediately updated with this new values, at time $t_1 = 0.02$ $[s]$. The estimates of the states are depicted in Fig.3(c) and 3(d), motor velocity and acceleration respectively. At this instant, every parameter and state are precisely estimated. The Coulomb's friction coefficient μ is directly obtained from (41). The estimation of this value is depicted in

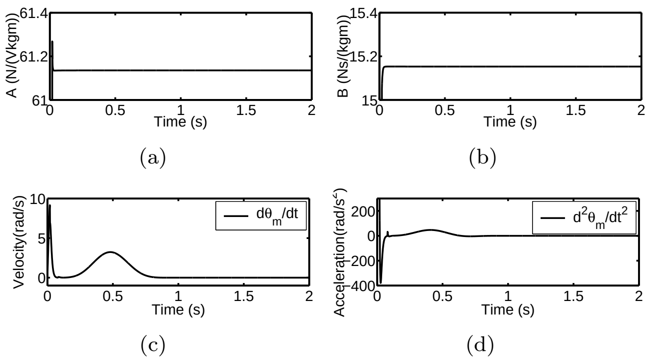

Figure 3: (a) Estimation of A.(b) Estimation of B. (c) Estimation of $\dot{\theta}_m$. (d) Estimation of $\ddot{\theta}_m$

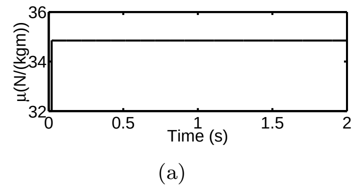

Figure 4: Estimation of the Coulomb's coefficient.

Fig.4(a). The estimated value is $\mu = 34.74$ $[N/(kg \cdot m)]$ also at time $t_1 = 0.02$ $[s]$. Whereas the algebraic parameters and states estimator finds the true values of A, B, $\dot{\theta}_m$ and $\ddot{\theta}_m$ the controller works with the initial parameters A_0 and B_0. When every of the motor parameters and states are obtained the controller is updated with the new parameters values A and B and in the control scheme a friction compensation term is included by switching ON the switch s_1 (see Fig.2). The controller makes the motor track the reference with good performance and null steady state error. Fig.5(a) shows the trajectory tracking. Note that the motor with position initial condition rapidly tracks the reference trajectory θ^*_m. Fig. 5(b) depicts the trajectory tracking error $\theta^*_m - \theta_m$. Note that the steady state error is null. Finally, in Fig.5(c), the control input voltage to the dc motor is presented. Until time $t_1 = 0.02$ $[s]$, the controller saturates the amplifier at ± 10 $[V]$. After that time, the control voltage is smooth and does not saturate the amplifier again.

On the other hand, in real life we always find noises and errors which corrupt the measuring data. In this case, the encoder is not an infinite precisely measure system, therefore, noises are included in the control system due to the limited precision of the apparatus. We consider a noise corrupting the data with zero mean and 10^{-3} standard deviation. We here compare the method proposed with an adaptive PD controller with numerical derivatives of fifth order instead of using the algebraic state estimations [3].

[3]Let us recall that we here use the same PD adaptive combined with algebraic estimated parameters. The difference is that, in this

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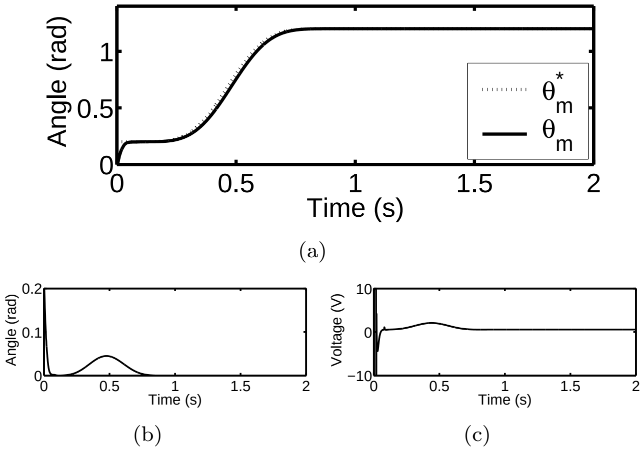


Figure 5: (a) Trajectory tracking.(b) Trajectory tracking error. (c) Control input voltage to the dc motor.

The estimation of the parameters are obtained with the same values that were estimated previously, however the estimator requires more time to estimate. In this case, the time at which the values A and B are obtained is $t_1 = 0.4$ $[s]$ (see Fig.6(a) and Fig.6(b) respectively). This period of time is small enough to be used in the control scheme. The algebraic estimate of the motor velocity, $\dot{\theta}_m$, is depicted in Fig.7(a). The numerical estimate of the motor velocity, $\dot{\theta}_{mn}$, is shown in Fig.7(b). The motor acceleration obtained with the observer $\ddot{\theta}_m$ is depicted in Fig.7(c) and the numerical acceleration $\ddot{\theta}_{mn}$ is represented in Fig.7(d). Note that the algebraic observer is quite robust with respect the noise. Nevertheless, the numerical estimations of the estates are strongly affected by it. This also affects the estimation of the Coulomb's friction coefficient μ when numerical derivatives are used. This is well estimated with the algebraic estimated signals $\dot{\theta}_m$ and $\ddot{\theta}_m$ (see Fig.7(e), where the estimation is the value $\mu \simeq 34.74$ $[N/(kg \cdot m)]$). In Fig.7(f) is depicted the estimation of the Coulomb's friction coefficient when the numerical derivatives are used. Note that no estimation is carried out in this case. Thus, the friction compensation term is not included in the control scheme.

The trajectory tracking when the algebraic observers

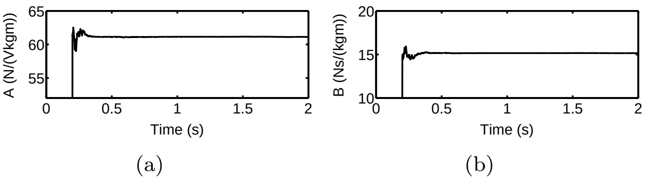


Figure 6: (a) Estimation of A with noise in the measure. (b) Estimation of B with noise in the measure.

are used is depicted in Fig.8(a). At time $t_1 = 0.4$ $[s]$

last case, we use the classical numerical derivatives of the motor position instead of the algebraic state estimations to compare both results.

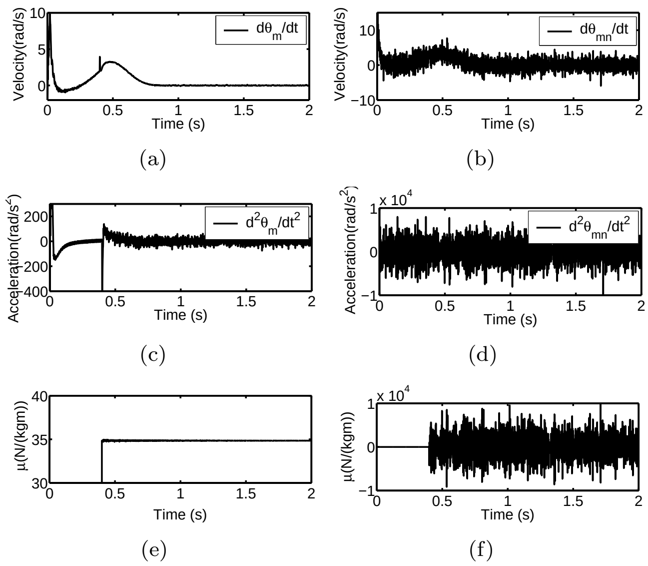


Figure 7: (a) Algebraic Estimation of $\dot{\theta}_m$. Numerical estimation of the motor velocity $\dot{\theta}_{mn}$. (c) Algebraic Estimation of $\ddot{\theta}_m$. (d) Numerical Estimation of the motor acceleration $\ddot{\theta}_{mn}$. (e) Estimation of μ with algebraic estimates of the states. (f) Estimation of μ with numerical estimates of the states.

the controller is updated with the estimated values of the motor parameters and also the observers, and the Coulomb's friction compensation term cancels the perturbation. See Fig.8(b) where a zoom of the steady state is represented. Note that there is not steady state error. Obviously appears noise in the measure because this is considered. Fig.8(c) depicts the trajectory tracking of the system when the PD adaptive controller is used with numerical estimates of the states. The tracking is affected by the noise and appears steady state error (see Fig.8(d)). Fig.9(a) shows the trajectory tracking error

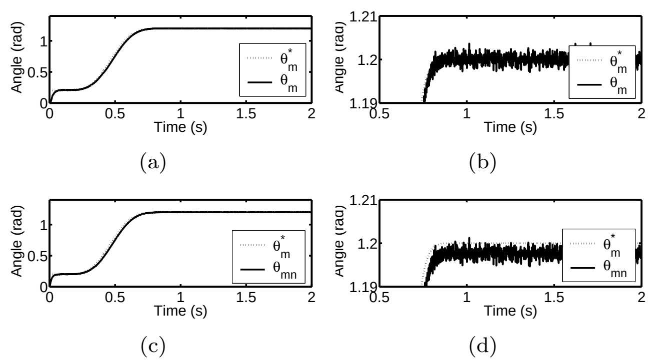
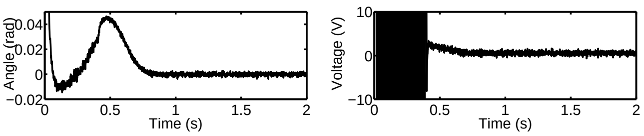


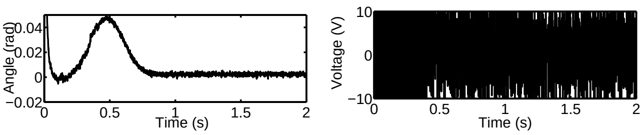
Figure 8: (a) Trajectory tracking with algebraic PD adaptive control. (b) Zoom of the trajectory tracking with algebraic PD adaptive control. (c) Trajectory tracking with adaptive PD control with numerical estimates $\dot{\theta}_{mn}$. (d) Zoom of the trajectory tracking with adaptive PD control with numerical estimates $\dot{\theta}_{mn}$.

of the control system with PD adaptive control and algebraic estimates of the states. Fig.9(b) represents the control input voltage of this proposed control scheme. From time $t = 0.4$ $[s]$ when the controller and observers are updated the control voltage does not saturates the amplifier. We can compare these signals with those of the adaptive PD controller with numerical estimates of the motor velocity and acceleration (see Fig.10(a) and Fig.10(b) respectively). Note that the error signal in steady state is not null, and that the control input voltage always saturates the amplifier. As a consequence, with the algebraic method proposed trajectory tracking tasks are more accurate and the control effort is small and smooth, as a consequence the amplifier does not suffer overheating.



(a) (b)

Figure 9: (a) Trajectory tracking error with algebraic PD adaptive control. (b) Control input voltage of the algebraic PD adaptive control.



(a) (b)

Figure 10: (a) Trajectory tracking error with PD control with numerical estimates $\dot{\theta}_{mn}$ and $\ddot{\theta}_{mn}$. (b) Control input voltage of the PD controller with numerical estimates of the states.

5 Conclusions

The parameter and state estimation method using a fast, non-asymptotic algebraic method, as well as its application to adaptive control has performed successfully on a DC motor model. The methodology only requires the measurement of the angular position of the motor and the voltage input to the motor. Among the advantages of this approach we find: it is independent of the motor initial conditions; the methodology properly compensate the Coulomb's friction torque. This is also robust with respect to zero mean high frequency noises; the estimation is obtained in a very short period of time and good results are achieved; a direct estimation of the parameters and the states are achieved without translation between discrete and continuous time domains; and the approach does not requires a specific design of the inputs for estimating the parameters of the plant.

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