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Behavioural Finance Or Efficient Market: Which Is Right ?

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Abstract—**Our main is to show that Behavioural Finance and Efficient Market arguments may coincide by using a simple anomaly model. Namely, we consider two identical assets A and B which have not the same price at any time. The asset price A reflects always its fundamental value while asset price B may be mispriced. We show that the mispricing for asset price B can be interpreted as a chance or as a limit of arbitrage explaining why it is difficult to distinguish between the two arguments. So this result gives a reason to find methods which try to reconcile the two theories by using dynamic models.**

Keywords: Behavioural Finance, Efficient Market, Optimal Asset Allocation.

1 Introduction

In traditional framework (rational agents, no frictions), a security's price equals its fundamental value i.e the discount sum of future returns. The underlying hypothesis that price reflects its fundamental value is known as the Efficient Market Hypothesis. If the market is efficient, there is no arbitrage opportunity. In fact, any deviation from no arbitrage values must trigger an immediate reaction from markets and the rapid disappearance of the mispricing. However, empirical evidence shows that assets can deviate from no arbitrage values for a long time. For example some stocks are overvalued or undervalued in the past. An often cited example of overpricing is the Internet stocks, see Ofek and Richardson [16]. These anomalies sometimes persist in financial markets before disappearing. The fundamental question is how to explain the presence and persistence of these anomalies ?
The first explanation supposes that all market participants are not fully rational. For example, there are participants who use their belief or sentiment to evaluate an asset instead of using only relevant information. We call these participants noise traders. The others who use only relevant information are called rational traders or arbitrageurs. The presence of noise traders sometimes can influence the asset price. A well-known objection to this point of view, see Friedman [7] or Fama [4], is that even if noise traders were to create such deviations, it would be for a small time. In fact, the argument given is that these noise traders often buy over- valued assets and sell under-valued assets. This behavior implies a loss of money and by the same diminish their influence. Some people are simply going to disappear by this process of loss of money leading them to failure and the others by experience are converted to rational investors so that for a bit, only rational agents remain. However, this argument is not shared by all.
First Figlewski [6] underlines that the process tending to lead the irrational towards failure may persist before being realized. In fact, even if a noise trader for example buys over-valued assets, one may have by chance that the realization of the first asset returns be higher than the average in the short term. So there is not a systematic loss of money for this agent who continues to be influent in the market. The possibility that the noise traders persist on the short term before disappearing constitutes a risk for the arbitrageurs. This form of risk is called *fundamental risk.* Some authors studied models where arbitrageurs are confronted to this *Fundamental Risk.* We refer to Shiller [18], Campbell and Kyle [2].
Another incurred risk for arbitrageurs is the no previsibility of the noise traders' behavior. In fact, in their valuation of assets' price, they incorporate their feeling or sentiment (optimism, pessimism, $\cdots$) instead of trusting only available information. As a result, the value of an asset may deflects with respect to its fundamental value. The sentiment or belief that they have, may diminish or augment in the short term. This fact constitutes also a risk for an arbitrageur who attempts to play against them. This form of risk (no possibility to forecast the noise traders' feelings) is called *Noise Trader Risk.* For more information on this concept, we refer to Shleifer and Summers [19].
Other facts which can limit arbitrage are synchronization risk, see Abreu and Brunnermeier [1] and transaction costs, see Tuckman and Vila [21], Mitchell and al. [15] .
All concepts mentioned above give no riskless arbitrage so that arbitrageurs are reticent to exploit them. Their risk is also accentuated by the fact that if their loss attains a certain level, one can demand them to liquidate their portfolios in the case where they are not owners. In general, it is the case as Shleifer and Vishny [20] underlined it. Of course, if an arbitrageur

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owns his portfolio or if he has a long time horizon, the arbitrage is sure since the asset prices will converge to its fundamental value. Let us note that models mixing arbitrageurs and noise traders begin to take largeness in the literature. It comes from to the appearance of certain anomalies in the financial markets which can not be explained with the hypothesis that all agents are fully rational as the crash of October, 1987.

An other way to explain anomalies is only by chance. Fama [5] underlines for example that both under-reaction and over-reaction are found on asset return's anomalies. He interprets these facts as just chance results. He shows also that over-valuation and undervaluation tend to disappear with changes in the way they are measured. Our objective is not to take part between Behavioural or Neo-classical Finance but to propose for example a model which shows that both arguments may be valid. The logic which is behind is to reconcile the both theory instead of trying to oppose them. This will be possible by supposing that efficient market is not a 0/1 property but varies continuously over time and across markets as mentioned by A. Lo [12]. Another way to reconcile them is to use a shrinkage approach where for example Efficient and Behavioural arguments appear. These models are base on heterogeneity and bounded rationality Hypothesis (Adaptive systems).

The outline of the paper is the following. In the section 2, we construct a model which simulates a market where Law of One Price is not satisfied. In section 3, we show that the failed of Law of One Price may be interpreted by chance (Efficient Market Hypothesis) or by limits to arbitrage(Behavioural Finance). Section 4 gives some comments on our model with respect to empirical observations. Section 5 concludes.

2 A market model without Law of One Price.

If we suppose that all agents are fully rational, a market does not admit arbitrage where arbitrage means earn some money without taking any risk. If such arbitrage exists, it is immediately exploited by all agents and it disappears by the Law of supply and demand. So the Law of One Price hold: two identical assets have the same price at any time. Here we want to develop a simple model of anomaly. Namely, we suppose that there exists an interval of time under which two identical assets have different prices. To do this, we consider a market with three assets. The first asset is no risky and the two latest are risky and similar. We denote the two risky assets by A and B[1].

[1] Assets A and B are identical. We may also consider asset B as a portfolio which replicates asset A.

We suppose the markets for assets A and B are segmented. Some agents, A-investors, can only invest in asset A and in the riskless asset while others, B-investors, can only invest in asset B and in the riskless asset. We take market segmentation as given. We simply assume that A-investors are fully rational so that the asset A reflects always its fundamental value. Alternatively, we suppose that B-investors are composed on two categories. The first category is fully rational traders and the second is noise traders (behavioural traders) category. There exists a third category of investors called arbitrageurs who can invest in both risky assets. Our model is similar to Gromb and Vayanos [9] except that we have supposed that the market where asset A is traded is efficient. Here, the two asset prices may defer because asset B can be influenced by the noise traders.
We choose the continuous time framework to model the dynamic of asset prices A and B. This allows to use the stochastic calculus for the optimal allocation for a given arbitrageur to exploit discrepancies between the two asset prices. We denote respectively $X = (X_t)_{t\in[0,T]}$ and $Y = (Y_t)_{t\in[0,T]}$ the price process of the two assets A and B where T is the horizon time. We suppose that $X_0 = Y_0 = C > 0$ that means assets A and B have the same initial price. Due to the fact that we treat with financial series, one chooses as dynamic for asset A a stochastic volatility model. The process X is modeled by:

$$dX_t = (r + \eta V_t)X_t dt + \sqrt{V_t} X_t dB_t^1 \ . \tag{1}$$

$$dV_t = (\alpha - \beta V_t)dt + \sigma\sqrt{V_t} dB_t^2 \ . \tag{2}$$

This dynamic for volatility was introduced by Cox, Ingersoll, Ross [3] and it assures that volatility is always strictly positive. Parameters r, η, α, β, σ are positive constants. The parameter β is the coefficient pointing out the speed on which volatility goes back in its long-term average represented by $\frac{\alpha}{\beta}$. B^1, B^2 represent two standard Brownian motions with correlation ρ . The correlation allows to take account the leverage effects seen on financial series.
Here r represents the rate for the riskless asset so that ηV_t is the risk premium associated to the asset A. This form of the risk premium specifies that the risk is proportional to the volatility. Some authors used this form of risk premium. We refer among others Merton [14], Pan [17].
Now we specify the dynamic of the process Y such that to violate the Law of One Price. We have several choices to do it but we have to link the dynamic with the process X since both assets are similar. We choose the dynamic of Y such that the two asset prices are equal until an unpredictable date T_1 and after they begin to differ. Also, we give the same expected returns for both processes to reflect the fact that they are similar. So the process (Y_t) may be seen as a modified version of the process (X_t).

The above comments motivate us to choose for Y, the following dynamic:

$$dY_t = (r + \eta V_t)Y_t dt + \sqrt{V_t} Y_t dB_t^1 + Z Y_{t-} dN_t \ , \qquad (3)$$

where N is a Poisson process with intensity λ.
The dynamic of V_t is given by (2). The random variable Z represents a random price-jump size. It has a support in $[-b_1, b_1]$ where b_1 is such that $-1 < b_1$ to guarantee the strict positivity of the process Y.
In our model, assets A and B having the same characteristics, they react therefore in the same way against news which arrive. It means that the instant of jumps introduced above is not link to the appearance of a basic information as it is supposed on models with jumps. The jumps mean only appearance of external noise or shock on the asset B. That's why, we have to make some particular hypotheses on the the random price-jump size Z. We suppose the following conditions

$$Z \ has \ a \ symmetric \ distribution. \qquad (4)$$

$$[inf\ Z, sup\ Z] = [-b_1, b_1] \ with \quad -\epsilon < -b_1 < 0 < b_1 < \epsilon \qquad (5)$$

where $\epsilon < 1$.
The condition (4) compels the random variable Z to have zero mean. If not, there is no hope to allow assets A and B having the same tendency. In fact, if $E(Z) = \mu \neq 0$, then we should deduct effect tendency that jumps would have caused. In that case, X and Y diffusions will not have the same drift and this is in contradiction with the hypothesis that assets are similar.
Condition (5) assures that the size jumps can not be too big. We recall that the jumps are interpreted as external effects independent from fundamental information and which touch the asset B.
The instants where external effects appear in the market are obtained by the following stopping times (T_k) defined by

$$T_0 = 0, \ T_{2k+1} \ = \ inf\left\{t > T_{2k}, \ \Delta N_t \neq 0\right\} \wedge T, \ \ (N_0 = 0)$$

$$T_{2k+2} \ = \ inf\left\{t > T_{2k+1}, \ \Delta N_t \neq 0\right\} \wedge T \ ,$$

where $k \in \mathbb{N}$.
The interval $[T_k, T_{k+1}[$ measures the length between the appearance of two shocks. We simulate the processes X and Y to see in a more clear manner the different scenarios that we may have. The figures are reported in the bottom. On FIG.1, we choose a uniform random variable $\mathcal{U}[-0.1, 0.1]$ for Z and for the model's parameters, we choose: $X_0 = Y_0 = 1, \ V_0 = 0.14, \ r = 0.05, \ \eta = 0.86, \ \alpha = 0.15, \ \beta = 1.1, \ \sigma = 0.02$. The process X is represented by a fill plot and the process Y by a plain plot.
FIG.2 is obtained for the same parameters except for the random variable Z which becomes $\mathcal{U}[-0.05, 0.05]$.
In both figures, the intensity of jumps is equal to $\lambda = 5$, the maturity is normalized to $T = 1$. For the simulation, we use as step, $\delta = 10^{-3}$.

We have defined a market model which takes account the fact that Law of One Price is not satisfied. The second step is to understand how can we interpret these anomalies. Generally, anomalies are found by specifying an asset pricing model. So giving some interpretation is not obvious since we have not idea on the real model which is behind. Here we can support our conclusion in the sense we know where anomalies come from.

3 Interpretation of the anomaly.

There is a long debate between proponents of Efficient Market Hypothesis and Behavioural Finance on what is behind anomalies found on financial times series. For efficient Market proponents, noise traders can not be influent by the presence of arbitrageurs. So if an asset is under-valued or over-valued during a time, it is only by chance. Behavioural Finance proponents say the contrary, noise traders may deviate asset price to its fundamental value for a long time since arbitrage is risky and costly giving a Limit to arbitrage. The anomaly model developed in the previous section allows to test these arguments.

3.1 Is Market Efficient ?

In our model, the first explanation we can give is that anomalies result by chance as mentioned by Fama [5]. The shocks which appear in the model are not predictable since they come from a jump of poisson process. So we can not find information which highlight when anomalies appear. Also, we have no signal which indicates when the shock would be positive or negative in other words when noise traders over-react or under-react. At last, the under-valuation or over-valuation for asset price B arrives randomly with our hypothesis that the random variable Z has a symmetric distribution, see (4. These assumptions imply that $E(Z) = 0$ and $P(Z \leq 0) = P(Z \geq 0) = \frac{1}{2}$. So in this market, we may suppose that market is efficient and anomalies (over-valuation and under-valuation) are due to chance since in mean asset B reflects always its fundamental value(Assets A and B have the same drift). So rational traders lead the market and noise traders even if they exist, have no influence (their effect is seen as a chance). The process Y may be seen as a noise of the process X. This fact is highlighted if we increase the intensity of jumps λ.

Behavioural Finance has shown some over-valuation or under-valuation of asset prices as in our example (with substitute) and explained it by Limits to arbitrage instead of chance. So in the next, we see if the model may be interpreted as Limits to arbitrage

3.2 Is there Limits to arbitrage ?

To respond on this question, we have to verify that arbitrage is risky.
The simulations made tend to underline this point. One notes on certain scenarios that the strategy which consists to play against the noise traders implies lost, see FIG.1, scenario (b). In others, we may win and lose money during the interval $[0,1]$. It is the case for the scenarios (a), (c) of $FIG.1$.
We find approximatively the same phenomena on $FIG.2$. So the introduction of Z creates no riskless arbitrage.
However this fact is it sufficient to discourage arbitrageurs to exploit arbitrage opportunities ? We have to respond on this question by studying the optimal asset allocation for an arbitrageur to exploit discrepancies between the two assets. The study is done on the interval $[0,T]$ where $T < \infty$. We suppose that any arbitrageur has a finite initial wealth $x > 0$. He constitutes with this wealth a portfolio containing the two risky assets A and B where the prices are represented by the processes X and Y. The portfolio does not contain the riskless asset since his objective is to exploit noise traders' misperceptions which appear when X_t and Y_t are different. If we note W_t the value of the portfolio at time t, ζ_t and ϕ_t respectively the fraction of wealth invested in the asset A and the asset B then the dynamic of the portfolio W is given by :

$$dW_t = \zeta_t W_t \frac{dX_t}{X_t} + \phi_t W_{t-} \frac{dY_t}{Y_{t-}} \ .$$

We have supposed that the wealth process satisfied the self-financing condition. There is nor income nor withdraw on the interval $[0,T]$. Using (1), (3) and the fact that $\zeta_t = 1 - \phi_t$, the dynamic of the wealth process can be expressed in terms of ϕ_t only. So, one has:

$$dW_t = W_t \Big(r + \eta V_t \Big) dt + W_t \sqrt{V_t} dB_t^1 + W_{t-} \phi_t Z_t dN_t \ .$$

The arbitrageurs being rational speculators, we suppose they all have preferences which can be materialized by utility functions. We denote by $\mathcal{U}$ the set of increasing, strictly concave functions which are $\mathcal{C}^2$ (twice continuously differentiable). We suppose that each arbitrageur has a utility function U in the set $\mathcal{U}$. Any arbitrageur maximizes the utility at time T that he gets through his portfolio. He solves the following problem:

$$\sup_{\phi_s, 0 \le s \le T} E_P[U(W_T)] \ . \tag{6}$$

We suppose also that any arbitrageur chooses ϕ such that, even a jump arrives, his portfolio remains always strictly positive. So he makes the following constraint $:W_t > 0 \ \forall t \in [0,T]$. The following proposition shows that the fraction of wealth invested by an arbitrageur in the asset B depends on the size length jump of the random variable Z.

Proposition
The positive constraint for wealth implies that the process (ϕ_t) is bounded and we have $\forall t, \ \ -\frac{1}{b_1} \le \phi_t \le \frac{1}{b_1}$.

Proof. The proof is similar to Liu et al. [11], we refer to them.

The above proposition shows that the arbitrageur has a lot of choice to take his fraction of wealth related to asset B. In fact, he can invest at time t the fraction ϕ_t which belongs to the interval $[-\frac{1}{\epsilon}, \frac{1}{\epsilon}]$. This last interval is however large since ϵ is small.
The next stage consists to find the optimal allocation associated to the arbitrageur's issue (6). We use the Hamilton Jacobi Bellman (HJB) approach. So we introduce the value function $J(W,V,t)$ defined by

$$J(W,V,t) = \sup_{\phi_s, t \le s \le T} E_P[U(W_T)] \ . \tag{7}$$

Note that the value function J inherits the proprieties of the utility function U. That's why it is also called the indirect utility function, see Merton [13].
If we note $J_t = \frac{\partial J}{\partial t}$ the derivative of J with respect to t and $J_{xy} = \frac{\partial^2 J}{\partial x \partial y}$ the second derivative of J with respect to x and y, the principle of optimal stochastic control leads to the following (HJB) equation for J,

$$\max_{\phi} \Bigg\{ J_t + W(r + \eta V) J_W + (\alpha - \beta V) J_V + \frac{W^2}{2} V J_{WW} + \frac{\sigma^2}{2} V J_{VV} + \rho\sigma V W J_{VW} + \lambda E\big[J(W(1+\phi Z),V,t) - J(W,V,t)\big] \Bigg\} = 0, \tag{8}$$

where the expectation is taken with respect to the distribution of Z.
We remark that, our (HJB) equation is simple to resolve. The control ϕ appears only in one term linked to the jump event. The explanation comes from to the arbitrageur's portfolio which contains only risky assets contrary to the general framework where a riskless asset is added to the portfolio. We recall that our objective is to see how to exploit optimally the arbitrage which exists in the market due to the fact X and Y have different price in some intervals.

Generally, to resolve an HJB equation, we have to specify a form for the value function J. Here, the optimal allocation ϕ^* can be found without this specification. A necessary condition for ϕ^* to be optimal is :

$$E\left[WZ\ J_W\left(W(1+\phi^* Z), V, t\right)\right] = 0\ . \qquad (9)$$

This condition is also sufficient since J is a concave function in W. It remains to determine the roots of the function

$$g:\ \phi \longrightarrow \quad E\left[WZ\ J_W\left(W(1+\phi^* Z), V, t\right)\right].$$

to obtain the optimal allocations.
The function g is a strictly decreasing function since

$$g'(\phi) = E\left[(WZ)^2\ J_{WW}\left(W(1+\phi^* Z), V, t\right)\right] < 0$$

So there exists at most one ϕ^* solution of (9).
The optimal ϕ^* is given by $\phi^* = 0$. In fact, we have for all $U \in \mathcal{U}$ and for Z satisfying (4),

$$g(0) = E\left[WZ\ J_W(W,V,t)\right] \ = \ WJ_W(W,V,t)\ E[Z] = 0.$$

This shows that the anomalies may be interpreted as limits to arbitrage since the optimal allocation is to invest only on asset A even if $X_t \neq Y_t$ for any arbitrageur.[2]
We end by some comments.

4 Comments

In the above model, we have seen that anomalies (over or under valuation) can be interpreted either by chance or by Limits to arbitrage. In the first case, asset price B reflects its fundamental value and the mispricing is due to chance. In the second case, anomalies are interpreted by noise traders effects who influence the asset price B such that it is riskier for rational traders to play against them. The reason is that, the dynamic of the asset B contains some factors appeared to limit arbitrage. *The noise trader risk* occurs in our model if for two successive stopping times, the realization of the random variable Z keeps a constant sign. This event corresponds to the case where the effects of the mispricing worsen.
Synchronization risk is modeled by the level of the random variable Z which can take any value in the interval $[-b_1, b_1]$ where $0 < b_1 < 1$. This level of Z may be seen as the level of interaction between rational and noise traders. The difference between $X_s - Y_s$ after a shock depends on the number of arbitrageurs who play against the noise traders. The correction only occurs when the arbitrageurs' action exceed those of the noise traders. exceeds the noise traders absorption.
Jarrow and Protter [10] constructed a complete marked model where arbitrages exist but are not exploited since they are hidden. They occur on a small set of time typically of Lebesgue measure zero. Here we have an incomplete market model where arbitrage opportunities occur on intervals (No hidden arbitrage) but are not exploited by the argument of Limits to arbitrage.
An other point is about our approach to model anomalies. We have to show that the proposed model is not contradictory to the empirical observations. In fact, as Friedman [7] said, a model is to be judged by its predictive power for the class of phenomena which it is intended to explain. Namely, we have to see if the model reproduces momentum and long time reversal effects since they are the most often cited anomalies in the literature. The momentum effect is integrated in our model by events where two successive shocks have the same sign (noise trader risk). For example if the two first shocks are positive, we have $X_{T_1} < Y_{T_1}$ and $X_{T_2} < Y_{T_2}$ where $(Y_{T_1} - X_{T_1}) < (Y_{T_2} - X_{T_2})$ (increase of mispricing). The reversal effect is integrated by the assumption that $E(Z) = 0$. If we have for the first stopping times events $\{Z > 0\}$, we will end by events $\{Z < 0\}$ since the random variable Z is centered. So we obtain a reversal effect. Also if we begin by negative shocks, we will end by positive shocks for the same reason.
At last, the model shows also the empirical observation that anomalies may be large. An often cited example is the twin shares composed by Royal Dutch (60 %) and Shell (40%). So the parity was 1.5. However Royal Dutch was sometimes 35% underpriced with respect to parity and sometimes 15% overpriced, see ([8]). In our model, the optimal solution $\phi^* = 0$ is independent to the support $[-b_1, b_1]$ of the random variable Z. So taking b_1 big gives a model with large mispricing.

5 Conclusion.

Several anomalies such that excess of volatility, under-valuation, over-valuation, etc..., have been found on the literature. The main question is how to explain these anomalies ? Since 1990 the proponents of Efficient Market Hypothesis and Behavioural Finance have developed models which comfort their position. So to highlight this conflict, it is useful or necessary to dispose a type of anomaly model to test the both arguments which are Chance for Efficient market proponents and Limits to Arbitrage for Behavioural Finance proponents. For this we use a continuous time framework to simulate a market where Law of One Price is not satisfied. This model allows to take account momentum and reversal effects often cited in behavioural models. We show that the two arguments are accepted. This result confirms why

[2] If $E(Z) \neq 0$, there is also no riskless arbitrage. However, the probability to obtain a gain is greater than a loss. So arbitrageurs act. Then Limits to arbitrage and Chance arguments require all the hypothesis $E(Z) = 0$.

dynamic systems are the more promising way to understand financial markets since they try to integrate the two arguments. Lo [12] has proposed the Adaptive Market Hypothesis concept to reconcile the two theories by using evolutionary systems.
Another literature uses Agent Based Models (adaptive or evolutionary systems) based on heterogeneity and bounded rationality . So they use Efficient Market Theory at a lesser level completed by Behavioural Finance arguments. Furthermore, equilibrium prices obtained with these models are consistent with financial time series since many of the stylized facts are reproduced.

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