Tests of Fit for Normal Variance Inverse Gaussian Distributions

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Abstract—**Goodness–of–fit tests for the family of symmetric normal variance inverse Gaussian distributions are constructed. The tests are based on a weighted integral incorporating the empirical characteristic function of suitably standardized data. An EM– type algorithm is employed for the estimation of the parameters involved in the test statistic. Monte Carlo results show that the new procedure is competitive with classical goodness–of–fit methods. An application with financial data is also included.**

Keywords: EM – type algorithm, Characteristic Function

1 Introduction

Applied researchers in the area of quantitative Finance have almost unanimously rejected the Gaussian hypothesis for the long term (unconditional) distribution of financial variables. Rather that this, they often operate in the context of the Mixture of Distributions Hypothesis under which it is assumed that the conditional sample returns are Gaussian, with a stochastic (latent) variance. Following the lead of Barndorff–Nielsen [1] many researchers have popularized the inverse Gaussian for modelling the stochastic behavior of the variance; see for instance [4], [10], [2], [9] and [6]. The resulting specification, called the normal variance inverse Gaussian (NIG) distribution, enjoys infinite divisibility, a feature which is desirable for financial modelling, and includes the Normal and the Cauchy distribution as special cases which shows the variety of peakness at the tails that this family encompasses.

In view of the popularity of the NIG model there is always a need to validate this hypothesis on the basis of real data. It should be noted that researchers avoid the use of classical goodness–of–fit procedures and often resort to explanatory data analysis techniques (such as Q–Q plots) to assess the fit of real data to the NIG law. This is due to the complicated structure of the cumulative distribution function and the density of the NIG distribution. The aim of this paper is to provide 'user friendly' goodness–of–fit tests for symmetric NIG (SNIG) distributions. The proposed test makes use of the characteristic function which, unlike the corresponding cumulative distribution function, can be written in a simple closed–form expression. In particular suppose that on the basis of independent copies $X_1, X_2, \ldots, X_n$, on a random variable X we wish to test the null hypothesis

$$H_0\text{: The law of } X \text{ is SNIG}(\delta, c, \lambda), \text{ for some } \delta \in \mathbf{R},\; c > 0 \text{ and } \lambda > 0,$$

where δ denotes location, c scale and λ shape, quantities that jointly define a specific parameterization of the SNIG law.

The remainder of the paper is structured as follows. In Section 2 we define the SNIG law, and use its characteristic function to compute the test statistic. Section 3 deals with estimation of parameters (which appear as nuisance in the test statistic). In Section 4 a Monte Carlo study is presented while Section 5 is devoted to an application with financial data.

2 The test statistic

Let $X|V = v$ be distributed as normal with mean δ and variance v, and assume that V follows an inverse Gaussian distribution with parameters (c^2, λ) and corresponding density

$$f(v) = \frac{c\lambda}{\sqrt{2\pi} v^{3/2}} e^{-\frac{1}{2}\frac{(v-c^2\lambda)^2}{c^2 v}}.$$

Then X follows a SNIG distribution with characteristic function $\varphi(t) = \mathbf{E}(e^{itX})$ given by

$$\varphi(t) = e^{i\delta t} e^{\lambda(1-\sqrt{1+c^2t^2})}. \tag{1}$$

Since (δ, c) are location and scale parameter, respectively, it is natural in the test statistic to consider the characteristic function $\phi(t; \lambda) = e^{\lambda(1-\sqrt{1+t^2})}$ corresponding to the standardized variable $Y = (X - \delta)/c$. Specifically we propose the test statistic

$$T_{n,w} = n \int_{-\infty}^{\infty} |\phi_n(t) - \phi(t; \hat{\lambda}_n)|^2 w(t) dt, \tag{2}$$

where

$$\phi_n(t) = \frac{1}{n} \sum_{j=1}^{n} e^{it\hat{Y}_j},$$

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is the empirical characteristic function of the standardized data $\hat{Y}_j = (X_j - \hat{\delta}_n)/\hat{c}_n,\ j = 1, 2, ..., n$, involving the parameter estimates $(\hat{\delta}_n, \hat{c}_n, \hat{\lambda}_n)$ and $w(t)$ denotes a non–negative weight function. Rejection of the null hypothesis H_0 is for large values of $T_{n,w}$.

By straightforward algebra we have from (2)

$$T_{n,w} = \frac{1}{n}\sum_{j,k=1}^{n}\int_0^{+\infty}\cos(t(\hat{Y}_j - \hat{Y}_k))w(t)dt \qquad (3)$$

$$+ne^{2\hat{\lambda}_n}\int_0^{+\infty} e^{-2\hat{\lambda}_n\sqrt{1+t^2}}w(t)dt$$

$$-2e^{\hat{\lambda}_n}\sum_{j=1}^{n}\int_0^{+\infty} e^{-\hat{\lambda}_n\sqrt{1+t^2}}\cos(t\hat{Y}_j)w(t)dt.$$

Any weight function $w(t)$ which renders the integrals in (3) finite could in principle be used in our test statistic. Particular appeal however lies with weight function which add the extra advantage of computational simplicity to the test statistic. To this end and by considering the formula of the modified Bessel function of the third kind of order zero (see [5], equation 3.961),

$$K_0(\alpha\sqrt{\beta^2+\gamma^2}) = \int_0^{+\infty} e^{-\gamma\sqrt{\alpha^2+t^2}}\cos(\beta t)\frac{1}{\sqrt{\alpha^2+t^2}}dt, \qquad (4)$$

we set $w(t) = (1+t^2)^{-1/2}$ in (3) and denote the resulting test statistic simply by T_n. Then using (4) we have

$$\hat{T}_n = \frac{1}{n-1}\sum_{j\neq k} K_0(\delta_{jk}) + ne^{2\hat{\lambda}_n}K_0(2\hat{\lambda}_n) - 2e^{\hat{\lambda}_n}\sum_{j=1}^{n}K_0(d_j) \qquad (5)$$

where $\delta_{jk} = |\hat{Y}_j - \hat{Y}_k|$, and $d_j = \sqrt{\hat{Y}_j^{\,2} + \hat{\lambda}_n^2}$.

3 ML Estimation of parameters

In this section, the representation $X|V \sim N(\delta, V)$ and $V \sim IG(c^2, \lambda)$ used in the previous section for the SNIG model is employed. This representation is similar to the standard derivation of the normal inverse Gaussian distribution [1] and the density function takes the form

$$f(x;\delta,c,\lambda) = \frac{\lambda e^{\lambda}}{\pi}K_1\left(\frac{r(x)^{1/2}}{c}\right)r(x)^{-1/2} \qquad (6)$$

where $r(x) = c^2\lambda^2 + (x-\delta)^2$. The null hypothesis H_0 is stated in this parametric form, with the corresponding distribution denoted by $SNIG(\delta, c, \lambda)$

The likelihood function is complicated, and therefore it is difficult to obtain the ML estimates by direct likelihood maximization. Instead, we use a EM type algorithm proposed by Karlis [6].

At the E–step on the basis of sample values $x_i,\ i = 1, 2, \ldots, n$, and using the current estimates δ, c, λ, calculate the quantities

$$\begin{aligned} s_i &= E(V \mid X = x_i) = cr(x_i)^{1/2}\frac{K_0(c^{-1}r(x_i)^{1/2})}{K_1(c^{-1}r(x_i)^{1/2})} \\ w_i &= E(V^{-1} \mid X = x_i) = \frac{1}{cr(x_i)^{1/2}}\frac{K_2(c^{-1}r(x_i)^{1/2})}{K_1(c^{-1}r(x_i)^{1/2})}, \end{aligned}$$

while at the M-step update the parameters using

$$\begin{aligned} \delta^{new} &= \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i} \\ \lambda^{new} &= (\bar{s}\bar{w} - 1)^{-1} \\ c^{new} &= \left(\frac{\lambda^{new}+1}{\bar{w}(\lambda^{new})^2}\right)^{1/2} \end{aligned}$$

where $\bar{s} = n^{-1}\sum s_i$ and $\bar{w} = n^{-1}\sum w_i$. The algorithm iterates between the two steps until a stopping criterion to be satisfied. We have used a rather strict stopping criterion, namely we stopped the iterations when the relative change of the log-likelihood was smaller than 10^{-8}.

It must be pointed out that the normal distribution results from the SNIG as $\lambda \to \infty$. Therefore estimating the parameter λ for normal samples (or samples that are too close to the normal distribution), often results in a divergent estimate of λ which may cause numerical instability.

4 Simulations

In this section we present the results of a Monte Carlo study for the new test given by (5), denoted by CF. The Monte Carlo study was implemented by drawing 1000 samples of size $n = 100$, $n = 200$ and $n = 500$. For comparison purposes we also include results on the classical Cramér–von Mises (CM) test; The CM statistic is computed as (see [3])

$$\mathrm{CM} = \frac{1}{12n} + \sum_{j=1}^{n}\left(F_{\hat{\lambda}_n}(\hat{Y}_j) - \frac{2j-1}{2n}\right)^2$$

where $F_\lambda(x)$ denotes the cumulative distribution function corresponding to the standard SNIG density. This distribution function was computed numerically, as

$$F_\lambda(x) = \int_{-\infty}^{x} f(u; 0, 1, \lambda)du,$$

where $f(\cdot; \delta, c, \lambda)$ is given by (6).

The null distribution of both test statistics depends on the value of the parameter λ, which is unknown. Therefore we resort to a parametric bootstrap procedure in order to obtain the critical point p_α of the test based on the observed values of $X_j,\ j = 1, 2, \ldots, n$:

Table 1: Percentage of rejection for 1000 Monte Carlo samples of size $n = 100$ at 5% (upper entry) and 10% (lower entry) level of significance (CF test)

	CF				
$\lambda =$	0.50	1.00	2.00	3.00	5.00
SNIG(λ)	5	5	5	6	5
	9	10	11	11	12
$\lambda =$	0.50	0.80	1.00	1.20	1.50
AL(λ)	90	19	7	16	51
	95	29	13	25	65
$\lambda =$	0.15	0.25	0.35	0.50	0.75
TU(λ)	11	22	39	73	99
	19	32	52	84	100
$\lambda =$	0.00	0.50	1.00	2.00	5.00
SN(λ)	6	5	6	14	55
	11	12	12	23	67
$\lambda_1 =$	1.00	1.25	1.50	1.75	2.00
ST($\lambda_1, 0.0$)	4	9	15	13	4
	10	15	22	19	12
ST($\lambda_1, 0.5$)	43	32	21	14	4
	56	43	32	21	12
ST($\lambda_1, 1.0$)	100	90	55	19	4
	100	95	70	30	12
$c =$	2.00	5.00	10.0	15.0	20.0
ML(c)	5	5	8	8	8
	10	10	13	15	14
$\lambda =$	0.25	0.50	0.75	1.00	2.00
NW(λ)	100	50	14	8	6
	100	63	22	13	11
$p =$		0.25	0.50	0.75	
NS($p, 1.25, 0.0$)		13	21	25	
		20	31	33	
NS($p, 1.50, 0.0$)		16	17	15	
		25	25	23	
NS($p, 1.75, 0.0$)		12	11	10	
		18	18	14	
NS($p, 1.25, 0.5$)		31	29	25	
		45	40	33	
NS($p, 1.50, 0.5$)		23	22	14	
		33	32	21	
NS($p, 1.75, 0.5$)		13	12	9	
		20	20	16	

Table 2: Percentage of rejection for 1000 Monte Carlo samples of size $n = 100$ at 5% (upper entry) and 10% (lower entry) level of significance (CM test)

	CM				
$\lambda =$	0.50	1.00	2.00	3.00	5.00
SNIG(λ)	4	4	4	5	6
	8	10	8	9	11
$\lambda =$	0.50	0.80	1.00	1.20	1.50
AL(λ)	99	35	5	29	80
	100	50	11	42	88
$\lambda =$	0.15	0.25	0.35	0.50	0.75
TU(λ)	18	38	60	90	100
	28	52	73	96	100
$\lambda =$	0.00	0.50	1.00	2.00	5.00
SN(λ)	5	5	8	24	78
	11	11	13	36	87
$\lambda_1 =$	1.00	1.25	1.50	1.75	2.00
ST($\lambda_1, 0.0$)	4	4	7	8	6
	9	10	14	16	12
ST($\lambda_1, 0.5$)	78	51	27	11	6
	85	67	41	20	12
ST($\lambda_1, 1.0$)	100	99	80	26	6
	100	100	91	39	12
$c =$	2.00	5.00	10.0	15.0	20.0
ML(c)	4	4	5	6	7
	9	9	11	14	12
$\lambda =$	0.25	0.50	0.75	1.00	2.00
NW(λ)	95	21	10	5	5
	99	36	20	12	10
$p =$		0.25	0.50	0.75	
NS($p, 1.25, 0.0$)		6	6	7	
		13	13	14	
NS($p, 1.50, 0.0$)		7	7	9	
		16	15	17	
NS($p, 1.75, 0.0$)		8	7	7	
		16	14	14	
NS($p, 1.25, 0.5$)		52	31	15	
		67	48	26	
NS($p, 1.50, 0.5$)		26	19	11	
		42	30	20	
NS($p, 1.75, 0.5$)		11	10	7	
		22	19	15	

- 1. Compute the observations $Y_j = (X_j - \bar{X}_n)/\hat{\sigma}_n$, $j = 1, 2, \ldots, n$, where $\bar{X}_n = n^{-1}\sum_{j=1}^n X_j$ and $\hat{\sigma}_n^2 = n^{-1}\sum_{j=1}^n (X_j - \hat{X}_n)^2$.
- 2. Based on $\{Y_j\}_{j=1}^n$, compute the estimates $(\hat{\delta}_n, \hat{c}_n, \hat{\lambda}_n)$ and then the observations $\hat{Y}_j = (X_j - \hat{\delta}_n)/\hat{c}_n$, $j = 1, 2, \ldots, n$.
- 3.a. Calculate the value of the test statistic, say $\hat{T}$, based on $\{\hat{Y}_j\}_{j=1}^n$ and $\hat{\lambda}_n$.
- 3.b.1. Generate a bootstrap sample $\{X_j^*\}_{j=1}^n$, from $SNIG(0, 1, \hat{\lambda}_n)$.
- 3.b.2. On the basis of $\{X_j^*\}_{j=1}^n$, compute the $Y_j^* = (X_j^* - \bar{X}_n^*)/\hat{\sigma}_n^*$, $j = 1, 2, \ldots, n$, where $\bar{X}_n^* = n^{-1}\sum_{j=1}^n X_j^*$ and $\hat{\sigma}_n^{*2} = n^{-1}\sum_{j=1}^n (X_j^* - \bar{X}_n^*)^2$.
- 3.b.3. On the basis of $\{Y_j^*\}_{j=1}^n$, compute the estimates $(\hat{\delta}_n^*, \hat{c}_n^*, \hat{\lambda}_n^*)$ and then the observations $\hat{Y}_j^* = (X_j^* - \hat{\delta}_n^*)/\hat{c}_n^*$, $j = 1, 2, \ldots, n$.
- 3.b.4. Calculate the value of the test statistic, say $\hat{T}^*$, based on $\{\hat{Y}_j^*\}_{j=1}^n$ and $\hat{\lambda}_n^*$.
- 3. Repeat steps 3.b.1 - 3.b.4., and calculate M values of $\hat{T}^*$, say $\{\hat{T}_j^*\}_{j=1}^M$.
- 4. Obtain p_α as $\hat{T}^*_{(M-\alpha M)}$, where $\{\hat{T}^*_{(j)}\}_{j=1}^M$ denote the ordered $\hat{T}_j^*$ - values.

The following distributions are simulated:

i. The standard SNIG distribution with density $f(\cdot; 0, 1, \lambda)$ given by (6), denoted by SNIG(λ).

ii. The asymmetric Laplace distribution of [7], $\mathrm{AL}(c, \lambda) = c \log U_1^\lambda U_2^{-1/\lambda}$, with U_1, U_2 independent uniform (0,1) variates.

iii. The skew normal distribution, $\mathrm{SN}(\lambda) = \vartheta \cdot |Z_1| + \sqrt{1 - \vartheta^2} Z_2$, with $\vartheta = \lambda(1 + \lambda^2)^{-1/2}$ and Z_1, Z_2 independent standard normal variates.

iv. Tukey's distribution, $\mathrm{TU}(\lambda) = (e^{\lambda Z_1} - 1)/\lambda$, with Z_1 standard normal.

Table 3: Percentage of rejection for 1000 Monte Carlo samples of size $n = 200$ at 5% (upper entry) and 10% (lower entry) level of significance (CF test)

	CF				
$\lambda =$	0.50	1.00	2.00	3.00	5.00
SNIG(λ)	4	4	6	5	6
	9	10	12	12	11
$\lambda =$	0.50	0.80	1.00	1.20	1.50
AL(λ)	100	43	11	34	88
	100	53	19	45	92
$\lambda =$	0.15	0.25	0.35	0.50	0.75
TU(λ)	13	41	79	99	100
	24	56	88	100	100
$\lambda =$	0.00	0.50	1.00	2.00	5.00
SN(λ)	5	4	7	23	89
	10	9	13	33	94
$\lambda_1 =$	1.00	1.25	1.50	1.75	2.00
ST($\lambda_1, 0.0$)	4	19	30	19	4
	10	29	38	28	9
ST($\lambda_1, 0.5$)	80	68	50	22	4
	87	79	61	32	9
ST($\lambda_1, 1.0$)	100	100	93	38	4
	100	100	97	50	9
$c =$	2.00	5.00	10.0	15.0	20.0
ML(c)	7	8	10	10	12
	13	14	17	18	19
$\lambda =$	0.25	0.50	0.75	1.00	2.00
NW(λ)	100	82	27	11	6
	100	90	39	19	13
$p =$		0.25	0.50	0.75	
NS($p, 1.25, 0.0$)		25	44	48	
		36	55	59	
NS($p, 1.50, 0.0$)		29	34	24	
		42	43	33	
NS($p, 1.75, 0.0$)		19	16	11	
		27	25	18	
NS($p, 1.25, 0.5$)		70	64	51	
		80	76	60	
NS($p, 1.50, 0.5$)		49	42	24	
		62	53	32	
NS($p, 1.75, 0.5$)		23	18	12	
		33	28	19	

Table 4: Percentage of rejection for 1000 Monte Carlo samples of size $n = 200$ at 5% (upper entry) and 10% (lower entry) level of significance (CM test)

	CM				
$\lambda =$	0.50	1.00	2.00	3.00	5.00
NIG(λ)	4	4	5	6	7
	9	9	9	10	12
$\lambda =$	0.50	0.80	1.00	1.20	1.50
AL(λ)	100	66	8	55	99
	100	78	18	68	100
$\lambda =$	0.15	0.25	0.35	0.50	0.75
TU(λ)	33	68	92	100	100
	45	80	97	100	100
$\lambda =$	0.00	0.50	1.00	2.00	5.00
SN(λ)	6	6	10	46	98
	12	10	17	60	99
$\lambda_1 =$	1.00	1.25	1.50	1.75	2.00
ST($\lambda_1, 0.0$)	5	6	12	13	5
	11	13	24	25	10
ST($\lambda_1, 0.5$)	97	86	58	23	5
	99	92	74	36	10
ST($\lambda_1, 1.0$)	100	100	99	54	5
	100	100	99	68	10
$c =$	2.00	5.00	10.0	15.0	20.0
ML(c)	5	7	9	8	10
	11	12	18	17	18
$\lambda =$	0.25	0.50	0.75	1.00	2.00
NW(λ)	100	39	15	8	5
	100	64	28	15	10
$p =$		0.25	0.50	0.75	
NS($p, 1.25, 0.0$)		7	13	26	
		14	27	44	
NS($p, 1.50, 0.0$)		12	18	18	
		25	30	28	
NS($p, 1.75, 0.0$)		13	13	10	
		24	23	19	
NS($p, 1.25, 0.5$)		84	68	37	
		93	82	53	
NS($p, 1.50, 0.5$)		59	41	21	
		73	58	32	
NS($p, 1.75, 0.5$)		23	18	11	
		37	29	19	

v. The standard stable distribution with shape parameter λ_1 and skewness parameter λ_2, ST(λ_1, λ_2); see [8] for more details on this model.

vi. The mixture of Laplace distributions, ML(c)= $0.5\text{AL}(1, 1) + 0.5\text{AL}(c, 1)$.

vii. The mixture of a standard normal distribution with a stable distribution, $\text{NS}(p, \lambda_1, \lambda_2) = pN(0, 1) + (1 - p)\text{ST}(\lambda_1, \lambda_2)$.

viii. The normal variance Weibull distribution, NW(λ), where $X|V = v$ is $N(0, v)$, and V is distributed as standard Weibull with shape parameter λ.

Some of these distributions have been extensively used by applied researchers for modelling skewed and heavy tailed data.

Note that calculating the CM statistic numerically is not always possible, because the numerical integration routine did not always converge. In fact we found this problem with a few samples either from the stable distribution or from a mixture of a standard normal with a stable distribution. What we did in practice is to discard the 'nonconvergent' sample, and replace it with another sample.

The figures in Tables 1–6 suggest that both tests CF and CM capture the nominal level of significance to a satisfactory degree, the CF–test having a slight edge in this respect. Powerwise the proposed test is seen to be less powerful than the CM–test for AL, TU, and SN distributions. On the other hand the performance of the new test for NS($p, \lambda_1, 0$), ST($\lambda_1, 0$), and NW distributions, coupled with the ease of computation, lead us to suggest the CF–test as a competitive and computationally efficient method for assessing the fit to a conditinally normal variable, with a stochastic variance following an inverse Gaussian distribution.

5 Application to currency exchange rates

In the current section, we apply the method to real data on daily currency exchange rates. These rates are generally known to be symmetric, but leptokurtic with respect to normality; refer to [2] and [4].

Table 5: Percentage of rejection for 1000 Monte Carlo samples of size $n = 500$ at 5% (upper entry) and 10% (lower entry) level of significance (CF test)

	CF				
$\lambda =$	0.50	1.00	2.00	3.00	5.00
SNIG(λ)	5	4	5	5	5
	10	9	10	11	10
$\lambda =$	0.50	0.80	1.00	1.20	1.50
AL(λ)	100	83	24	73	100
	100	90	35	82	100
$\lambda =$	0.15	0.25	0.35	0.50	0.75
TU(λ)	37	90	100	100	100
	50	94	100	100	100
$\lambda =$	0.00	0.50	1.00	2.00	5.00
SN(λ)	5	4	7	51	100
	10	9	12	64	100
$\lambda_1 =$	1.00	1.25	1.50	1.75	2.00
ST($\lambda_1, 0.0$)	4	57	68	43	5
	9	67	75	52	8
ST($\lambda_1, 0.5$)	100	100	94	56	5
	100	100	97	65	8
ST($\lambda_1, 1.0$)	100	100	100	86	5
	100	100	100	93	8
$c =$	2.00	5.00	10.0	15.0	20.0
ML(c)	6	12	19	23	25
	12	22	29	34	36
$\lambda =$	0.25	0.50	0.75	1.00	2.00
NW(λ)	100	100	65	22	5
	100	100	74	35	11
$p =$		0.25	0.50	0.75	
NS($p, 1.25, 0.0$)		66	91	86	
		76	94	89	
NS($p, 1.50, 0.0$)		68	71	47	
		76	79	57	
NS($p, 1.75, 0.0$)		42	38	19	
		53	46	26	
NS($p, 1.25, 0.5$)		99	98	89	
		100	100	93	
NS($p, 1.50, 0.5$)		92	85	50	
		96	91	60	
NS($p, 1.75, 0.5$)		53	47	20	
		66	61	27	

Table 6: Percentage of rejection for 1000 Monte Carlo samples of size $n = 500$ at 5% (upper entry) and 10% (lower entry) level of significance (CM test)

	CM				
$\lambda =$	0.50	1.00	2.00	3.00	5.00
SNIG(λ)	5	4	5	6	5
	10	9	9	10	10
$\lambda =$	0.50	0.80	1.00	1.20	1.50
AL(λ)	100	97	19	92	100
	100	99	32	96	100
$\lambda =$	0.15	0.25	0.35	0.50	0.75
TU(λ)	74	99	100	100	100
	84	100	100	100	100
$\lambda =$	0.00	0.50	1.00	2.00	5.00
SN(λ)	5	5	13	85	100
	10	10	21	91	100
$\lambda_1 =$	1.00	1.25	1.50	1.75	2.00
ST($\lambda_1, 0.0$)	5	14	40	39	6
	10	29	59	51	11
ST($\lambda_1, 0.5$)	100	100	99	61	6
	100	100	100	74	11
ST($\lambda_1, 1.0$)	100	100	100	95	6
	100	100	100	98	11
$c =$	2.00	5.00	10.0	15.0	20.0
ML(c)	6	10	16	18	20
	13	18	27	31	32
$\lambda =$	0.25	0.50	0.75	1.00	2.00
NW(λ)	100	93	41	19	5
	100	98	60	33	10
$p =$		0.25	0.50	0.75	
NS($p, 1.25, 0.0$)		20	53	77	
		37	77	87	
NS($p, 1.50, 0.0$)		43	57	43	
		61	72	58	
NS($p, 1.75, 0.0$)		38	36	18	
		52	49	27	
NS($p, 1.25, 0.5$)		100	99	88	
		100	100	94	
NS($p, 1.50, 0.5$)		96	89	51	
		99	95	61	
NS($p, 1.75, 0.5$)		60	47	20	
		72	61	29	

The data consist of daily currency spot exchange rates for USA, UK and Japan, covering the period from January 1, 1975 to December 31, 2005. The variable of interest is the daily return, computed as $(r_t - r_{t-1})/r_{t-1}$, for consecutive days $t-1$ and t. For each pair of currencies, the corresponding data–set was proken to 31 subsamples (one subsample for each year) of approximately 250 observations each. We apply the test on every subsample, to see whether the law of daily rates of return of currencies follows the SNVIG distribution. Our results indicate a rate of acceptance of 80.65% for the USD–GBP rate, of 83.87% for the YEN–USD rate and of 80.65% for the YEN–GBP rate. Consequently, we suggest the SNIG as a good model for currency exchange rates, a conclusion reached by earlier researchers based on explanatory data techniques.

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