N-G Approach for Solving the Multiresponse Problem in Taguchi Method

Abbas Al-Refaie, Ming-Hsien Li, and Chun-Wei Tsao

***Abstract*—The Taguchi method aims at reducing response variation from target so as to increase yields and product lifetime, reduce defects, and improve performance. However, the Taguchi method is only efficient for optimizing a single quality response. This research, therefore, proposes a neural network–grey relational analysis (hereafter abbreviated as N-G) approach for solving the multiresponse problem in the Taguchi method. The proposed approach includes two phases. The first phase utilizes neural networks to predict the normalized multiresponse values for all combinations of factor levels. Whereas, the second phase employs grey relational analysis to decide optimal combination of factor levels for multiresponse problem. A case study is provided for illustration. In conclusion, the N-G approach may provide a great assistance to practitioners for solving the multiresponse problem in real life applications on the Taguchi method.**

***Index Terms*—Neural Networks, Grey analysis, Taguchi method, multiresponse problem.**

I. Introduction

In practice, a great deal of engineering time is spent generating information bout how different design parameters affect performance under different usage conditions. The Taguchi [1] method is a widely accepted approach for robust design. The overall goal of robust design is to find settings of the controllable factors so that the response is least sensitive to variations in the noise variables, while still yielding an acceptable mean level of the response. Generally, a process's or a product's quality response can be divided into three main types: the smaller-the-better (STB); the nominal-the-best (NTB); and the larger-the-better (LTB) type responses. To optimize a quality response by the Taguchi method, an orthogonal array (OA) is utilized to reduce the number of experiments under permissive reliability. Then, signal-to-ratio (S/N) ratio is employed as a quality measure to decide the optimal combination of factor levels.

Manuscript received March 9, 2008. This work was supported by the Faculty of Engineering/ the Dept. of Industrial Engineering and Systems Management in Feng Chia University.

A. Al-Refaie is currently pursuing the Ph.D. degree at the Dept. of Industrial Engineering and Systems Management in Fen Chia University, Taiwan. (Corresponding author e-mail: eng_jo_2000@ yahoo.com).

M.H. Li is a professor in the Dept. of Industrial Engineering and Systems Management in Fen Chia University.

C.W. Tsao is currently pursuing a M.Sc Degree at the Dept. of Industrial Engineering and Systems Management in Fen Chia University.

In today's highly competitive markets, however, customers are concerned about more than one quality response. The Taguchi method has been extensively employed in manufacturing to robustly design a product or process with only one quality response [2-3]. Recently, the multiresponse problem in the Taguchi method has received an increasing research attention. For example, Phadke [4] employed pure engineering judgment for optimizing concurrently three quality responses in a very-large-scale integrated circuit-manufacturing process. However, human judgment increases uncertainty in decision-making process. Tai *et al*. [5] assigned a weight to the S/N ratio of each response then combined the multiresponses into a single performance index, which was then used to identify optimal combination of factor levels. In reality, it still remains difficult to determine and define a weight for each response. Pignatello [6] discussed a manufacturing process with five responses. They used data-driven transformations for each response variable in a multiple-univariate or one-at-a-time manner. Then, the regression technique based approaches were utilized to determine tentative optimal factor levels. Also, Reddy *et al*. [7] employed regression techniques based approaches during unifying goal programming to optimize simultaneously several responses. Regression approaches, however, increase the complexity of computational process. Thereafter, Antony [8] utilized principal component analysis (PCA) to transform the multiresponses in few uncorrelated ones, which were then employed for solving the multiresponse problem. However, PCA is based on some rigid assumptions, such as the error terms are multivariate normally distributed random variables, which may limit its use in practical applications. Jeyapaul *et al*. [9] utilized genetic algorithm for determining a weight for the S/N ratio of each response. Then, the weighted sum of S/N ratios was used to decide optimal factor levels. However, the genetic algorithm is a search heuristic that provides near optimal solutions for complex search spaces, such as scheduling and transportation problems.

Artificial neural networks (NNs) and grey relational analysis have been broadly used for optimizing many a manufacturing process or a product. This research, therefore, provides a combined approach of artificial NNs and grey relational analysis for solving the multiresponse problem in the Taguchi method. Relevant background of NNs and grey relational analysis are introduced in Section II. The proposed approach is outlined in Section III. A case study is provided for illustration in Section IV. Finally, conclusions are made in Section V.

II. RELEVANT BACKGROUND

A- Neural Networks

The NNs mimic human brains to learn the relationships between certain inputs and outputs from experience. They are considered as information processing systems that have the abilities to learn, recall and generalize from training data. An NN is a parallel computing system consisting of many processing elements connected from layer to layer. An elementary neuron with *R* inputs is shown in Fig 1.

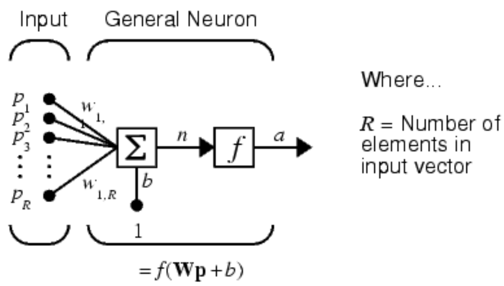

Fig. 1. Illustration of an elementary neuron.

Each input is weighted with an appropriate *w*. The sum of the weighted inputs and the bias forms the input to the transfer function *f*. Neurons may use any differentiable transfer function *f* to generate their output. As shown in Fig .2, the transfer function can be *logsin*, *tansig*, or *purelin*.

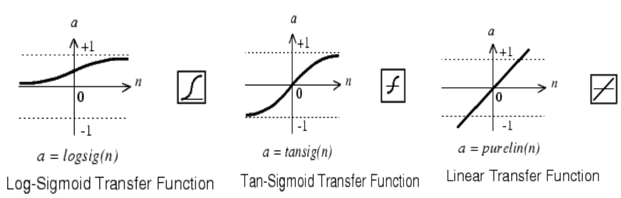

Fig. 2. Types of transfer functions.

In this research, a feedforward network will be used. A single-layer network of *S logsig* neurons having *R* inputs is shown in full detail on the left of Fig. 3 and with a layer diagram on the right.

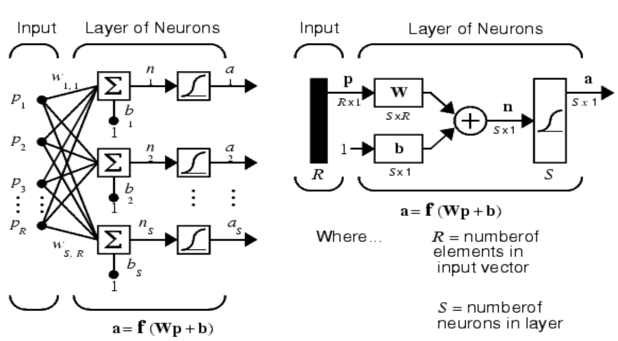

Fig. 3. A single-layer network.

Two types of learning NNs are respectively supervised and unsupervised. For supervised learning NNs, a set of training input vectors with a corresponding set of target vectors is trained to adjust the weights in artificial NNs. For unsupervised learning NNs, a set of input vectors is proposed; however no target vectors are specified. For solving the multiresponse problem, therefore, the supervised learning NNs is appropriate for dealing with the multiresponse problem in the Taguchi method. The most extensively-used supervised learning network is the backpropagation (BP) model, which has been successfully applied to solve many problems [10-11]. Standard BP is a gradient descent algorithm in which the network weights are moved along the negative of the gradient of the performance function. Fig. 4 illustrates a basic BP neural network with three layers.

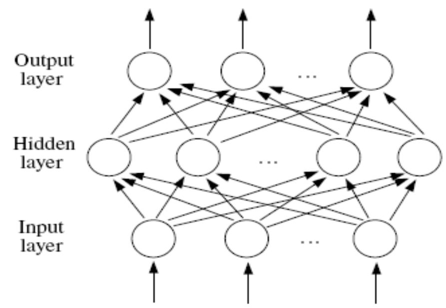

Fig. 4. A basic BP neural network.

The BP neural network initially receives the input vector and directly passes it into the hidden layer. Each element of the hidden layer is used to compute an activation value by summing up the weighted input. The sum of the weighted inputs will be transformed into an activity level by applying a transfer function. Each element of the output layer is then used to calculate an activation value by summing up the weighted inputs attributed to the hidden layer. Next, a transfer function is used to estimate the network output. The actual network output is then compared with the target value. The BP algorithm refers to the propagation of errors of the nodes from the output to the nodes in the hidden layers. These errors are used to update the weights of the network. The most widely learning rule is the BP rule, in which the generalization property makes it possible to train a network on a representative set of input/target pairs and get good results without training the network on all possible input/output pairs. To test the network, a set of data are presented to the trained BP, and then the outputs are evaluated.

Artificial NNs have been widely used for solving the multiresponse problem in many business applications. For example, Hou *et al*. [12] used neural networks and immune algorithms to find the optimal parameters for an IC wire bonding process. Huang and Tang [13] adopted NNs and genetic algorithms to optimize the multiresponse in As-Spun polypropylene yarn. Liao [14] proposed an approach based on NNs and data envelopment analysis (DEA) to solve the multiresponse problem in Taguchi method.

B- Grey Relational Analysis

Grey relational analysis, proposed by Deng [15], is a method of measuring degree of approximation among sequences according to the grey relational grade. Grey relational analysis is part of grey system theory, which is suitable for solving the complicated interrelationships between multiple factors and variables. The major advantage of Grey theory is that it can handle both incomplete information and unclear problems very precisely. The grey relational analysis has been widely employed for solving the multiresponse problem in many manufacturing applications [16-18].

Based on the above introduction, this research proposes a combined approach using BP networks and grey relational analysis for solving the multiresponse problem in the Taguchi method.

III. THE PROPOSED APPROACH

The proposed N-G approach to solve for the multiresponse problem in Taguchi method is displayed in Fig. 5.

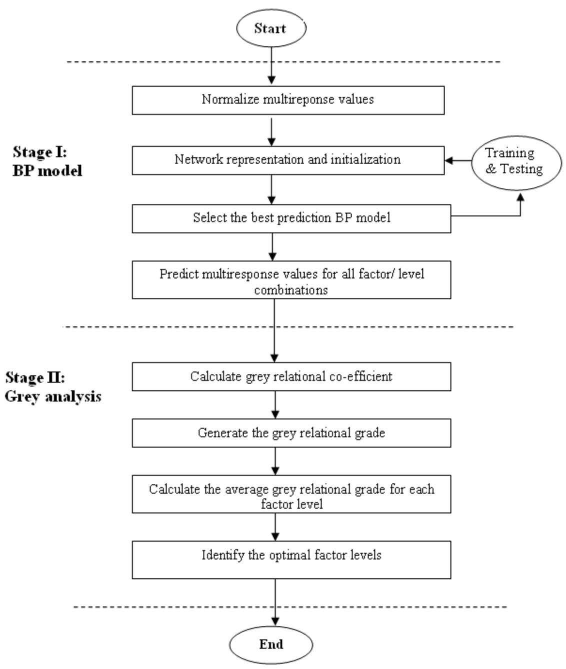

Fig. 5. The steps of N-G approach.

The main steps of the proposed approach are described as follows:

Step 1: Normalize each quality response to avoid the effect of adopting different units and to reduce the variability. This is a necessary step before analyzing the data with the grey relational analysis. Let y_{ij} represents a response i (i=1,…, m) of experiment j (j=1,…, n). Let z_{ij} be the normalized value of y_{ij}, where z_{ij} lies between zero and one. Calculate z_{ij} using the appropriate formula from the Eqs. (1) to (3).

$$z_{ij} = \frac{y_{ij} - \min\ (y_{ij}, j=1,...,n)}{\max\ (y_{ij}, j=1,...,n) - \min\ (y_{ij}, j=1,...,n)} \tag{1}$$

to be used for LTB response. While,

$$z_{ij} = \frac{\max\ (y_{ij}, j=1,...,n) - y_{ij}}{\max\ (y_{ij}, j=1,...,n) - \min\ (y_{ij}, j=1,...,n)} \tag{2}$$

to be used for STB response. Finally,

$$z_{ij} = \begin{cases} \dfrac{y_{ij}}{T} & y_{ij} \le T \\ -\dfrac{y_{ij}}{T} + 2 & y_{ij} > T \end{cases} \tag{3}$$

to be used for NTB response, where T is the desired target value.

Step 2: Build a BP network to predict the z_{ij} values for all combinations of factor levels. Set the number of inputs equal to the number of controllable factors and set the input values as the values of factor levels. Set output values as the multiresponse values. Define the number of hidden layers, transfer functions, training parameters, and input data for prediction. Usually, the network parameter including the learning rate and momentum will be set to assist the trained network to attempt the convergence and stabilization in prediction behavior. An adaptive learning rate will attempt to keep the learning step size as large as possible while keeping learning stable. The learning rate is made responsive to the complexity of the local error surface. The stopping criterion is set to lower the root mean square error (RMSE) in training and testing process.

Step 3: Let z_{oi} denotes the reference sequence for response i; where the z_{oi} value is equal to one in most applications, and z_{ij} be the specific comparison sequence. Let $\gamma(z_{oi}, z_{ij})$ be the grey relational co-efficient for z_{ij}, which is calculated as

$$\gamma(z_{oi},\ z_{ij}) = \frac{\Delta_{\min} + \xi\Delta_{\max}}{\Delta_{ij} + \xi\Delta_{\max}} \tag{4}$$

where ξ is the distinguishing coefficient of a value between zero and one, and commonly set value of 0.5. The Δ_{ij} is the absolute deviation of the z_{ij} value from the reference z_{oi}, or mathematically

$$\Delta_{ij} = \left| z_{0i} - z_{ij} \right| \tag{5}$$

The $\Delta_{\min}$ and $\Delta_{\max}$ are the minimum and maximum values of Δ_{ij}, respectively, from m responses for n experiments. That is,

$$\Delta_{\min} = \min_{\forall i} \min_{\forall j} \Delta_{ij} \tag{6}$$

and

$$\Delta_{\max} = \max_{\forall i} \max_{\forall j} \Delta_{ij} \tag{7}$$

Step 4: Let $\bar{\gamma}_j$ be the grey relational grade for each experiment j. Calculate $\bar{\gamma}_j$ using Eq. (8).

$$\bar{\gamma}_j = \frac{1}{n} \sum_{j=1}^{n} \gamma_{ij} \qquad j = 1, \dots, n \tag{8}$$

Step 5: Let TGV_l be the sum of the $\bar{\gamma}_j$ values at factor level l. Calculate the TGV_l for all factor levels. Typically, larger value of TGV_l indicates better performance. Consequently, identify the factor level with the largest TGV_l as the optimal level for that factor.

A case study will be employed to illustrate the proposed approach in the following section.

IV. Illustration

This case study aimed at improving the performance of wire drawing process for SUS 304 using grey relational analysis based Taguchi method [19]. The main objective was to maximize concurrently the die life (hour abbreviated by hr) and wire tensile strength (kg/mm^2). The orthogonal array L_9 (3^4) was utilized to provide an experimental layout. In SUS 304 wire drawing process, three controllable process factors, including reduction ratio (A), lubricant temperature (B), and drawing speed (C), were investigated. The experimental data utilizing L_9 (3^4) array are displayed in Table 1. The proposed approach was adopted to optimize the die life and wire tensile strength and described as follows:

Step 1: Let z_{1j} and z_{2j} denote the normalized averages of die life ($\bar{y}_{1j}$) and wire tensile strength ($\bar{y}_{2j}$), respectively, for each experiment j; j = 1, ..., 9. Since both quality responses are LTB type responses, the z_{ij} value is calculated using Eq. (1) and displayed in Table 1. For example, the z_{11} for die life in the first experiment is calculated as follows. The maximum values of $\bar{y}_{1j}$ and $\bar{y}_{2j}$ are 75.37 hr and 250.3 kg/mm^2, respectively. Whereas, the minimum values of $\bar{y}_{1j}$ and $\bar{y}_{2j}$ are 45.45 hr and 206.6 kg/mm^2, respectively. Then, the z_{11} is

$$z_{11} = \frac{65.58 - 45.45}{75.37 - 45.45} = 0.673$$

The other z_{ij} values are calculated similarly.

Step 2: To investigate all the combinations of the three-level factors, twenty-seven (= 3^3) experiments are needed as shown in Table 2. The BP network, shown in Fig. 6, is found the most efficient model to predict the z_{ij} values for all the 27 factor level combinations.

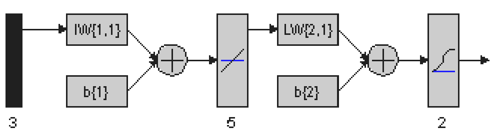

Fig. 6. A BP network used for SUS case study.

In Fig. 6, the BP network consists of three layers, including the input layer. The input layer contains the three process factors. Five neurons are assigned for the hidden layer. Finally, the output layer contains two outputs; the normalized averages of die life and wire tensile strength. Training is set for 10000 runs. The adaptive learning rate and momentum were initially set at 0.01 and 0.9, respectively. The mean square error (*MSE*) is used as stopping criteria. The network was then tested on actual inputs. The predicted z_{ij} values are also displayed in Table 2.

Step 3: The Δ_{ij} values are calculated as $|1 - z_{ji}|$ and also listed in Table 2. From Table 2, the $\Delta_{\min}$ and $\Delta_{\max}$ are equal to 0.997936 and 0.05768, respectively. The $\gamma(z_{oi}, z_{ij})$ values are then calculated using Eq. (4). For example, $\gamma(z_{o1}, z_{11})$ for die life at the first experiment is calculated as

$$\gamma(z_{o1}, z_{11}) = \frac{0.05768 + 0.5 * 0.997936}{0.5163 + 0.5 * 0.997936} = 0.548277$$

The other $\gamma(z_{oi}, z_{ij})$ values are obtained in a similar manner.

Step 4: The $\bar{\gamma}_j$ values are calculated by Eq. (8) for all experiments and listed in Table 2. For example, the $\bar{\gamma}_1$ (= 0.461284) is the average of the $\gamma(z_{o1}, z_{11})$ and $\gamma(z_{o2}, z_{21})$ values; or (0.548277+0.37429)/2. The other $\bar{\gamma}_j$ values are obtained similarly.

Step 5: The TGV_l values are obtained for all factor levels and depicted in Fig. 7.

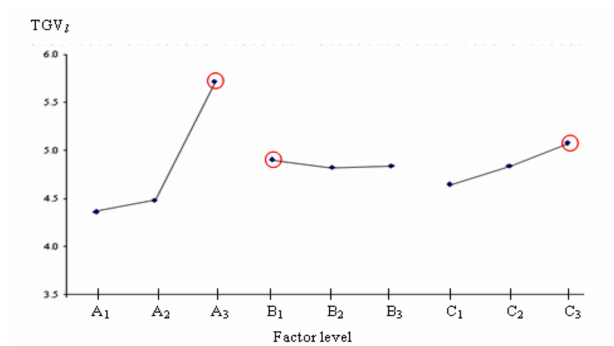

Fig. 7. The plot of TGV_l value versus factor level.

From Fig. 7, the combination of factor levels that optimizes the die life and wire tensile strength concurrently is identified as $A_3B_1C_3$. In Table 1, the die life and wire tensile strength at $A_3B_1C_3$ are 63.8 hr, which is above the overall average, and 250.3 (kg/mm^2), which is the largest wire tensile strength obtained among all combinations of factor levels. This shows the efficiency of the proposed approach for solving the multireponse problem in SUS 304 wire drawing process.

V. Conclusions

An N-G approach is proposed and illustrated for solving the multiresponse problem in the Taguchi method. The main advantage of this approach is that it evaluates all of factor level combinations, and thus provides a global optimal combination is obtained with minimum resources.

ISBN:978-988-98671-9-5

Table 1. Experimental results of L_9 (3^4) array for SUS 304 wire drawing process.

Exp. j	Control factor			**Die life** (hr)						**Wire tensile strength** (kg/mm^2)					
	A (%)	B (°C)	C (m/min)	Replicates (y_{1jk})				$\bar{y}_{1j}$	z_{1j}	Replicates (y_{2jk})				$\bar{y}_{2j}$	z_{2j}
1	1	1	1	66.3	64.86	65.34	65.82	65.58	0.673	207.6	208.4	209.2	210.0	208.8	0.050
2	1	2	2	45.71	48.11	47.31	46.51	46.91	0.049	216.2	215.0	215.8	215.4	215.6	0.206
3	1	3	3	74.87	75.86	75.53	75.20	75.37	1.000	209.8	214.9	211.5	213.2	212.4	0.133
4	2	1	2	62.44	64.57	63.86	63.15	63.51	0.604	216.6	213.0	215.4	214.2	214.8	0.188
5	2	2	3	48.24	42.66	44.52	46.38	45.45	0.000	222.8	224.3	223.3	223.8	223.6	0.389
6	2	3	1	69.47	72.05	71.19	70.33	70.76	0.846	209.6	203.6	207.6	205.6	206.6	0.000
7	3	1	3	61.01	66.59	64.73	62.87	63.80	0.613	245.6	251.8	248.7	254.9	250.3	1.000
8	3	2	1	49.35	53.52	52.13	50.74	51.44	0.200	239.6	232.8	236.2	229.4	234.5	0.638
9	3	3	2	65.62	62.14	63.3	64.46	63.88	0.616	223.5	225.5	227.5	229.5	226.5	0.455
Overall average								60.74						221.5	

Table 2. Grey analysis for predicted normalized responses.

Exp. j	Control factor			**Phase I**		**Phase II**				
				Normalized values		absolute deviation		Grey relational coefficient		Grey grade
	A (%)	B (°C)	C (m/min)	z_{1j}	z_{2j}	Δ_{1j}	Δ_{2j}	$\gamma(z_{o1}, z_{1j})$	$\gamma(z_{o2}, z_{2j})$	$\bar{\gamma}_j$
1	1	1	1	0.4837	0.011759	0.5163	0.988241	0.548277	0.37429	0.461284
2	1	1	2	0.45849	0.027543	0.54151	0.972457	0.534993	0.378305	0.456649
3	1	1	3	0.43349	0.063158	0.56651	0.936842	0.52244	0.387689	0.455064
4	1	2	1	0.58047	0.004937	0.41953	0.995063	0.606042	0.372581	0.489311
5	1	2	2	0.55565	0.011671	0.44435	0.988329	0.590096	0.374268	0.482182
6	1	2	3	0.53054	0.02734	0.46946	0.97266	0.574795	0.378253	0.476524
7	1	3	1	0.67142	0.002064	0.32858	0.997936	0.672647	0.371866	0.522257
8	1	3	2	0.64872	0.0049	0.35128	0.995101	0.654689	0.372572	0.51363
9	1	3	3	0.62533	0.011584	0.37467	0.988416	0.637161	0.374246	0.505704
10	2	1	1	0.43174	0.15629	0.56826	0.84371	0.521583	0.41458	0.468082
11	2	1	2	0.4071	0.30599	0.5929	0.69401	0.509812	0.466604	0.488208
12	2	1	3	0.38292	0.51207	0.61708	0.48793	0.498767	0.564038	0.531403
13	2	2	1	0.52875	0.071694	0.47125	0.928306	0.573735	0.390008	0.481871
14	2	2	2	0.50349	0.15528	0.49651	0.84472	0.559177	0.414269	0.486723
15	2	2	3	0.4782	0.30438	0.5218	0.69562	0.545323	0.465975	0.505649
16	2	3	1	0.62365	0.031196	0.37635	0.968804	0.635938	0.379247	0.507592
17	2	3	2	0.59962	0.071189	0.40038	0.928811	0.618946	0.38987	0.504408
18	2	3	3	0.57509	0.15429	0.42491	0.84571	0.602512	0.413964	0.508238
19	3	1	1	0.38123	0.7425	0.61877	0.2575	0.498013	0.735851	0.616932
20	3	1	2	0.35766	0.87283	0.64234	0.12717	0.487728	0.889018	0.688373
21	3	1	3	0.33476	0.94232	0.66524	0.05768	0.478134	1	0.739067
22	3	2	1	0.47641	0.54592	0.52359	0.45408	0.544368	0.584071	0.56422
23	3	2	2	0.45125	0.74105	0.54875	0.25895	0.531296	0.734444	0.63287
24	3	2	3	0.42634	0.87198	0.57366	0.12802	0.518957	0.887813	0.703385
25	3	3	1	0.57334	0.3339	0.42666	0.6661	0.601373	0.477782	0.539577
26	3	3	2	0.54842	0.54403	0.45158	0.45597	0.585607	0.582915	0.584261
27	3	3	3	0.52326	0.73958	0.47674	0.26042	0.570507	0.733022	0.651764

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