

Process Capability Index for Off-line Application of Product Life Cycle

Angus Jeang, Chien-Ping Chung, Huan-Chung Li and Mei-Hui Sung

Abstract—In the past, the process capability index (PCI) was the only one used in on-line quality management; there were no researches on off-line applications, such as product design or process planning. In using conventional PCI for off-line application, designers normally established the process mean close to the design target, and minimized the process variance so that the PCI value would be maximized. The process variance is determined by the process tolerance, which affects the production cost. This factor must be considered because cost will increase as process tolerance is minimized. Simply averting small tolerance for the sake of cost reduction is not always a rational choice, because a great tolerance value generally results in poor quality. Hence, the conventional PCI value does not truly represent the measurement score for off-line applications during product design or process planning. In this regard, the off-line PCI expression is developed with consideration of the balance between product quality and production cost. The product quality is represented by quality loss function, and production cost is expressed by tolerance cost function. These two functions are simultaneously related to process mean and process tolerance. Thus, the new PCI expression can be used to determine appropriate process mean and process tolerance, as well as a measurement score for comparison and selection among candidates. Consequently, an economical and quality of product design and process planning can be achieved during off-line applications.

Index Terms—Process capability index (PCI); Process Mean; Process tolerance; Off-line application.

I. INTRODUCTION

Today's manufacturing industry is facing intensive competition, so both the cost and quality aspects have become important issues among management concerns. Thus producers thrive on providing economical processes which are also capable of meeting the customer's quality requirement. In recent years, as the concept of concurrent engineering has become widely accepted, design engineers hope to achieve simultaneous product design and process planning, side by side, at an early stage of product development [4]. The goals are: to shorten the time span required for introducing the new product onto the

market and to attain the lowest production cost and premium product quality. Hence, what is needed is a way to measure the degree of the producer's process capability, in satisfying the customer's quality requirement. More importantly, a growing number of producers include this measurement value in their purchase contracts with customers, as a documentation requirement. One such measurement is the process capability index (PCI).

The process capability index (PCI) is a value which reflects real-time quality status. The PCI acts as the reference for real-time monitoring that enables process controllers to acquire a better grasp of the quality of their on site processes [6,7]. Although the PCI is considered as one of the quality measurements employed during on-line quality management, several authors have pointed out that the PCI should be addressed at the beginning of the design stage rather than at the production stage, where process capability analysis is typically done [12]. For the sake of convenience, let us call the PCI for the former one, off-line PCI, and the latter one, on-line PCI. The on-line PCI has process mean and process variance that are obtained from the existing process. Conversely, the off-line PCI has the process mean and process variance as two unknown variables, which the process planner would have to determine. When cost is not considered as a factor for off-line PCI analysis; normally the process planners would do their best to set the process mean close to the design target, and minimize the process variance. Because the additional cost incurred for tightening the variance is not considered, obviously, the establishment of mean and variance values will result in a high PCI scale [9]. Thus, this research intends to develop an Off-line PCI that expression which contains cost factors.

The PCI value is typically defined as the ability to carry out a task or achieve a goal. In process capability analysis, the lower and upper limits are initially assumed to be firm and non-negotiable, unless it can be proven that the quality of the final product will not be lessened by changing these limits. The controllable factors are the process mean and process variance [8]. The deviation between process mean and design target can be reduced by locating the process mean close to the design target without additional cost being incurred. The process variance can be lowered by tightening the process tolerance, with extra cost incurred. In case the conventional on-line PCI is used for process capability analysis during the product development, designer engineers naturally intend to raise the PCI value by locating the process mean near the target value, and by reducing the tolerance value to ensure a better product quality. However, simply increasing the PCI value can easily create additional and unnecessary production costs that result from extra efforts and expensive devices for ensuring tolerance control. Hence, there is a need to balance customer demands for quality and production costs. In this regard, the off-line PCI value is introduced, in consideration of quality loss and production cost, simultaneously in this research. The quality loss

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is expressed by quality loss function, and the production cost is represented by tolerance cost function. Then, this new PCI expression can be used as linkage for concurrent product design and process planning, prior to actual production. The rationale will be discussed in the latter sections.

II. BACKGROUND REVIEW

A. Quality loss and tolerance cost

In the product life cycle, quality values will vary under different circumstances. Let y_1, y_2, \dots, y_n be the quality values appearing in different situations. The average quality loss in its symmetric quality loss function is the following [11,14]:

$$E[L(y)] = K[(U - T)^2 + \sigma^2] \quad (1)$$

U represents the average value of the quality characteristic, while σ^2 is the variance of the quality characteristic. Equation (1) tells us that there are two sources of quality loss. (a) $K(U-T)^2$ is the deviation between the process mean and the design target. Examples are blade damage, machine breakdown, problems with raw materials and worker carelessness, which are usually easy to handle without cost becoming a factor. (b) $K\sigma^2$ is the loss resulting from process variances. To eliminate this source, better equipment, materials, and processes are usually required. There is cost containment in this effort. Hence, we usually improve (a) first, and then (b), for economic considerations.

Usually, a high tolerance cost is associated with a tight process tolerance, while a low tolerance cost results from a loose process tolerance. The tolerance cost can be formulated in various function expressions. To evaluate the tolerance cost, this paper adopts the tolerance cost function as developed in the literature [3].

$$C_M(t) = a + b \cdot \exp(-c \cdot t) \quad (2)$$

where a , b , and c are the coefficients for the tolerance cost function, and t is the process tolerance.

From the above cost expression, it can be noted that a tight process tolerance results in a higher tolerance cost, due to additional manufacturing operations, more expensive equipment needed and slower production rates, while a loose process tolerance results in a lower tolerance cost [8-10].

B. Process Capability Indices (PCI)

The frequently seen PCI includes C_p , C_{pk} , and C_{pm} expressions. C_p can be defined as follows [1,2,5,6,7]:

$$C_p = \frac{USL - LSL}{6\sigma} \quad (3)$$

The expression $(USL-LSL)$ refers to the difference between the upper and lower limits which are specified by the customer's quality requirement; σ is the standard deviation which is actually incurred in the production process. However, during the production process, the process means U can be located at positions other than design target. If the process variance σ^2 did not change, the above C_p value would also remain unchanged; this was the major defect owing to the facts that only the spread of the process is reflected, and the deviation of process mean can not be reflected in the measurement. These are the main reasons why C_{pk} was developed; the C_{pk} expression is defined as below:

$$C_{pk} = \text{Min}\left(\frac{USL - U}{3\sigma}, \frac{U - LSL}{3\sigma}\right) \quad (4)$$

There is still a deficiency for C_{pk} expression: the same C_{pk} values may be constituted with different process means and variances. This situation has created a great deal of confusion, and uncertainty as to which would be the best process capability among the alternatives. To cope with the above arguments, another form of PCI, C_{pm} , was developed. C_{pm} is defined as follows:

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (U - T)^2}} \quad (5)$$

When the process mean is equal to design target, the C_{pm} can be simplified as C_p . For the purpose of comparison, three processes: A, B, and C are depicted in Fig. 1. The C_p , C_{pk} , and C_{pm} values from processes A, B, and C are shown in Table 1. Because process C has the greatest C_p value, it might be mistakenly concluded that process C had the best process capability among processes A, B, and C, when C_p is considered as a reference value. However, this erroneous conclusion originates from the fact that the C_p value is solely based on the magnitude of variance, and disregards the negative impact from the process deviation. Similarly, when C_{pk} was used in representing the levels of process capability, the process C_{pk} values for processes A, B, and C, are all just equal to one. Thus again, quite obviously, there is difficulty in ordering the superiority of process capability of the three processes. To overcome the defects appearing in C_p and C_{pk} expressions, another PCI expression, C_{pm} , is introduced. Unlike the previous two expressions, C_{pm} can simultaneously reflect the impact from process deviation and process variance. This feature is particularly important because process mean and process variance are generally changed at the same time in most production process. Unfortunately, with C_{pm} , processes A and B are the best two choices. The non-different outcomes between processes A and B result from the fact that the contribution of C_{pm} value, from process mean deviation and process variance magnitude, is identical. Hence, there must be a way of measurement being provided to make mean deviation and process variance magnitude distinguishable in PCI expression.

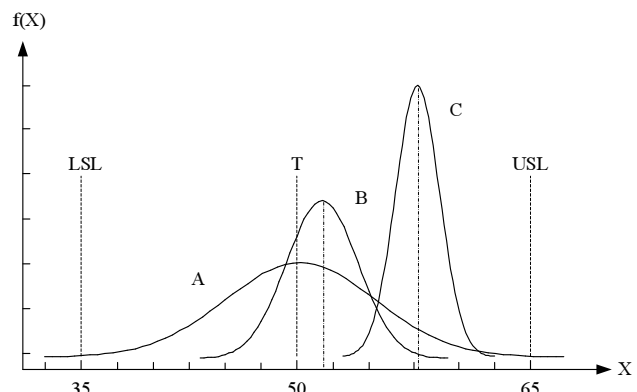


Fig. 1. The distribution for process A, B and C

Note: Process A: $U_A = 50.0$, $\sigma_A = 5.00$, $C_M(t) = \$2000$,

Process B: $U_B = 53.0$, $\sigma_B = 3.83$, $C_M(t) = \$3500$,

Process C: $U_C = 57.5$, $\sigma_C = 2.50$, $C_M(t) = \$6000$

Table 1 PCI values for process A, B, and C

Process	C_p	C_{pk}	C_{pm}	C_{pc}
A	1	1	1	0.051298
B	1.25	1	1	0.047673
C	2	1	0.632456	0.031782

III. SIMULTANEOUS DETERMINATION OF PARAMETER AND TOLERANCE VALUES

Product functionality is achieved through the assignment of appropriate design target and design tolerance, at the stage of product design and process planning. Generally, when the manufacturer attempts to attain a high process capability with a small process tolerance, a higher manufacturing cost will usually result. But when the design tolerance exceeds the process tolerance, additional space or process distribution is provided for a possible shift or drift. Consequently, the parameter value may be set at various positions within a feasible range, which in turn, can lead to quality improvement and cost reduction. The mathematical relationship of the design target T , design tolerance S , parameter value U and process tolerance t , can be represented as [10]:

$$|T - U| \leq S - t \tag{6}$$

where t is constrained by t_L and t_U which refer to upper and lower process capability limits, respectively. However, the parameter value is also contained within an acceptable range, between U_U and U_L . That is,

$$t_L \leq t \leq t_U \tag{7}$$

$$U_L \leq U \leq U_U \tag{8}$$

The values of t_L , t_U , U_U , U_L , S , and T are known in advance. Equation (3) states that the deviation between design target and parameter value should be less than the distance between design tolerance and process tolerance. In this paper, the process tolerance t is called tolerance. By satisfying (3), manufacturability for all possible combinations will be ensured. As a result, the feasible combinations increase the flexibility of both product design and process planning, to a great amount. In addition, the effort of obtaining the precision required for the control of the process is reduced. Quality improvement and cost reduction are thus achieved. In general, there are no costs associated with the parameter design which changes the nominal value of a parameter value; conversely, there are costs involved in the tolerance design. Hence, the best setting of the nominal values is often determined prior to the tolerance design, for the sake of economics. However, based on the discussion regarding (3), in some situations it is important to simultaneously determine parameter and tolerance values, due to the dependency between these two values on the quality characteristics of interest.

IV. PROCESS CAPABILITY INDEX FOR OFF-LINE APPLICATION

The conventional PCI values are mainly applied to measure the on-line spread resulting from actual process deviation and process variance. The reality of this spread is a consequence of the production process, in which there are no more controllable elements. Hence, the conventional PCI expression is valid only in the on-line application. In this regard, there is an attempt to

explore the process capability analysis, at the pre-production stage, to enable designers to integrate the aspects of product design and process planning at an early time. According to the preceding discussion, unlike the other two PCI expressions, C_{pm} is able to simultaneously reflect the influences of process deviation and process variance. However, this is only legitimate at the post-production stage, due to the fact that U and t are realized values which are not controllable for design. However, when C_{pm} is used as a measurement scale in the pre-production stage, U and t become controllable variables. Then, it is possible that various combinations of U and t will result in the same C_{pm} value. Thus, it is difficult to make a distinction among alternatives, in order to make a correct choice from among them. As is known, the process mean U can be adjusted at no additional cost. The designers would most likely establish the process mean U as close as possible to the design target T , within the process feasible range, and attempt to decrease the process variance as much as possible within the process capability limits in order to attain a higher PCI value. In other words, with the exclusive use of the process mean and process tolerance as the determinants of conventional PCI, regardless of the cost impact on customer and production, there is a tendency for designers to position the process mean as close to the target value as possible, and solely cut down the process tolerance to lower capability limit in order to increase the PCI value. Apparently, the found PCI value is erroneous.

The degree of proximity reflects the different quality loss according to the customer's view. Reducing the process variance is normally completed by tightening the tolerance value through tolerance design which usually involves additional cost. Therefore, in addition to the constraints from feasible ranges and capability limits, the influence exerted by the relevant costs representing the selected process mean and process tolerance, should be considered as well. This brings us to the next section, a discussion on the requirement that costs related to process mean and process tolerance must be contained in PCI expression, when referred to as off-line process capability analysis, during product design and process planning.

As indicated in Fig. 2, various combinations of U and t will result in the same C_{pm} value, 1.2. This unhelpful facet will prevent the conventional C_{pm} from being a suitable index for differentiating possible alternatives during product design or process planning. To overcome the above weakness, the lack of consideration of the cost influence from various U and t values, should be resolved. As is known, all costs incurred within a product life cycle, include the material and production costs which are incurred before the product reaches the consumer, and quality loss, which occurs after a sale. In these regards, let the denominator of C_{pm} be replaced with the sum of quality and production related cost, which includes quality loss $K[\sigma^2 + (U - T)^2]$ and tolerance cost $C_M(t)$. Have both U and t be controllable variables so that a maximum C_{pc} can be achieved. This cost effectiveness PCI expression C_{pc} is shown as follows.

$$C_{pc} = \frac{USL - LSL}{6\sqrt{K[\sigma^2 + (U - T)^2] + C_M(t)}} \tag{9}$$

where σ is $\frac{t}{P}$ [10].

Fig. 2 depicts that there are infinite combinations of U and t which have C_{pm} as 1.2. Table 2 shows that different C_{pc}

values can be obtained under various combinations of U and t , when C_{pm} is fixed as 1.2. Apparently, the conventional C_{pm} is not capable of representing all possible alternatives for off-line application during design and planning stage. Table 1 shows that processes A and B are identical choices based on C_{pm} ; however, process A is the only selection based on C_{pc} value. Different selections are being made because C_{pm} lacks consideration of the combined influence from quality loss for customers and tolerance costs for production.

Let design target value $T = 30$ mm, design tolerance $S = 0.05$ mm, quality loss coefficient $K = 1200$, coefficients of tolerance function $a = 50.11345$, $b = 119.3737$, $c = 31.5877$, $P = 3$, $t_L = 0.024$ mm, $t_U = 0.086$ mm. Substitute these values into (6), (7), (8), and (9), to proceed the following discussion.

The optimal C_{pc}^* is 0.0019. The optimal process mean U^* is 30.0000 and the process tolerance t^* is 0.05 mm. Figs. 3 and 4 show that when the process mean is located at the target with fixed tolerance value, the maximum C_{pm} and C_{pc} can be reached. The explanation is discernable by looking into the common expression, $(U-T)^2$, in the denominator of (5) and (9). On the other hand, in Fig. 5, with fixed process mean at the target value, the maximum value C_{pm} , which is infinite, is reached when t is near to zero and the maximum C_{pc} , which is finite, arrives when t is 0.04. The same conclusions are derived by examining Figs. 5 and 6. The fact behind different optimal t values being found when the PCI is at its maximum, is comprehensible because the

variance in C_{pc} is cost related, while the variance in C_{pm} is cost unrelated. The t value in C_{pm} can be any small value regardless of the cost impact resulting from tolerance cost.

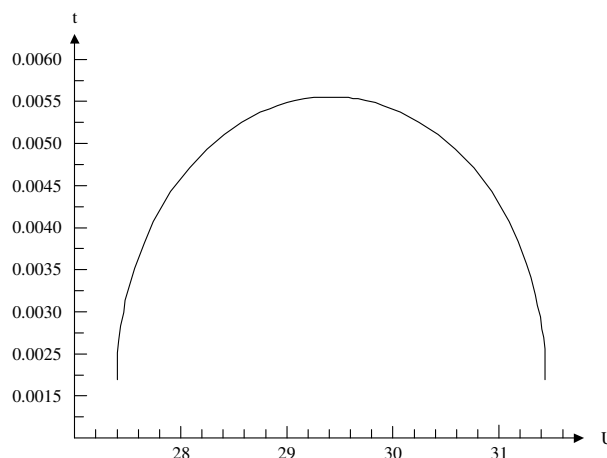


Fig. 2. Feasible range for various combination of U and t with $T=30$, $S=0.05$, and $P=3$ when $C_{pm}=1.2$

Table 2 Various C_{pc} when C_{pm} is 1.2

(U,t)	(50.00,12.50)	(50.05,12.49)	(51.00,12.13)	(51.05,12.09)	(52.00,10.96)	(52.05,10.88)
C_{pm}	1.2000000	1.2000000	1.2000000	1.2000000	1.2000000	1.2000000
C_{pc}	0.0345994	0.0346019	0.0356385	0.0357504	0.0394258	0.039727

V. CONCLUSION

Conventionally, production related issues are usually dealt with via process analysis after the product design or process planning has been completed. These approaches will likely result in poor product quality and high production cost, as a consequence of a lack of consideration concerning quality and production related costs. These are some of the reasons that concurrent engineers suggesting that possible issues occurring in the production stages should first be considered at the time that the new product is developed. That can reduce the time span for introducing a new product onto market, and increase the chance for obtaining a superior edge among competitors. Thus, the present research introduces a PCI measurement, C_{pc} , for the process capability analysis, to ensure that lower production cost and high product quality can be achieved at the earlier time of the blue print stage. As a result, an effective PCI for product life cycle becomes actualized.

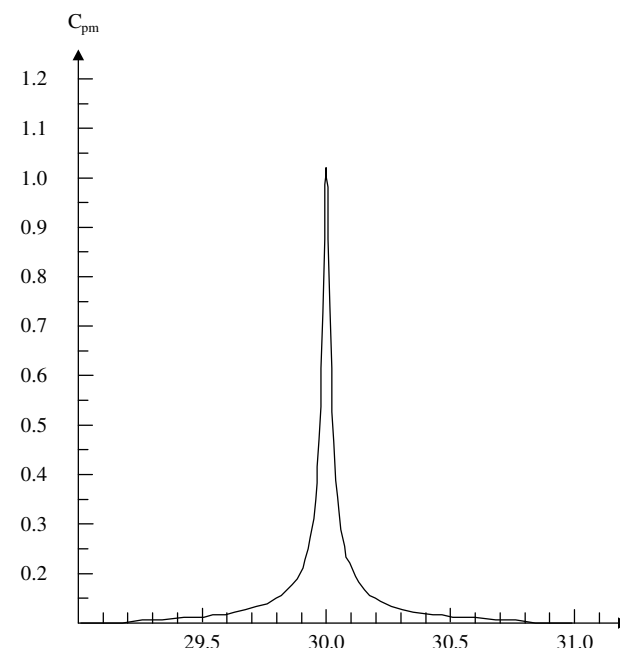


Fig. 3. C_{pm} vs U with $T=30$, $S=0.05$, $t=0.399863$, and $P=3$

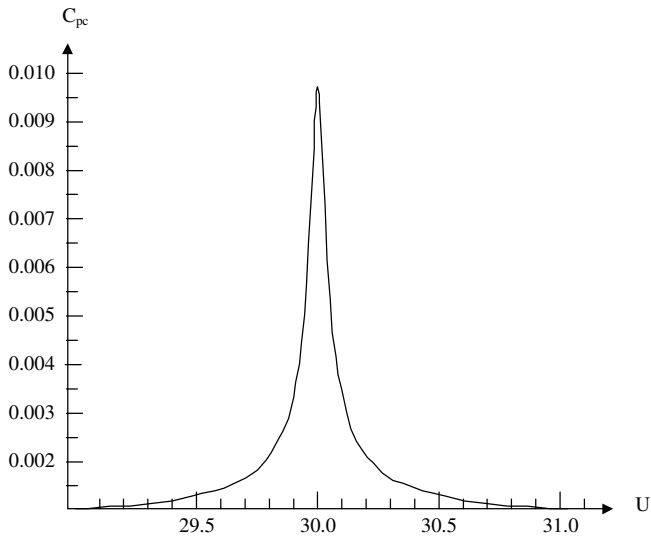


Fig. 4. C_{pc} vs U with $T=30$, $K=1200$, $S=0.05$, $t=0.0399863$,
 $a=50.1134$, $b=119.3737$ and $c=31.58775$

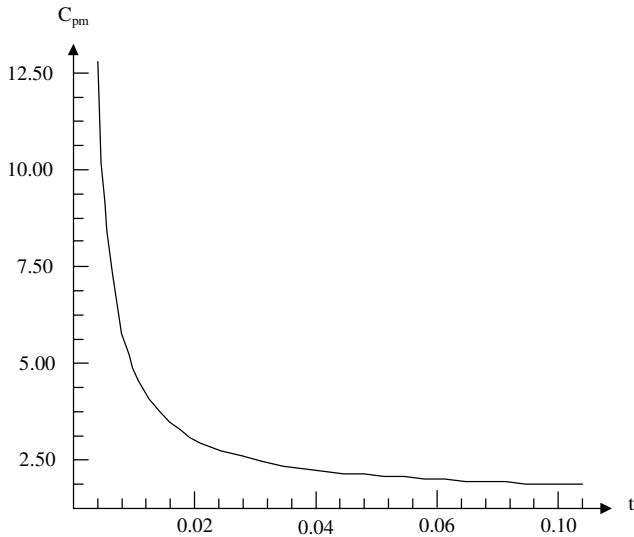


Fig. 5. C_{pm} vs t with $T=30$, $U=30$, $S=0.05$, and $P=3$

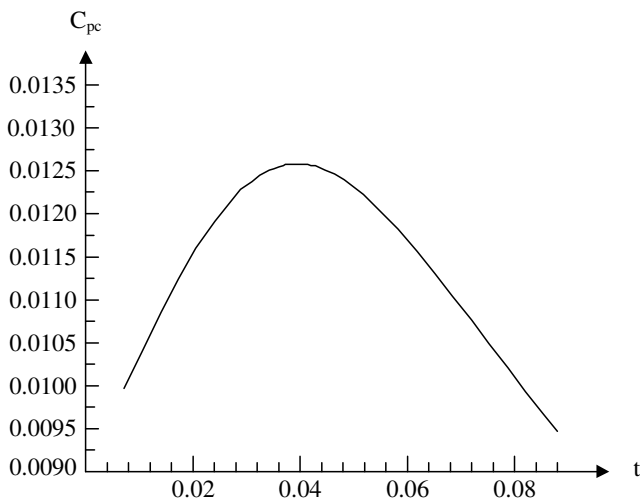


Fig. 6. C_{pc} vs t with $T=30$, $K=1200$, $S=0.05$, $a=50.11345$,
 $b=119.3737$, and $c=31.58775$

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