Development of a Statistical Model for the Ultimate Concrete Shrinkage

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Abstract—**The aim of this paper is to present a statistical model for the prediction of the ultimate shrinkage of concrete as a function of the ration of moisture loss, volume of the paste and compressive strength of concrete at the time of starting shrinkage. The model is based on laboratory tests for 500 empirical data points. At each test, the shrinkage and weight loss of the concrete prisms have been measured at different drying times for different mix proportions and different aggregate types. The results from the developed model are compared with practical test results which show satisfactory results.**

Index Terms—**Statistical Modeling, Concrete, Ultimate Shrinkage**

I. Introduction

The literature of the concrete technology has resulted in that the major part in concrete shrinkage is played by water[1]. Thus when concrete is exposed to dry condition, moisture slowly diffuses from the interior mass to the surface where it is lost by evaporation. During drying the pore water forms menisci causing stress by loads imposed by water surface tension. This stress is acting on the concrete bulk and tending to compress it, hence concrete shrinks. According to Power and Brownyard (1946) there are three types of water occurring in cement paste, which are: non evaporable water, gel water and capillary water [1].

ACI model code (1996) predicted the ultimate standard shrinkage of concrete to be equal to 780 micro strain. Whereas for condition other than standard, the ultimate shrinkage is corrected; different factors are taken into account, for example: relative humidity of the ambient air, volume to surface ratio of the member and factors related to the composition of the mix [2].

Bazant (1978, 1991, 1995) has proposed different models to predict shrinkage of concrete at any drying time. These models involved the influence of temperature[3]. Bazant and Kim (1991) introduced a mathematical formula to estimate the overall shrinkage for the case of one dimension diffusion in a wall based on the consequence of diffusion theory for drying one dimension and shrinkage of concrete[4]. Bazant and Baweja (1995) proposed another simple model for the characterization of concrete creep and shrinkage called B3 model[5].

This paper presents statistical model based on empirical results. The model relates the shrinkage to moisture loss, volume of the paste and compressive strength at the age when concrete start dry.

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II. Empirical test setting

In this study, an ordinary portland cement produced by Tasluja factory in Silemani, Kurdistan region has been used. The sand with a low sulfate content (SO_3=0.244 percent by weight of sand) has been used which is within the limit of Iraqi standard specification. Naturally available uncrushed gravel was used (with SO_3 content of 0.08% by weight of the gravel). Other types of natural local rocks were crushed and washed to be used as a coarse aggregate for the preparation of concrete specimens. Maximum size of coarse aggregate selected was 12.5mm for all the types.

For the concrete mixes, different mix proportions using different types of coarse aggregate as shown in tables 1 and 2 and water –to-cement ratio fixed to be equal to 0.5. For the cement paste spacemen's different water –to-cement ration were selected (0.35, 0.4 , 0.45, 0.5). Materials were mixed in a container mix. In the present work different type s of coarse aggregate were used also to obtain different strength levels which are related to the compressibility of the aggregate. The mixes then were cast in to the steel moulds; prism with dimension (75*75*285mm) for the concrete and (25*25*285mm) for cements paste spacemen are used for volume change measurements while for the compressive strength determination 100 mm cubes for concrete and 50 mm cubes for cement paste are used. The mixes were cast in two layers; vibrating table was used until full compaction was achieved, and then trawled to maintain and even surface . The curing was achieved firstly by covering the prisms in their moulds for a period of about twenty four hours with polythene shits. Then after demoulding the specimens pounded in water bath for six days.

The concrete and cement paste prisms were subjected to a temperature of about 41 ^oC and R.H. reached down to 27% by an electrically controlled room. However some of the concrete prisms were subjected to a natural drying condition in the room started from the March of 2006. The temperature was ranged between 26^{o}C and 30 ^{o}C and R.H. about 50-60%.

Table 1 Mix proportions of the concrete specimens

Mixes	I	II	III	IV	V	VI
Mix Proportion	1:1.90:2.80	1:1.80:2.70	1:1.70:2.62	1:1.59:2.60	1:1.50:2.54	1:1.42:2.49
Cement Content Kg/m3	385	395	405	415	425	435

Table 2 Mix proportions of the mixes for different types of coarse aggregate material

Mixes	Mix Proportion	Coarse Aggregate Content (Kg/m3)	Aggregate Type
L	1:1.53:2.56	1047	Limestone
M	1:1.42:2.59	1084	Marble
G	1:1.61:2.59	1065	Gravel
D	1:1.52:2.57	1078	Dolomite
GR	1:1.53:2.55	1060	Granite
B	1:2.26:1.64	690	Bauxite

III. EMPIRICAL TEST RESULTS

The relationships between the water lost through evaporation and shrinkage of concrete prisms and cements paste are shown in Fig.1 and 2 respectively. In these Figs approximately two stage can be distinguished. The first stage of the evaporated water, which is established by the loss in spacemen's weight, is the free water occupies the capillary pores. This cased low shrinkage of concrete. The second stage of the evaporated water in the physically bound water which is either gel water or inter-crystalline gel water absorbed by their surface , cases high shrinkage. It was stated that there are third stage also can be distinguished by the increase in shrinkage with no substantial moisture loss[6]. This last phenomena is explained by water migration within the concrete bulk from the inside towards the outside. Fig.3 shows that light weight aggregate concrete as mixes B result in low initial rate of shrinkage i.e. their shrinkage is low at the first stage of moisture loss. This can be explained on the bases that porous aggregates contain more water than normal weight aggregate. This is a consequence of the greater volume of the pores whose loss of water through evaporation cased no or low shrink age. The shrinkage cased by water loss is called hydraulic or drying shrinkage and it is considered to be proportional to the amount of the evaporated water. Based on these data points presented in these Figs, it can be concluded that the relationship of shrinkage versus moisture loss must be explained by an exponential or power equation.

The ultimate shrinkage of concrete formulated presently as a function of moisture loss, volume of the paste and compressive strength of the spacemen's. Moisture loss is a function of the relative humidity of the air surrounding the concrete. Moisture content distribution at each point in the section of concrete follows the same shape of the relative humidity distribution curve. Moisture content in the pores of concrete whiled be lowering with time due to the diffusion from the internal parts of the concrete towards the surface and from the surface into the air.

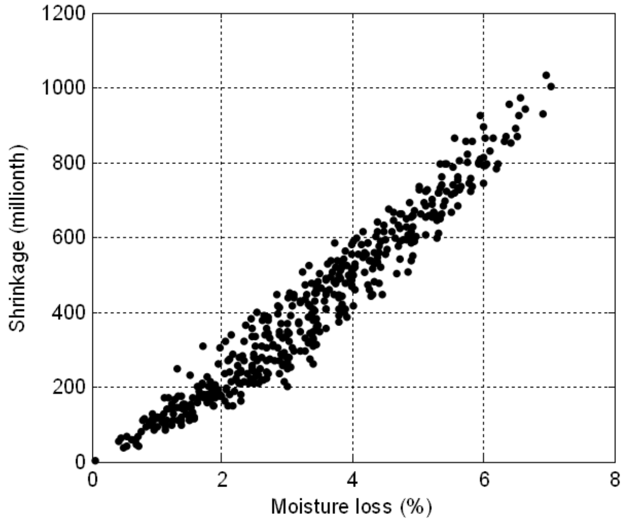

Fig.1 Concrete shrinkage vs. the percentage moisture loss for concrete mixes

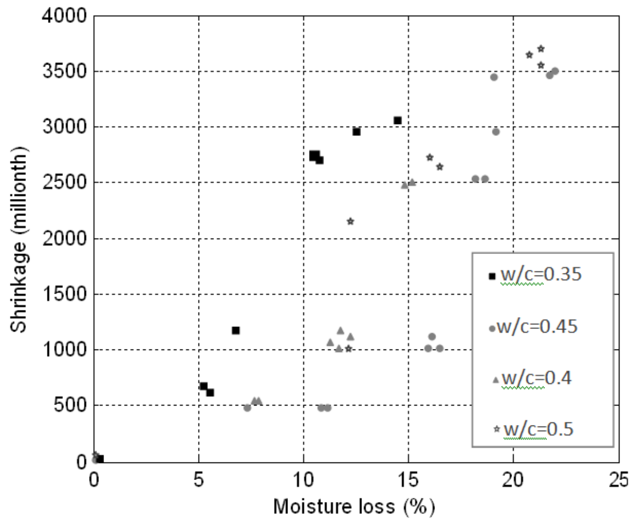

Fig.2 Cement paste shrinkage vs. the percentage moisture loss

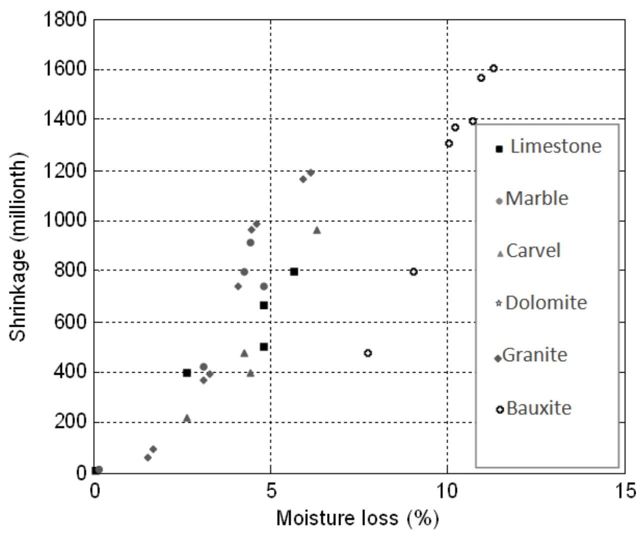

Fig.3 Concrete shrinkage vs. the percentage moisture loss for concrete mixes for different aggregate types

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The moisture loss can be determined, theoretically from the diffusion theory, while empirically the average moisture loss of the cross section of concrete determined at any drying time. The total moisture content is determined by drying the prisms in a ventilated oven from six days at a temperature of about 60 ^oC. The mean relative humidity was measured by hygrometer from the oven it was about 15%. Theoretically, the initial or total moisture content can be determined by the model of moisture capacity, which is established by X_i, Y and W, Z[7]. This was based on the adsorption isotherm equation concede ring the relative humidity to be 100% firstly for initial moisture content, and then moisture content is calculated again considered the relative humidity to be 15% . Then the total moisture content is determined by subtracting the moisture content at relative humidity of 15% from the moisture content at relative humidity of 100%. The other important factor is the volume of the paste which is the shrinkage face in concrete. In the present work the volume of the cement space which is included, for the estimation of ultimate shrink age of concrete is considered to be:

$$V_{cp} = 1 - V_a - V_{uc} \tag{1}$$

$$V_{uc} = V_c (1 - h) \tag{2}$$

Where,
V_a is the volume fraction of aggregate in concrete,
V_{uc} is the volume of unhydrated cement.
h is the hydration degree of cement.

IV. STATISTICAL MODEL

It is clear from the experimental results that there is a strong relationship between concrete shrinkage and moisture loss. To determine whether both ranges of data (shrinkage and moisture loss) move together for all the mix, the coefficient of variation, Cov(x,y) and correlation coefficient r(X,Y) are calculated and determined for both cement paste and concrete prisms with different type of aggregate.

For different concrete mixes there are different amount of moisture content and moisture loss and so, there are different proportion of the capillary to gel water within the pores of concrete. The initial rate of shrinkage for light weight or low strength concrete is low relatively, while the ultimate shrink age after the proud of drying time appeared relatively higher than normal strength concrete. So for the same amount of moisture loss there will not be always to same amount of shrinkage for different concrete mixes. That is due to the existing of different type of pores un the structure of concrete and difference in the effect between capillary and gel water on the shrinkage of concrete. To avoid these phenomena and to increase the reliability of the results of different concrete mixtures, the ration of moisture loss RML was used instead of the moisture loss to estimate the ultimate shrinkage model of concrete. The ratio of moisture loss RML is determined from the following equations:
Empirically:

$$M.Loss_{RH} = \frac{W_i - W_{RH}}{W_d} \tag{3}$$

$$M.Loss_T = \frac{W_i - W_d}{W_d} \tag{4}$$

Theoretically, using absorption isotherm equation,

$$M.Loss_{RH} = M_{c_{(i)}} - M_{c_{(RH)}} \tag{5}$$

$$M.Loss_T = M_{c_{(i)}} - M_{c_{(15)}} \tag{6}$$

$$RML_{RH} = \frac{M.Loss_{RH}}{M.Loss_T} \tag{7}$$

Where,
$W_{(i)}$ is the initial weight of the specimens at the saturated condition (RH=100%).
$W_{(RH)}$ is the weight of the specimens at any relative humidity.
W_d is the weight of dried specimens at nearly relative humidity (15%).
$RML_{(RH)}$ is the ratio of moisture loss at any relative humidity.
$M.Loss_{(RH)}$ is the moisture loss at any relative humidity.
$M.Loss_{(T)}$ is the total moisture loss of the specimen.
$Mc_{(RH)}$ is the moisture content at any relative humidity.
$Mc_{(i)}$ is the moisture content at the saturated condition (RH=100%).
$Mc_{(15)}$ is the moisture content at dried condition humidity (15%).

This has proved by the values of both the coefficient of variation and correlation coefficient for cement paste and concrete prisms. The presented results as shown in table 3 indicated that using RML is better than the percentage of moisture loss. This will allow the model to be used for the higher range of concrete porosity.

Table 3 Measured values for coefficient of variation and correlation between shrinkage and both moisture and RML for cement and concrete mixes.

Data (x,y)	Mix type	Cov(x,y)	r(x,y)
Moisture loss and shrinkage	Cement paste	6431	0.85
	Concrete mixes	611.06	0.87
RML and shrinkage	Cement paste	304.8	0.90
	Concrete mixes	72.90	0.91

The ultimate shrinkage of concrete is formulated as a function of RML volume of the paste, and compressive strength. These are estimated statically for 500 empirical data points using STATISTICA 5. The best fit non-linear exponential function suggested to express this ultimate shrinkage at different humidity and different concrete types. The assumed relation is as shown below:

$$SH_{ult} = b_o.\exp(b_1.X_1^{b_2}.X_2^{b_3}.X_3^{b_4}) \tag{8}$$

Where
X_1 : Ratio of moisture loss RML, its value ranges between 0 to 1.
X_2: Volume of the paste per unit volume of concrete (0-1).
X_3: Cube compressive strength at the age when concrete begins to dry (Moa).

b_o, b_1, b_2, b_3, b_4 are constants their values are estimated statistically using quasi Newton method for the best fit non linear curve estimation with highest coefficient of determination R^2 and lowest loss function expressed by residual mean squares. These coefficient values are b_o= 55.9, b_1=7.49, b_2=0.61, b_3=0.45, b_4=0.144 with R^2=0.908.

From this equation it can be observed that increasing both volume of the paste and moisture loss or RML will increase the shrinkage of the concrete, while increasing the strength will decrease the shrinkage, see Fig. 4 which shows the measures versus the predicated values using the proposed model.

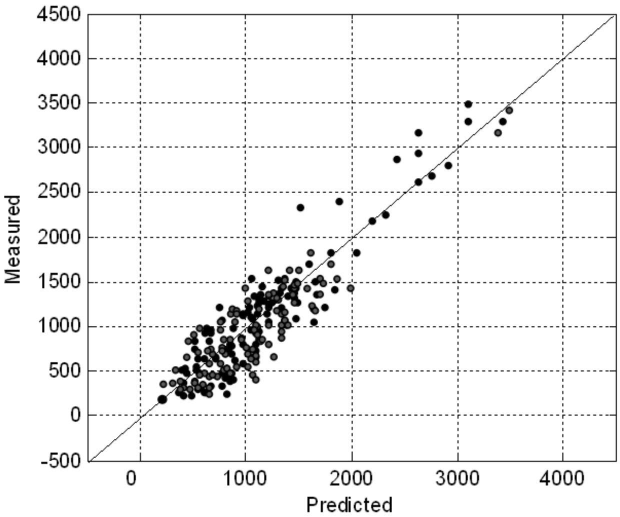

Fig.4 Measured vs. predicted values for the ultimate shrinkage using the proposed model

V. Conclusions

A statistical model for the prediction of the ultimate shrinkage of concrete as a function of the ration of moisture loss, volume of the paste and compressive strength of concrete at the time of starting shrinkage has been developed in this paper using empirical data. The measured and calculated performance have been compared. The results show that:

1. There is a strong relationship between drying shrinkage of the concrete and moisture loss from it. This relationship is expressed by and exponential equation.
2. The empirical data point is transformed to the graphical model using negative exponential smoothing curves for the lines.
3. The resulted model indicate that ACI model code 209R-92 gives the reliable results for the ultimate shrinkage of concrete.
4. Ultimate shrinkage of concrete can be estimated by measuring the total weight loss of testing specimens using an accurate balance after storing the specimens in a humidity chamber at any relative humidity needed and testing the compressive strength of the specimens in the labs.

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