Mass Efficiency in Mechanical Design

Subbiah Ramalingam

Abstract—**Using the maximum strain energy density in a linear mechanical element stressed in tension as the mass efficiency reference, a direct method for determining the mass efficiencies in transversely loaded and in torsionally stressed mechanical elements is developed and presented. It is shown that this method also allows the determination of relative contribution of each geometric feature in a cross section to mass efficiency of the element. Common loading conditions and structural geometries are analyzed and it is shown that the mass efficiencies realized in practice offer great many opportunities for improvement. It is suggested that more efficient use of materials and energy is feasible by paying closer attention to mass efficiency considerations during mechanical design.**

***Index Terms*— Mass and volumetric efficiency, Sustainable and green design**

I. INTRODUCTION

Global consumption of engineering materials in the early years of the 21st Century is estimated at 10^{13} kg per year [1]. Since nearly all engineered products and systems are implemented with materials drawn from non-renewable resources it will be difficult to sustain this level of material consumption and its likely growth in the coming years. Embedded energy in the material, energy requirements to process the materials into useful forms and the use energy of engineered systems also make major demands on available energy resources. Efficient use of materials is therefore a fundamental requirement to sustain the global community and its rapidly improving living standards.

By replacing gaseous electronics with solid state electronics, major economies have been achieved both in material and energy consumption in computer, control and communication industries. Comparable progress has also been achieved in a few segments of transportation industries and in some parts of civil construction industry. Similar progress is now necessary in industries related to consumer durables and individual/group mobility.

Many of the components and structures used in engineered products and systems are primarily linear, i.e., their linear dimensions are much greater than their cross sectional dimensions. In such structures, four distinct loadings can be identified - axial tension, axial compression, torsion and transverse loading. Major progress has been made in design when linear structures and elements are subjected to axial loads either in tension or in compression. Many efficient designs can be cited to withstand tensile loading. Suspension bridges and various forms of cable-stayed bridges are among examples. Two- and three-dimensional trusses and frames, and stiffened panels and shells used for airframes and in spacecrafts are examples where axial compression is efficiently handled.

This need is still largely unfulfilled in both transversely- and torsionally loaded elements and structures. Mass or volumetric efficiency measures are used in this paper to evaluate the solutions now in use to withstand transverse and torsional loads. Maximum strain energy storable in a given amount of material is taken as the reference measure. With this reference measure, common transversely loaded and torsionally loaded designs and components are evaluated to assess relative efficiency in the use of materials and energy. It is shown that most common designs offer significant opportunities for improvements.

II. REFERENCE MEASURE FOR MASS EFFICIENCY

Structures and elements subject to tensile loads offer no difficulty in developing efficient designs since it is only necessary to distribute the material and the applied load uniformly across the load-carrying section so that yielding or plastic collapse can occur everywhere at the same time when a limiting stress is reached. Mass efficiency can then be defined in terms of **strain energy density**, i.e., strain energy per unit volume $\boldsymbol{U_o}$ in J, permissible in a structure/element subject to pure tensile loading. If the maximum permissible design stress is σ_m, $\sigma_m < \sigma_y$ where σ_y is the yield stress, the strain energy density in uniaxial tensile loading $\boldsymbol{U_o}$ is a reasonable reference measure for mass efficiency. With E representing the elastic modulus, the reference measure for mass efficiency can now be taken as

$$U_o = \frac{\sigma_m^2}{2E} \tag{1}$$

III. MASS EFFICIENCY OF TRANSVERSELY LOADED STRUCTURES

Unlike uniaxial tensile loading where the displacement under load is proportional to length, displacement is proportional to the third power of length in transverse loading. Larger elastic displacements and larger stresses, and a non-uniform stress distribution across the cross section occur. One way to improve mass efficiency is to replace transverse loading with axial loading when feasible, as in trusses. Although this and other empirical principles have been developed to handle transverse loading, quantitative measures are rarely used to evaluate efficient use of materials and energy for the solutions advanced.

In this work, it is proposed that **strain energy density**

Manuscript received January 17, 2008. Subbiah Ramalingam is with the Institute of Technology, University of Minnesota, MN 55455 USA (phone: 612-625-4017; fax: 612-625-4344; e-mail: ram@ me.umn.edu).

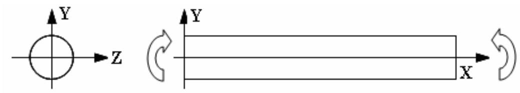

Figure 1. A cylindrical shaft or beam subject to an end moment M.

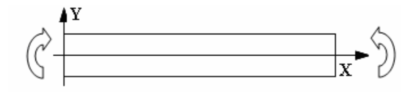

Figure 2. A Square beam of length 2l subject to a moment M.

ratio $\eta = U/\boldsymbol{U_o}$, with the strain energy density $\boldsymbol{U_o}$ in uniaxial tensile loading as the mass efficiency reference, is an appropriate measure for the objective evaluation for the **mass efficiency** $\boldsymbol{\eta}$ of a design solution. Green design is facilitated by requiring high mass efficiency both from the material and the energy conservation perspective.

A. Cylindrical Shaft Subject To Pure Bending

Consider a cylindrical shaft/beam of radius R and length *2 l* subjected to an end moment ***M***, shown in Figure 1. According to Euler-Bernoulli beam theory, the state of stress in the beam is planar. And the bending stress σ_x generated by the moment ***M*** applied is solely a function of *y*, i.e., the distance from the neutral axis. For the strongest beam with maximum permissible design stress $\pm\ \sigma_m$ and moment of inertia *I*, the bending stress at any point between the two ends, with the axis system shown, is given by:

$$\sigma_x = M\left\{\frac{y}{I}\right\} \text{ and } \sigma_m = M\left\{\frac{R}{I}\right\} \tag{2}$$

so that, $$\sigma_x = \sigma_m\left\{\frac{y}{R}\right\} \tag{3}$$

It is enough to consider the first quadrant of one-half of the cylindrical volume of length *2 l* to calculate strain energy density due to problem symmetry with respect to $y = 0$, $z = 0$ and $x = 1$. For a volume *V*, the strain energy density ***U*** due to the moment ***M*** is:

$$U = \left\{\frac{1}{V}\right\}\int_0^R \frac{\sigma_x^2}{2E}dy = \left\{\frac{1}{V}\right\}\frac{\sigma_m^2}{2E}\int_0^R \frac{y^2}{R^2}dy \tag{4}$$

Using polar coordinates, a volume element $r\,dr\,d\theta\,dx$, and a volume $V = (4/\pi R^2 l)$ for 1/8th of the cylinder, strain energy density is:

$$U = \left\{\frac{4}{\pi R^2 l}\right\}\left\{\frac{\sigma_m^2}{2E}\right\}\int_0^l\int_0^R\int_0^{\pi/2}\frac{r^2\sin^2\theta}{R^2}\,r\,dr\,d\theta\,dx \tag{5}$$

Integrating and applying limits, strain energy density ***U*** is:

$$U = \frac{1}{4}\left\{\frac{\sigma_m^2}{2E}\right\} \tag{6}$$

Mass efficiency $\boldsymbol{\eta}$ of the strongest cylindrical beam subjected to the end moment ***M*** is calculated by dividing strain energy density given by (6) with the strain energy density reference measure given by (1). Mass efficiency of the strongest cylindrical beam stressed by an end moment is determined to be $\boldsymbol{\eta} = U/\boldsymbol{U_o} = 1/4$ or *25.0 %*

Yielding failure will ensue due to the non-uniform stress distribution generated by the moment applied, if the strain energy density exceeds *25.0 %* of the maximum possible within the shaft or beam volume. Non-uniform stress distribution imposed leads to inefficient use of material.

B. A Square Shaft/Beam Subject To Pure Bending

Consider a square beam, *2h x 2h* in cross section, and of length *2 l* subjected to an end moment ***M*** shown in Figure 2. Bending stress σ_x generated is a function of *y*, as in the case of the cylindrical beam. In the strongest beam with maximum design stress $\pm\ \sigma_m$ and moment of inertia *I*, the bending stress σ_x at a point between the two ends is:

$$\sigma_x = M\left\{\frac{y}{I}\right\} \text{ and } \sigma_m = M\left\{\frac{h}{I}\right\} \tag{7}$$

so that, $$\sigma_x = \sigma_m\left\{\frac{y}{h}\right\} \tag{8}$$

It is enough to consider *1/4* of the square beam volume to calculate strain energy density due to problem symmetry with respect to $y = 0$ and $x = l$. Considering a ***unit length*** and a ***unit width*** of the beam and calculating as before, strain energy density ***U*** is:

$$U = \left\{\frac{1}{h}\right\}\frac{\sigma_m^2}{2E}\int_0^h\frac{y^2}{h^2}dy = \frac{1}{3}\left\{\frac{\sigma_m^2}{2E}\right\} \tag{9}$$

Mass efficiency $\boldsymbol{\eta}$ of the strongest square beam subjected to the end moment ***M*** is $\boldsymbol{\eta} = U/\boldsymbol{U_o} = 1/3$ or *33.33 %*. The higher mass efficiency in the square beam, when compared with the cylindrical beam, is due to better distribution of load carrying material along the *y* direction. Still, the non-uniform stress distribution imposed, leads to inefficient use of material.

C. A Rectangular Beam Subject To Pure Bending

Consider a rectangular beam of height *2 h* and unit width along *z* and of length *2 l* along *x*, subjected to an end moment ***M***, as in the case of the square beam of the previous section. Bending stress and strain energy density equations for this case are identical to those shown in (7) to (9). Mass efficiency $\boldsymbol{\eta}$ of the strongest rectangular beam subjected to the end moment ***M*** is therefore the same as that for a square beam, i.e., $\boldsymbol{\eta} = U/\boldsymbol{U_o} = 1/3$ or *33.33 %*.

It is well known that rectangular beams are preferred over square beams to withstand transverse loads and moments. The notion of a shape factor has been introduced to emphasize the preference for rectangular beams over square beams to carry such loads [2], [3]. However, it is seen that in terms of mass efficiency, there is little to choose between a rectangular beam and a square one.

D. Mass Efficiency Of A Mid-Span Loaded Rectangular Beam.

Consider now a simply-supported rectangular beam of height *2 h*, unit width and length *2 l* along *x* with a mid span load normal to the longitudinal axis. The bending stress σ_x generated is a function of both *x* and *y*, so that in the strongest beam,

$$\sigma_x = M(x)\left\{\frac{y}{I}\right\} = \sigma_m\left\{\frac{x}{l}\right\}\left\{\frac{y}{h}\right\} \tag{10}$$

As before, just ¼ of the beam volume is considered to calculate the strain energy density ***U***.

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$$U = \left\{\frac{1}{h}\ \frac{1}{l}\right\}\frac{\sigma_m^2}{2E}\int_0^l\int_0^h\left\{\frac{x^2}{l^2}\right\}\left\{\frac{y^2}{h^2}\right\}dx\,dy \tag{11}$$

$$U = \left\{\frac{1}{3}x\frac{1}{3}\right\}\frac{\sigma_m^2}{2E} = \frac{1}{9}\frac{\sigma_m^2}{2E} \tag{12}$$

Mass efficiency of the rectangular beam with a mid span load is $\boldsymbol{\eta} = U/\boldsymbol{U_o}$ = *1/9* or *11.11 %*. Since the maximum bending moment occurs at mid span, most of the material used in the beam is inefficiently deployed for the load carrying function.

Since the choice of height and width of the rectangular beam is of no consequence in obtaining (10) and (11), mass efficiencies of square beams and of rectangular flats, i.e., in beams where the beam width >> depth, remain at the same value. That is, $\boldsymbol{\eta} = U/\boldsymbol{U_o}$ = *1/9* or *11.11 %*.

E. Mass efficiencies of cylindrical beams with a mid span load

With a similar analysis as in the last section, it is easily shown that the mass efficiency of a mid span loaded and simply supported strongest beam of solid circular cross section is $\boldsymbol{\eta}$ =*1/12* or *8.33 %*. For an identically loaded tubular beam with a tube wall thickness t << tube diameter D, mass efficiency can be shown to be twice as large as that for a solid circular beam, i.e., $\boldsymbol{\eta}$ =*1/6* or *16.66 %*.

Beams of circular cross section, commonly used in gear boxes, power transmission systems and mechanical shafting, are less efficient than square beams since more of the load carrying material is placed closer to the neutral axis. In contrast, positioning the material away from the neutral axis, as in tubular shafts, is a more effective approach to raise mass efficiency.

F. Mass Efficiency Of A Rectangular Beam With An Intermediate Load Between Supports.

Using an identical analysis, it can be shown that the mass efficiency of a rectangular beam subjected to a load normal to the beam span, applied at ***any*** location between the two simple supports, is the same as for the beam with mid span load. The same result, namely $\boldsymbol{\eta}$ = *U/Uo = 1/9* or *11.11 %* also holds for mass efficiencies of square beams and for rectangular flats carrying a normal load applied anywhere between the two supports.

For beams/shafts of solid circular cross section, mass efficiency is $\boldsymbol{\eta}$ =*1/12* or *8.33 %* when the transverse load is applied any where between its two simple supports. Mass efficiency is twice as large as that for an identically loaded solid circular beam, i.e., $\boldsymbol{\eta}$ =*1/6* or *16.66 %* when the element is a (circular) tubular beam.

G. Mass efficiency of a rectangular beam with a distributed transverse load.

When the transverse load applied to a rectangular beam is a distributed load of constant magnitude per unit length, bending moment rises more rapidly with position x along beam length. The bending stress generated is a function of both x and y so that for the strongest beam of rectangular cross section,

$$\sigma_x = M(x)\left\{\frac{y}{I}\right\} = \sigma_m\left\{\frac{x^2}{l^2}\right\}\left\{\frac{y}{h}\right\} \tag{13}$$

Mass efficiency for this case is determined by replacing σ_x of (10) with σ_x of (13) and calculating strain energy density as with (11). Mass efficiency is determined to be $\boldsymbol{\eta}$ = *1/15* or *6.66 %*. With no change in beam geometry, application of distributed load is found to greatly reduce mass efficiency when compared with the application of a single mid span load.

H. Mass efficiency of a cylindrical beams with a distributed load.

When the transverse load applied to a circular shaft is a distributed load of constant value per unit length, bending stress σ_x and strain energy density $\boldsymbol{U}$ are given by (Fig. 1):

$$\sigma_x = \sigma_m\left\{\frac{x^2}{l^2}\right\}\left\{\frac{y}{R}\right\} \tag{14}$$

$$\begin{aligned} U &= \left\{\frac{4}{\pi R^2 l}\right\}\left\{\frac{\sigma_m^2}{2E}\right\}\int_0^l\frac{x^4}{l^4}dx\int_0^R\int_0^{\pi/2}\frac{r^2\sin^2\theta}{R^2}r\,dr\,d\theta \\ &= \frac{1}{20}\left\{\frac{\sigma_m^2}{2E}\right\} \end{aligned} \tag{15}$$

Mass efficiency is $\boldsymbol{\eta} = U/\boldsymbol{U_o}$ = *1/20* or *5.0 %*. With a change in cross section from a rectangle/square/flat to circle, the application of distributed load is found to reduce the beam mass efficiency $\boldsymbol{\eta}$ from *1/15* to *1/20*. It can be shown that when the solid cylindrical beam is replaced with a tubular beam with a tube wall thickness t << tube diameter D, mass efficiency $\boldsymbol{\eta}$ rises from *1/20* to *1/10*.

Poor mass efficiency obtained in bending of solid circular shafts leads to reduced critical speeds in simply supported rotating shafts (self-aligning bearings or universal joints), where the self-weight of the rotating shaft represents the uniformly distributed load. This problem is encountered in the design of automotive propeller shafts. Raising the mass efficiency by resorting to tubular propeller shafts is a remedy when higher critical speeds are needed.

IV. MASS EFFICIENCY OF TORSIONALLY LOADED ELEMENTS

In torsionally loaded elements, maximum displacement due to the torsion applied is a function both of length and radius or the largest distance from the axis of torsion. Since the stresses induced are a function of position with respect to rotational axis, mass efficiency of the torsionally loaded structure is also low.

A. A circular rod subjected to a torque T

Consider a circular rod of length l and radius R fixed at one end and subjected to a torque T at the other end. Shear stress τ is a function of radius with maximum shear stress τ_m at $r = R$. Shear strain energy $\boldsymbol{dU}$ in an element $r\,d\theta\ \ dr\,dz$ at a distance r from the torsion axis is

$$dU = \frac{\tau^2}{2G}r\,dr\,d\theta\,dz = \frac{\tau_m^2}{2G}\left\{\frac{r^2}{R^2}\right\}r\,dr\,d\theta\,dz \tag{16}$$

so that the strain energy stored per unit length $\boldsymbol{U_s}$ is

$$U_s = \left\{\frac{1}{V}\right\}\frac{\tau_m^2}{2G}\int_0^{R}\int_0^{2\pi}\frac{r^3}{R^2}\,dr\,d\theta = \left\{\frac{1}{\pi R^2}\right\}\frac{\tau_m^2}{2G}\frac{1}{4}(2\pi R^2) \quad (17)$$

Hence, the calculated strain energy density in torsion is

$$U_s = \frac{1}{2}\left\{\frac{\tau_m^2}{2G}\right\} \approx 0.20\left\{\frac{\sigma_m^2}{2E}\right\} \quad (18)$$

The last term on the right hand side of (18) is obtained by replacing the maximum shear stress τ_m with the yield stress σ_m, replacing the shear modulus *G* with an expression using Young's modulus *E* and Poisson's ratio ν, and taking ν as *0.3*.

Mass efficiency of a circular cylinder subject to torsion is determined to be $\boldsymbol{\eta} = \boldsymbol{U_s / U_o} \approx 1/5 = 20.0\ \%$. Thus, torsionally stressed machine elements such as coil springs, mechanical and power transmission shafting, torsion bars, etc., are found to have mass efficiencies not much different from those for transversely loaded mechanical elements.

When torsionally stressed elements are also subjected to bending, mass efficiency attainable becomes even lower. Raising mass efficiency in torsionally stressed structures now requires use of tubular elements where the torsional mass efficiency $\boldsymbol{\eta}$ is $\boldsymbol{U_s / U_o} \cong 2/5 = 40.0\ \%$. It is shown in a following section that this value is much lower than that possible by geometric shaping when rectangular sections are subjected to bending !

V. Geometry changes to improve mass efficiency

One way to improve mass efficiency in transversely loaded elements or structures is to place the load carrying material farther away from the neutral axis in the plane of bending. Higher mass efficiencies can also obtained by placing the available material such that the amount of load carrying material increases with the increase in bending moment applied. In this instance, material is deployed preferentially in the plane of the beam so that a ***constant stress*** beam geometry results - cross section remains rectangular but the bending stress no longer varies with distance along the longitudinal axis.

A. Modification of beam width to improve mass efficiency

In a transversely loaded beam with a mid span load, if the beam width *2 w* is allowed to vary with bending moment, it is possible to make the maximum bending stress at the upper and lower surfaces of the beam to remain constant. Bending stress σ_x is now solely functions of distance *y* from neutral axis. Plan view of a rhomboidal beam with linearly varying width is shown in Figure 3 below. Bending stress σ_x generated by the mid span load applied in this variable-width beam is:

$$\sigma_x = M\left\{\frac{y}{I}\right\} = \sigma_m\left\{\frac{y}{h}\right\} \quad (19)$$

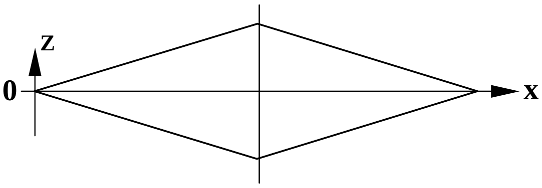

Figure 3. Plan view of a 'constant stress' beam of variable width

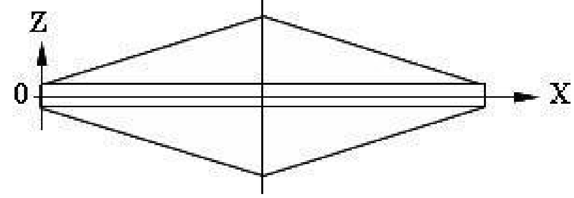

Figure 4. Plan view of a constant stress beam of variable width modified for ease of mounting

Bending stress σ_x given by (19) is the same as that in a beam of constant width subject to pure bending. Mass efficiency for the ***constant stress*** beam shown in Figure 3 is therefore the same, i.e., $\boldsymbol{\eta} = U/\boldsymbol{U_o} = 1/3$ or *33.33 %*. The geometry shown is representative of leaf springs constructed by slicing the rhombus parallel to x axis into a set of fixed width elements of variable length, stacking and clamping the elements to produce the familiar wagon and automotive springs [4].

Mounting requirements make it necessary to modify the variable width beam as shown in Figure 4 (in plan view). Leaf spring structure is now an assembly of one rectangular element and a set of sliced, fixed width, elements of variable length, all clamped together.

If $\boldsymbol{f_1}$ and $\boldsymbol{f_2}$ are the weight fractions of central mounting element and that of ***all*** the sliced elements, mass efficiency $\boldsymbol{\eta}$ of the assembled leaf spring is the ***weighted sum*** of two mass efficiencies $\boldsymbol{\eta_1}$ and $\boldsymbol{\eta_2}$ and is given by:

$$\eta = (\eta_1 \mathbf{f}_1 + \eta_2 \mathbf{f}_2) = 100(0.111\mathbf{f}_1 + 0.333\mathbf{f}_2)\ \% \quad (20)$$

Mass efficiency of this transversely loaded beam of variable width is a function both of the loading and of the beam cross section (geometry). It is lower than the theoretical maximum of *33.33%*.

Automotive suspensions with mass efficiency less than 33% have been in use for over a hundread years even though it is known from vehicles for mass transport (passanger buses) that pneumatic suspensions provide very much greater mass efficiencies.

B. Changing beam geometry to improve mass efficiency

It is known that ***variable depth*** beams of constant width can be made to yield ***constant stress*** beams much like the ***variable width*** beams [4]. But, constant stress beams, designed whether by varying width or by varying depth, do not sufficiently displace the load carrying material away from the bending axis to raise mass efficiency beyond 33.33 %.

A better geometry can now be visualized in which all of the beam material is disposed within a pair of uniaxially loaded, thin rectangular regions symmetrically disposed in the plane of bending but much farther removed from the neutral axis. Such a structure, with a pair of uniaxially stressed flanges *A* and *B*, each of thickness t_f, shown in Figure 5, is an ***idealized I-Beam*** [5] or an ***open web I-Beam***.

In an ideal I-Beam subjected to pure bending of the form shown in Figure 1 or 2, the flange *A* above the neutral axis at a mean distance h is in uniform compression and flange *B*, below the neutral axis at the same mean distance h, is in uniform tension.

For the strongest beam with design stress ± σ_m, as each flange is under a simple uniaxial stress state in compression or

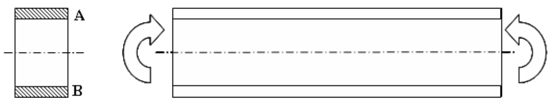

Figure 5. An *Ideal I-Beam* subjected to a moment M and its section (left)

tension with a stress σ_m , the strain energy density ***U*** is

$$U = \frac{\sigma_m^2}{2E} \tag{21}$$

Mass efficiency of an ***ideal I-beam*** is therefore seen to be its ***maximum*** possible value, i.e., $\eta = 1 = 100$ %. An ideal I-Beam with 100% mass efficiency is, of course, unattainable, since a shear web necessary to generate the two different uni-axial flange stress states is not present.

C. Mass efficiency of a real I-Beam in pure bending.

A real I-Beam of total depth *2 h* can now be constructed with a pair of flanges, each of thickness t_f, and a singe web of thickness t_w. When the I-Beam meets all the requirements of an Euler-Bernoulli beam, the flanges are ***mainly*** in uniaxial tension or compression. A variable bending stress state, same as that in a transversely loaded rectangular beam, prevails in the shear web. Mass efficiency of the beam is then given by the ***weighted sum*** of mass efficiencies of the flanges and of the rectangular shear web, which is *11.11%*.

To get an accurate estimate of flange efficiency, define half-depth h_f as $h_f = (h - t_f)$ and nondimensionalise it with *h* as $h_f^* = (h_f / h)$. Then, by following the same steps as for derivation of (7) and (8), it can be shown that without the shear web, the flange efficiency η_f of the I-beam (i.e., for a flange width less shear web thickness) is

$$\eta_f = \frac{1-(h_f^*)^3}{3(1-h_f)} = \frac{1-(h_f^*)^3}{3(1-h_f)} \times 100\,\% \tag{22}$$

To derive this expression, bending stress in the flange is allowed to vary as a function of distance from neutral axis (ideal flange assumption is not used). A similar expression can also be derived for tubular beams of rectangular cross section.

Shear web efficiency η_s is already known to be *11.11%*. Hence the ***weighted sum*** of mass efficiencies of the flanges and of the shear web is readily determined. High mass efficiencies are obtained in pure bending with beam shaping.

D. Mass efficiency of an I-Beam under other loading conditions.

If the I-Beam were mid-span loaded with a single load, bending stress in the flange is a function of position along the span - hence the ***best mass efficiency*** attainable in an ***ideal I-Beam*** of **constant flange width** and **thickness** is *33.33%* (no shear web present). With distributed loading of constant magnitude or of triangular loading with the peak load at mid span, attainable best mass efficiency in an ideal I-Beam, again with a flange of constant width and thickness, will be much lower than 33.33 %.

In all these instances, both the ideal mass efficiencies and the real mass efficiencies in the presence of a shear web are calculable by following the same steps as those shown for real

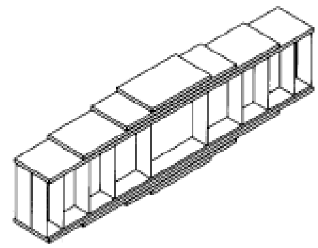

Figure 6. An I-Beam with variable flange thickness and high mass efficiency

I-beams in pure bending.

E. Other methods for improving mass efficiency

As noted earlier, for the mid-span loaded I-beam, bending stress is a function of distance *x* along the span. Varying the flange width offers one means of raising mass efficiency. It is also possible to vary the flange thickness as a function of position *x* along the span when rolled I-Beams are used. This is accomplished by plate welding or riveting on to the flanges so that the flange thickness varies from zero at the beam extremities to its maximum value at mid span as shown in Figure 6. Mass efficiency η attained with such flange modifications can be substantially above the mass efficiency limit value of 0.333. Calculation of the mass efficiency follows the same steps as in previous examples.

Tapered beam*s,* i.e., beams of ***variable depth***, offer an another means for raising mass efficiency. Tapered beam approach is accessible for solid and tubular rectangular beams as well as I-beams and ***castellated*** I-beams. Reliable welding technology is available to commercially implement variable depth or tapered beams.

As long as the taper is not too great, Euler-Bernoulli beam modeling is adequate for the analysis of variable depth beams [6]. Numerical calculations are now necessary to determine mass efficiency, especially for end-mounted mechanical elements of complex geometry such as turbine blades (which are basically cantilevers). Fabricated, variable depth beams in wide commercial use in the form of ***fabricated metal building structures*** also require numerical calculations. Methods outlined here are sufficient for this purpose.

VI. Summary and Conclusions

A direct method to determine the mass efficiencies possible in transversely loaded and in torsionally stressed mechanical elements is presented here. Maximum strain energy per unit volume stored in a linear mechanical element stressed in uniaxial tension, is used as the mass efficiency reference. This method of determining the mass efficiency also allows the evaluation of contribution of each geometric feature in a cross section to mass efficiency of the entire mechanical element. Common loading conditions and several beam geometries have been analyzed.

Loading modes are frequently not taken into account in modeling for minimum weight design in transversely and torsionally loaded elements. Taking the loading mode into consideration and evaluating the mass or volumetric efficiency, it is found that the mass efficiencies realized in practice are rather low. In nearly all cases, local maximum stresses generated invariably lead to grossly inefficient use of engineering materials. Efficient use of available materials and energy requires significant improvements in mass efficiency. This requires much greater attention to mass efficiency considerations during mechanical design.

If life cycle energy requirement [1] is taken to be comprised of embodied energy in the material H_m, processing energy H_p, the use-energy of energy using products H_{use} (including the energy associated with maintenance and service over the useful life of the product) and the energy of disposal H_{disp}, it is seen that most mechanical designs make very inefficient use of available energy resources. Superior shaping, preferred geometric designs and materials are necessary to make better use of available material and energy resources.

Mass efficiency can be significantly raised in transversely loaded mechanical elements by selective placement of load carrying material. Shaped and tapered elements offer means of improving efficient use of materials. Doing so is feasible using several different cost effective and energy efficient processing methods. Increasing use of metal buildings, castellated beams, and other structures suggest that this is a viable approach.

Significantly raising mass efficiency is not straightforward when a design is dominated by torsion. Large reduction in mass efficiency accompanies torsion when it is accompanied by transverse loading. This condition, combined torsion and bending, is encountered in many mechanical elements such as shafts used in gear boxes, power transmission systems and drive trains. Use of such mechanical elements and structures now calls for closer examination and rigorous engineering justification.

Where efficiency improvements by shape or geometric modifications do not appear feasible, sustainable or green design demands adaptation of alternate designs or even alternate product implementation technologies. Direct drive, as in robotic systems and hard disk drives (both for read-head actuation using a voice coil driver and for platen-drive with axial field motors) [7] is one option. In general, direct drive systems dispense with gears/drive trains to raise overall mass efficiency especially in instances where torsion predominates in the presence of bending. Other examples where direct drive systems have served to raise total mass efficiency include refrigeration systems (directly driven scroll compressors as opposed to reciprocating compressors) [8] and wind energy converters (gearless, directly-coupled multi pole synchronous generators)[9].

Inability to improve mass efficiency in torsionally loaded elements will also force acceleration of the on-going trend towards hybrid systems and other direct drive systems (hydraulically-driven earth moving and farm machinery). Efforts underway to develop electric propulsion for surface and submerged vessels [10]as well as aircrafts [11] tends to reinforce this view.

Mass efficiency is considered here primarily from the static loading perspective. It is noted in closing that poor mass efficiencies will invariably lead to lowered natural frequencies (in bending) and reduced critical speeds (in torsion) in virtually all dynamically excited mechanical systems. Improved mass efficiency offers a more rational path to better material and energy conservation in all such instances when dynamic loading is a cause for concern.

REFERENCES

[1] M. F. Ashby, H. Shercliff, and D. Cebon, , "Materials : Engineering, Science, Processing and Design", Elsevier Butterworth-Heinmann, UK, 2007, ch. 20.
[2] M. F. Ashby, "Materials Selection in Mechanical Design", Elsevier Butterworth-Heinmann , UK 2005, pp. 283-337
[3] F. R. Shanley, "Weight-strength analysis of aircraft structures", McGraw-Hill NY 1952, pp. 16-23
[4] S. Timoshenko , S., "Strength of Materials", 3rd Ed., Van Nostrand Co Princeton NJ 1955, pp. 210-214
[5] M. J. French, "Conceptual design for engineers", Berlin, Springer, 1985, pp. 91-93
[6] B. A. Boley, "On the accuracy of Bernoulli-Euler theory of beams of variable section", Journal of Applied Mechanics, Vol. 30, No. 3, 1963, pp 373-378
[7] J. S. Heath, "Design of a Swinging Arm Actuator for a Disk File", IBM J . Res. Develop., Vol. 20, No.7, 1976, pp. 386-397
[8] E. Morishita, M. Sugihara and T. Nakamure, "Scroll Compressor Dynamics : 1st Report, The Model for the Fixed Radius Crank", Bulletin of JSME, Vol. 29, No. 248, 1986, pp. 476-482
[9] E. Hau and H. von Renouard, "Wind Turbines", 2nd ed., Springer , Heidelberg, 2006, Ch. 9.
[10] D. S. Parker and C. G. Hodge, "The electric warship", Power Engineering Journal, Vol.12, No. 1, 1998, pp. 5-13
[11] P J Masson, G V Brown, D S Soban and C A Luongo, " HTS machines as enabling technology for all-electric airborne vehicles", Super Conductor Science and Technology, Vol 20, 2007, pp. 748–756