Two-Stage Stochastic Programs with Mixed Probabilities

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Abstract—**We extend the traditional two-stage linear stochastic program by probabilistic constraints imposed in the second stage. This adds nonlinearity such that basic arguments for analyzing the structure of linear two-stage stochastic programs have to be rethought from the very beginning. We identify assumptions under which the problem is structurally sound and behaves stable under perturbations of probability measures.**

Keywords: Stochastic programming, two-stage models, stability analysis

1 Introduction

This is an extended abstract of a recently published paper in SIAM Journal on Optimization, Vol. 18, No.3, see [6].

Motivated by the study of stochastic programming problems coming from planing and operational management decisions in power generation companies, in [6], was introduced the following parametric family of mixed-probability stochastic programs

$$P(\mu,\lambda) \qquad \min\{c^T x + \int_{\mathbb{R}^s} Q(z - Ax, \lambda)\, \mu(dz) \ : \ x \in C\} \tag{1}$$

where $(\mu, \lambda) \in \Delta \times \Lambda$ and

$$\begin{array}{rcl} Q(t,\lambda) & := & \min q^T y \\ & \text{s.t} & Wy = t \\ & & y \geq 0 \\ & & \lambda(H_j(y)) \geq p_j, \quad j = 1, \ldots, d. \end{array} \tag{2}$$

Here $H_j, j = 1, \ldots, d$, are set-valued mappings from $\mathbb{R}^m$ to $\mathbb{R}^r$ with closed graph, and $p_j, j = 1, \ldots, d$, are predesigned probability levels. If $\mathcal{P}(\mathbb{R}^s)$, $\mathcal{P}(\mathbb{R}^r)$ denote the sets of all Borel probability measures on $\mathbb{R}^s$ and $\mathbb{R}^r$, respectively, we assume that Δ, Λ are subsets of $\mathcal{P}(\mathbb{R}^s)$, $\mathcal{P}(\mathbb{R}^r)$. Moreover, C is a closed subset of $\mathbb{R}^{\bar{m}}$, and all remaining vectors and matrices have suitable dimensions.

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This model was introduced for the first time, by the author in his thesis of doctoral degree, see [4] or [5] chapter 4, altogether with Professors Alejandro Jofré and Rüdiger Schultz.

The model $P(\mu, \lambda)$ extends the traditional two-stage linear stochastic program introducing some probabilistic constraints $\lambda(H_j(y)) \geq p_j, j = 1, \ldots, d$ in the second stage of the problem. These types of constraints add nonlinearities to the problem so that basic arguments to analyze the basic well-posedness of $P(\mu, \lambda)$ had to be rethought from the very beginning, see our work in [6].

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