WCE 2008, July 2 C. 2008, London, Parameterization for Shape Optimization of Aerofoils

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Abstract— As shape parameterization defines the design variables for the optimization of some object (geometric knowledge representation), it is very important to apply parameterizations with a low number of control points in order to reduce the dimensionality of the search space. A parameterization for 2D and 3D geometries based on piecewise Bezier curves and surfaces is proposed here. The requested C¹ inter- segment continuity is accomplished by automatically generating additional control points without increasing the number of optimization variables. The computational procedure takes the initially given complex surface or points cloud (2D or 3D), adaptively splits the domain into 2D or 3D patches and iteratively tries to reduce the necessary number of control points while satisfying the requested modeling accuracy. This adaptive parameterization procedure can serve as a geometric data-set compression utility and fits well into evolutionary optimization.

Index Terms—Geometry parameterization, piecewise Bezier patches, shape optimization, aerofoils and blades

I. INTRODUCTION

The traditional engineering approach to design is more or less one of trial-and-error. Geometric shapes are iteratively proposed as candidate designs and subsequently verified for given design requirements by applying engineering analysis. Optimization is a process of determining the best-possible values for the free variables x (continuous or discrete) of a problem, [1], [2], where some objective function defines excellence criteria such as weight, cost, net-present-value (NPV), etc, and constraints provide requests related to sustaining loads, permissible deflections, stresses, eigenfrequences, dimensions, technological constraints, technical regulations, etc, [3], [4].

Optimum design for given functionality is generally an inverse problem, more specifically one of synthesis, since the shape being generated arises as a consequence of the required functionality. The objective is to create the best possible design for the given design specification consisting of a set of excellence and criteria я set of design requirements-constraints. With this respect, optimum design is a process where the excellence criteria and design constraints steer the change of shape towards optimality in an evolution-like procedure. However, full-scale automatic design synthesis based on design specifications is yet to become a routine and mature general methodology.

Integrating the worlds of engineering design (geometric modeling), engineering analysis and optimization also

imposes difficulties as the corresponding tools need to be applies primarily harmonized. This to mutual synchronization, respective communication and transfer of data as well as coordination of all processes. The corresponding numerical packages must be coupled by data exchange (with data mining) processes as well as synchronized execution in sequential and parallel modes with branching when necessary. Appropriate data mining is necessary since different analysis and/or synthesis programs are automatically executed during processing whereby they mutually write and read from each other's native input and output files, possibly having a changing structure. In design optimization, the optimizer changes the values of the optimization variables and consequently both the model parameterized geometric in CAD (computer-aided-design) and the data input on geometry, conditions loading and boundary in FEA (finite-element-analysis), which steers the search process by evaluation of constraints and objective functions.



Fig. 1. Optimization logistics

The complexity of the design process makes it computationally very intensive. In order to obtain solutions in reasonable computer time, numerical provisions such as parallelization (especially suited with evolutionary algorithms) and approximate and surrogate models (such as response surfaces or neural networks) are applied. Moreover, efficient description of 2D and 3D geometric objects is of critical importance in engineering design since it reduces the dimensionality of the search space.

Design optimization is usually viewed as consisting of three different stages [5], [6], [7]: topology optimization, shape optimization and dimensional (sizing) optimization. Shape optimization is a difficult issue, since it involves a substantially changing geometry, the current state of which in each iteration must be communicated to the simulation package (eg. FEA). Typically the entire geometry of the domain changes with boundaries and nodes having new locations and possibly new FE meshing to be undertaken. These difficulties and the need to define efficient shape parameterizations for complex 3D geometries have so far

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prevented shape optimization from being a routine procedure in engineering design in the industry. Shape optimization can also be part of the reverse engineering process, where some object's (undocumented) geometry is scanned in high-density 3D optical technology, post-processed and parameterized to reduce the resulting point clouds into a compact set of data, and then subjected to shape optimization to improve performance.

The fact that in shape optimization the optimization variables also have to steer the forming of the FE mesh led to several approaches in practical numerical implementations, [8], [9], [10]. One is the concept of design elements whose control nodes are directly linked to the optimization variables. Based on the current positions of the control nodes of the design elements, new mesh generation can be carried out in FEA simulation packages. Changes in shape by the optimizer may cause topological or dimensional conflicts with other objects if the optimized object is part of a complex multi-object system.

Evolutionary optimizers and genetic-algorithms [11] are also applied, [12], [13], [14], [15] in shape optimization, providing for efficient multi-objective shape design and Pareto-set generation. Some authors combine topological design and shape optimization in a single integrated process. Binary representation of elements of 2D shape with successive refinement and with subsequent smoothening is another option which is sometimes applied with GA.

The purpose of parameterization of the geometry is to provide for compact representation and coding of shape, simple storage, interactive visualization, geometric transformations, animation of response and simulation of interaction with the environment, etc.

Several approaches to shape parameterization have been proposed [6], [7], [8], [9]. They include simple sets of points along the boundary, design elements, parametric curves or surfaces, and superposition of component shapes (modes).



Fig. 2: Parameterization of 3D shape

The CAD approaches use feature-based solid modeling to define shape, in other words parametric CAD capabilities are used [16], [17], [18], [19]. Standardized graphics exchange formats are used, however up to date they don't offer full provision for parametric and rule-based geometry. Other approaches also use the computer graphics operators to define shape [20], [21].

Unfortunately, all these approaches to parameterization are still insufficiently mature for routine industrial usage. For example the CAD-based approach, in addition to not supplying sensitivity derivatives, lacks robustness since inconsistent (in terms of topology, geometry constraint rules, interference) geometries can be generated by the optimizer, making the grid generation in the FEA simulator impossible. None of the approaches provides fully for all the conflicting requests listed above, and there is no superior generic parameterization scheme that couples well and exchanges data with CAD and simulation packages (remeshing), provides sensitivity derivatives, provides faithful geometric modeling properties with local control, etc, and does all this with a compact dataset to constrain the dimensionality of the search space.

A number of papers present specific cases of coupling the optimizer with CAD software [18], [19]. Optimization methods offer tools for creating the best designs for given criteria, subject to given constraints. Hence, combining the two, CAD and optimization, offers prospects for the classical 'wishful thinking' of the practicing engineer: tools that automatically create the best shape for the specified design functionality, i.e. optimal synthesis.

With the recent developments of the CAD software that include parametric design and feature-based geometry this coupling is becoming increasingly possible, where some of the parameters in the CAD database can be assigned to optimization variables. Feature-based parametric CAD provides the capability of automatically updating the designs based on new values of the parameters, while preserving the integrity and the design intent based on the rules and relations that the user has defined during the initial design process. The parameters linked to the shape optimization variables can be any of the feature-based CAD parameters such as properties of the basic solid modeling primitives, control points of 2D contours, control parameters of the operators evolving 2D contours into 3D shapes, locations of some elements, etc.

There are many examples of parameterizations for shape optimization in recent literature, partial surveys of numerous papers can be found in [7]- [9]. They include curves for shape optimization of airfoils, where multi-point and multi-criteria optimization is typically applied for variable operating regimes. Other examples include shape design of structures, for example plates, bicycle frames, machine elements, vehicle components and body parts, aircraft components, etc. Applications of shape optimization also include die shape design in sheet metal forming, casting shape optimization, optimization of metal forming processes such as forging, shape optimization for fatigue behaviour, tool design optimization, etc.

II. PARAMETERIZATION USING CHAINED BEZIER CURVES AND SURFACES

In shape optimization using parametric curves, Bezier curves, B-splines and NURBS are typically employed because of their favorable properties [20], [21]. They act as approximation curves defined by corresponding control points, which is different to cases where interpolation curves (such as cubic splines) pass through interpolation points. For reasons mentioned above, Bezier curves (Fig. 3) can conveniently be used for 2D shape parameterization due to the following reasons:

- they pass through initial and final control points
- the tangent in the initial point is defined by the initial two control points, the tangent in the final point is defined by the final two control points,
- the n-th derivative of the curve in the initial and final point is defined by the (n+1) initial and final control points respectively

These characteristics of Bezier curves are convenient in providing for inter-segment continuity when chained Bezier curves are applied.



Fig. 3: Bezier curve of degree 3

The equation of the Bezier curve of *n*-th degree for (n+1) control points follows using Bernstein polinomials B

$$\mathbf{P}(u) = \sum_{i=0}^{n} B_{i,n}(u) \cdot \mathbf{P}_{i} = \sum_{i=0}^{n} {n \choose i} \cdot u^{i} \cdot (1-u)^{n-i} \cdot \mathbf{P}_{i}$$
(1)

where $\binom{n}{i} = \frac{n!}{i!(n-i)!}$, P_i are the coordinates of control

points-nodes. Introducing multiple coincident control points has the effect of pulling the curve closer towards the control points. Bezier curves possess the convex hull property. A closed Bezier curve is obtained by specifying coincident initial and final control point. Bezier curves are invariant under affine transformations and they are transformed by transforming their control points. The degree of the curve is directly linked to the number of control points. For example, for n=3 as used here:

$$\mathbf{P}(u) = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \cdot (1-u)^3 \cdot \mathbf{P}_0 + \begin{pmatrix} 3 \\ 1 \end{pmatrix} \cdot u \cdot (1-u)^2 \cdot \mathbf{P}_1 + \begin{pmatrix} 3 \\ 2 \end{pmatrix} \cdot u^2 \cdot (1-u) \cdot \mathbf{P}_2 + \begin{pmatrix} 3 \\ 3 \end{pmatrix} \cdot u^3 \cdot \mathbf{P}_3$$

The derivatives are obtained as [16]:

$$\frac{d\mathbf{P}(u)}{du} = n \cdot \sum_{i=0}^{n-1} \binom{n-1}{i} \cdot u^i \cdot (1-u)^{n-1-i} \cdot A_i \quad , \quad A_i = P_{i+1} - P_i$$
(2)

A Bezier curve does not posses the property of locality since a change in a single control point changes the entire curve. As a consequence, if a higher number of control points are needed for satisfactory description of a certain shape, a high-degree Bezier curve is generated. In addition to not possessing the property of locality, high-degree Bezier curves can also oscillate between control points as they are based on high-degree polinomials.

Bezier curves can be extended to 3D Bezier surfaces with corresponding properties, [16]:

$$\mathbf{P}(u,v) = \sum_{i=0}^{n} \sum_{j=0}^{m} B_{i,n}(u) \cdot B_{j,m}(v) \cdot \mathbf{P}_{i,j} , \quad u,v \in (0,1)$$
(3)

where $P_{i,j}$ are control points- nodes of the control polihedron.



Fig. 4: Control polyhedron for Bezier surface of degree 3

The degree of the surface is determined by the respective numbers of control points in the respective directions, like

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with Bezier curves. The Bezier surface is essentially obtained by blending Bezier curves; Bezier control points of a Bezier curve are replaced by Bezier curves in the orthogonal direction.

$$\mathbf{P}(u,v) = \sum_{i=0}^{n} B_{i,n}(u) \cdot \left(B_{0,m}(v) \mathbf{P}_{i,0} + B_{1,m}(v) \mathbf{P}_{i,1} + \dots + B_{m,m}(v) \mathbf{P}_{i,m} \right)$$
(4)

III. PROPOSED PARAMETERIZATION PROCEDURE

The objective is to develop an automatic procedure that adaptively determines the necessary number of control parameters (as compact as possible for given requirements for accuracy of representation) and splits the domain into patches with sufficient continuity. This procedure could also act as an interface that can accept some given dense points-cloud as input, and reduce the necessary number of control points to a minimum for requested accuracy ('data-set compression'). These control parameters will subsequently be used as a compact set of shape optimization variables. The whole process would resemble reverse engineering.

The approach in this paper is limited in its objectives to proposing a compact optimization-oriented 2D and 3D parameterization by automatically chaining low-order Bezier curves and surfaces into complex shapes, which will provide numerical simplicity, good geometric modeling capabilities with localized control, and compact dimensionality of the resulting search space. It can also be seen as a lossy data-set compression technique where the trade-off between data-set compression and representation accuracy is adjustable and controllable. Since the numerical effort is strongly correlated with the number of variables (design space) and the number of objective function evaluations in the search space, a reduction of the dimensionality of the search space can be critically important for the efficiency of the optimization process.

The parameterization procedure developed here is based on a simple method of chaining Bezier curves with C^1 continuity, while it can also be extended to C^2 and higher-order continuity (C^0 = continuity of function value, C^1 = continuity of slope, C^2 = continuity of curvature)



Fig. 5: Chained Bezier curves, three curves of degree 3

The simple procedure for parameterization developed here is based on automatic generation of additional control points in segments where neighboring Bezier curves join, such that it does not increase the number of optimization variables. Since the additional control points are automatically generated from existing ones, the number of optimization variables (original control points) is not increased while continuity is fully provided for.

Similar computational procedures are developed for the representation of complex 3D surfaces, based on chaining individual Bezier surfaces as patches in both directions. Arrays of additional control points are generated along the segments where individual patches join, as shown with dashed lines on Fig. 6.



Fig. 6: Control polyhedron for a chained Bezier surface with generated additional points at segments where individual surfaces join (dashed lines)

The computational procedure is shown in Fig. 7:



Fig. 7: Flowchart of the process

The procedure interpolates a number of additional Bezier points in addition to the original control points, hence, it does not increase the number of optimization variables. On the other hand it provides full flexibility in automatic splitting of complex surfaces into low-order Bezier surfaces with C^1 continuity without the need for user interventions.

The procedure in Fig. 7 can itself be part of an optimization process. It can serve in determining the optimal subdivision of some given complex surface (or a given points cloud) into patches by optimizing the parameters listed as inputs in Fig. 8: number of patches in x and y directions and number of

control points in x and y directions (degree of Bezier curves). This optimization of the respective parameterization of the chained surface patches is performed using the minimum offset from original surface or points cloud as the objective and possibly expressing the permissible total offset as a constraint.



Fig. 8. Determining the values of parameters for optimal subdivision of a complex surface or points cloud into Bezier patches

IV. RESULTS

In the first example, the data compression aspect of the developed parameterization is demonstrated on the example of the airfoil NACA 4413. A 'complete' data set consisting of 82 points defining the shape is imported, and some of those points (with some arbitrarily chosen increment) are selected to be the initial control points (design variables), which is shown as 'initial parameterization in Figs. 9-11. The values of these variables are then optimized to yield a curve with a minimum offset from the given complete data-set (Figs. 9-11), using non-gradient (Nelder-Mead), gradient (BFGS), and evolutionary (GA) unconstrained optimization. All three optimizers are found to converge properly, BFGS naturally needing far less iterations. The 'offset' is evaluated as the cumulative distance between the points on the given airfoil and corresponding ones on the parameterized curve.

It is shown that a significant reduction of the number of control points can be achieved with acceptable values of the total offset. It illustrates that by applying this procedure before shape optimization a significant decrease in optimization computer time due to reduced dimensionality of the search space can be achieved. It also leads to simplification in the data exchange between the optimizer, the CAD software and the FE analysis software and accelerated data mining.

For the cases in Figs. 9-11, the following applies:

Number of points defining NACA 4413 contour =82

Objective function: minimum offset between the given set of points on the contour of the NACA airfoil and the chained Bezier approximation

Optimizers: BFGS, Nelder – Mead, GA, [22] and in-house development

The cases in Figs. 9-11 present different combinations of number of chained curves, their degree related to respective

numbers of control points and positions of Bezier control points. Both original control points corresponding to optimization variables and interpolated additional control points are shown. The values of control points and total offsets are listed below.

Case 1:

Degree of individual chained Bezier curves =3

Number of design variables defining the chained Bezier curves =12Expanded number of control points for chained Bezier curves =16Number of chained Bezier curves =5

Optimized values of Bezier points (x / y):

2.999 2.902 2.703 2.232 2.083 1.973 1.986 2.357 2.589 2.896 2.951 3.028

0.000 0.030 0.086 0.129 0.082 -0.002 -0.047 -0.036 -0.021 -0.005 -0.002 -0.000

Total offset: 2.3751e-006



Fig. 9. Chained Bezier curves for NACA 4413 airfoil parameterization, case 1

Case 2:

Degree of individual chained Bezier curves =4

Number of design variables defining the chained Bezier curves =11 Expanded number of control points for chained Bezier curves =13 Number of chained Bezier curves =3

Optimized values of Bezier points (x / y):

2.9393 1.9544 2.0122 3.1383 2.1923 2.1863 1.9700 2.9443 3 2130 2.8483 0.1254 0.1291 3.0190 0.0002 0.0417 -0.0002 -0.0426 -0.0242 -0.0066 -0.0002 -0.0001 -0.0003 Total offset: 0.0018



Fig. 10. Chained Bezier curves for NACA 4413 airfoil parameterization, case 2

Case 3:

Degree of individual chained Bezier curves =5 Number of design variables defining the chained Bezier curves =14 Expanded number of control points for chained Bezier curves =16 Number of chained Bezier curves =3 Optimized values of Bezier points (x / y): 2.899 3.153 2.453 2.431 2.165 1.935 2.090 1.771 2.611 2.907 2.857 2.931 2.928 3.071 0.000 0.009 0.114 0.149 0.113 -0.002 -0.043 -0.062 -0.022 -0.004 -0.002 0.000 -0.001 -0.001 Total offset: 1.2705e-004



Fig. 11. Chained Bezier curves for NACA 4413 airfoil parameterization, case 3

In the second example shown in Figs. 12-14, the proposed parameterization is applied to wind turbine blades with a complex 3D geometry. A similar approach can be applied to describing a general 3D geometry. In the following case, a wind turbine blade based on NACA 4413 airfoils is the object to be described using a reduced number of control points. The objective is to apply low-order Bezier patches and have C^0 and C^1 continuity in both directions.

The initial description of the blade is given by the NACA 4413 contour points (as in example 1) and the following control data which describe the scaling and rotation of the airfoils along the radial axis:

Radii = [0.344 0.472 0.600 0.728 0.856 0.984 1.112 1.240 1.368 1.496 1.505 1.510];

Length = [0.177 0.168 0.160 0.151 0.143 0.134 0.126 0.117 0.109 0.100 0.08 0];

Angle = [11.0 9.4 8.0 6.7 5.6 4.6 3.8 3.1 2.5 2.1 2.1 2.1];

which gives a total of 82 * 12 points on the 3D surface. In the case shown below, 6 different radii are selected with 10 airfoil control points. The following figures show the resulting control points (Fig.12) and the corresponding 3D visualization (Figs.13, 14) obtained with chained Bezier surfaces of degree 3 in both directions:



Fig. 12. Control points for chained Bezier patches for NACA 4413 based wind turbine blades (both original points- design variables and generated additional points)



Fig. 13. NACA 4413 based wind turbine parameterized using chained Bezier patches



Fig. 14. NACA 4413 based wind turbine parameterized using chained Bezier patches, alternative view (complex 3D shape due to different aerodynamic conditions along the blade)

V. CONCLUSION

This paper presents an approach to parameterization of complex 2D and 3D shapes based on adaptive chaining of Bezier curves and surfaces. The continuity is imposed by automatically interpolating auxiliary control points in segments where neighbor surfaces join. In general terms, the representation of a complex 3D shape (or points cloud) can be reduced to a compact set of design variables, and the procedure proposed optimizes the necessary number of Bezier patches, their degree and positions of control points for given requirements of accuracy. The procedure leads to efficient shape optimization due to resulting low dimensionality of the search space. The procedure developed can serve as a geometric data-set compression utility and as an interface between the geometric model and the evolutionary optimization procedure.

The procedure is illustrated on 2D NACA 4413 airfoils and 3D wind turbine blades based on the same airfoil.

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