# A Simple Approach for Modeling Shapes and Objects with Irregular Boundaries and Cavities by Combining and Interpolating Parametrically Defined Contours

A.Arokiasamy

***Abstract:*** **Modeling shapes of surfaces and objects with irregular boundaries is not so easy as it will not follow any direct mathematics. So, an attempt is made to model surfaces and objects by combining contours that are defined parametrically. The method proposed here leads to the generation of objects and surfaces without specifying control points. This approach also eliminates need for inputting any data. The interpolation is done between two contours that are parametrically defined. The boundary of the surface can be changed from one contour to the other. By controlling the normal to the surface some interesting objects can also be modeled.**

**Key words:** cavity**,** contour, linear interpolation, non-linear interpolation, surface modeling

## I. Introduction

Surface modeling is now widely used in all engineering design disciplines from consumer goods products to ships, automotive and aerospace industries. In technical applications of 3D computer graphics such as computer-aided design and computer-aided manufacturing, surfaces are one way of representing objects. In my earlier work [1], I have used single contours to develop surfaces and objects with irregular boundaries and cavities. Even though, interpolation of single contours will result in different types of surfaces and objects, all designs are to be carried out by just changing the curvature of a single profile. While a circular or a square plate is extruded to form an object, this will not result in a desired bottom. Moreover, if a cavity is to be introduced, with the single contour technique only a cavity as determined by the selected contour can be introduced. There is no possibility of introducing a different cavity on it. For example, with single contour techniques, a washer with a circular cavity in a square plate cannot be produced. These limitations are overcome by interpolating two different contours.

The objects that can be generated with this technique, are flower vases of different top and bottom, bowls of any shape, a cylinder of any cross-section etc. A pipe connector between two pipes of different cross-section can be designed. For example, a circular pipe and a square pipe can be connected.

Here, two parametrically defined contours are used as controlling curves rather than boundary curves. Because, these curves may not necessarily be the boundaries of the generated objects, though they control the form of the generated object. [2].

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## II. THEORY OF INTERPOLATION OF TWO CONTOURS

Now, the concept of interpolation in generating three dimensional objects will be discussed. Here, one can start with two given contours, say C1 and C2, which are space curves in 3-D, and generate interpolated contours to form an object. [2]
Let the parametric equations of the x,y,z coordinates of the two contours be given as

$$C_1: \quad x = f_1(t), \quad y = f_2(t), \quad z = f_3(t), \tag{1}$$

$$C_2: \quad x = g_1(p), \quad y = g_2(p), \quad z = g_3(p) \tag{2}$$

where $f_i$ and $g_i$ for i = 1,2,3, are the functions of the parameters t and p respectively. One can generate x, y and z coordinates of a 3D object from these contours as

$$\begin{aligned} x &= a_1 . f_1(t) + b_1 . g_1(p), \\ y &= a_2 . f_2(t) + b_2 . g_2(p), \\ z &= a_3 . f_3(t) + b_3 . g_3(p) \end{aligned} \tag{3}$$

One can refer to the $a_i$ 's and $b_i$ 's in (3) as the interpolating functions. These interpolating functions may be linear or non-linear of some parameter(s) resulting into linear or non-linear interpolation.
Equation (3) can be written in matrix form as

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{bmatrix} \begin{bmatrix} f_1(t) \\ f_2(t) \\ f_3(t) \end{bmatrix} + \begin{bmatrix} b_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & b3 \end{bmatrix} \begin{bmatrix} g_1(t) \\ g_2(t) \\ g_3(t) \end{bmatrix} \tag{4}$$

One can now clearly observe that $a_1$ and $b_1$ scale the x-coordinates of the given contours to give the x coordinate of the generated object. There is no contribution from y or z coordinates of the given contours towards the x coordinate of the object. Similar observations can be made about the y and z coordinates of the object. In general we can introduce non zero elements in the rows of both a, and b matrices to allow contribution from the y and z coordinates. Then the equation 4

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{32} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} f_1(t) \\ f_2(t) \\ f_3(t) \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} g_1(t) \\ g_2(t) \\ g_3(t) \end{bmatrix}$$

can be written as

(5)

In (5), $a_{i\ j}$ and $b_{i\ j}$ are functions of some parameter(s). Following analogous treatment for 3-D generalized homogeneous transformation matrix, we can introduce the fourth coordinate H and represent the most generalized equation of the interpolated object as

$$\begin{bmatrix} x \\ y \\ z \\ h \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} f_1(t) \\ f_2(t) \\ f_3(t) \\ f_4(t) \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} \begin{bmatrix} g_1(t) \\ g_2(t) \\ g_3(t) \\ g_4(t) \end{bmatrix} \quad (6)$$

Or compactly written as

$$P = A \,.\, F + B \,.\, G. \quad (7)$$

Each individual components of A and B may be

- a constant in which case P will represent only one resultant contour,
- a linear or non- linear function of one parameter which may or may not be the same for all elements of A and B.
- a linear or non- linear function of more than one parameter which may or may not be the same for all the elements of A and B.

## III. LINEAR INTERPOLATION

For the linear interpolation $a_i$ and $b_i$ in (3) for i = 1,2,3, are set as

$$a_i(s) = A_i s + B_i$$
$$b_i(s) = P_i s + Q_i \quad (8)$$

where $a_i$, $b_i$, $p_i$ and $q_i$ are constants. Substituting (8) into (3) and rearranging we get

$$x = [\, A_1 f_1 . (t) + P_1 . g_1(p) \,]\, s + [B_1 f_1(t) + Q_1 . g_1(p)\,]$$
$$y = [\, A_2 f_2 . (t) + P_2 . g_2(p) \,]\, s + [B_2 f_2(t) + Q_2 . g_2(p)\,]$$
$$z = [\, A_3 f_3 . (t) + P_3 . g_3(p) \,]\, s + [B_3 f_3(t) + Q_3 . g_3(p)\,] \quad (9)$$

If one varies the parameter s between 0 and 1, (9) gives two end contours of the object at s = 0 and s = 1. At s = 0, the x,y,z coordinates of the contour say $C_3$ are

$$C_3 : x = B_1 . f_1(t) + Q_1 . g_1(p),$$
$$y = B_2 . f_2(t) + Q_2 . g_2(p),$$
$$z = B_3 . f_3(t) + Q_3 . g_3(p), \quad (10)$$

These can be written in matrix form as $C_3$:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} B_1 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & B_3 \end{bmatrix} \begin{bmatrix} f_1(t) \\ f_2(t) \\ f_3(t) \end{bmatrix} + \begin{bmatrix} Q_1 & 0 & 0 \\ 0 & Q_2 & 0 \\ 0 & 0 & Q3 \end{bmatrix} \begin{bmatrix} g_1(t) \\ g_2(t) \\ g_3(t) \end{bmatrix} \quad (11)$$

At s =1, the x,y,z coordinates of the contour, say $C_4$ are

$C_4$ :

$$X = (A_1+B_1) . f_1(t) + (P_1+Q_1) . g_1(p)$$
$$Y = (A_2+B_2) . f_2(t) + (P_2+Q_2) . g_2(p)$$
$$Z = (A_3+B_3) . f_3(t) + (P_3+Q_3) . g_3(p) \quad (12)$$

These can be written in the matrix form as C4:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} A_1+B_1 & 0 & 0 \\ 0 & A_2+B_2 & 0 \\ 0 & 0 & A_3+B_3 \end{bmatrix} \begin{bmatrix} f_1(t) \\ f_2(t) \\ f_3(t) \end{bmatrix} + \begin{bmatrix} P_1+Q_1 & 0 & 0 \\ 0 & P_2+Q_2 & 0 \\ 0 & 0 & P_3+Q_3 \end{bmatrix} \begin{bmatrix} g_1(t) \\ g_2(t) \\ g_3(t) \end{bmatrix} \quad (13)$$

The contours $C_3$ and $C_4$ can be identified from the above equations as the results of the linear combinations of the contours $C_1$ and $C_2$ although the weighting factors for individual x,y,z coordinates are different. Therefore the contours $C_3$ and $C_4$ may not in general, have any similarity with the given contours $C_1$ and $C_2$. If one compares each of the equations for the coordinates x,y,z from (9) with (5), it becomes evident that (9) represents the standard linear interpolation between the contours $C_3$ and $C_4$. Thus this standard interpolation is similar to the notion of the generalized interpolation, introduced in the previous section between the given two contours $C_1$ and $C_2$. The interpolated end contours $C_3$ and $C_4$ ( for s = 0 and s = 1, respectively, with s varying between 0 and 1 inclusive ) form the caps or the ends or limits of the generalized object.

If we choose the functions $a_i(s)$ and $b_i(s)$ as

$$a_1(s) = a_2(s) = a_3(s) = As + B,$$
$$b_1(s) = b_2(s) = b_3(s) = Ps + Q \quad (14)$$

then, the weighting factors in each of the x,y,z coordinates become the same.

When A = -1, B=1, P =1 and Q = 0 in (14) , i.e. when $a_1(s) = a_2(s) = a_3(s) = 1-s$ and $b_1(s) = b_2(s) = b_3(s) = s$, the object is formed by the standard interpolation between $C_1$ and $C_2$ , i.e. $C_3$ reduces to $C_1$ and $C_4$ reduces to $C_2$ .

The equation of the object in this case is

$$x = (1-s) . f_1(t) + s . g_1(p),$$
$$y = (1-s) . f_2(t) + s . g_2(p),$$
$$z = (1-s) . f_3(t) + s . g_3(p) \quad (15)$$

The process of object generation involves selecting two points from the given two contours and connecting them through the corresponding interpolated points obtained by (4). This curve defines the shape of the object in the z-direction at these particular points. One is to note that this shape is not the same, in general, as one move from one point to the next on the given contours.

## IV. SURFACES WITH Z-COMPONENT ZERO OR CONSTANT

Based on the above theory of interpolation between two different contours, say a circle and a square i.e. when f(t) is a circle and g(p) is a square , a flat surface with the desired cavity is generated if the z-coordinate is set to zero. The inner and outer boundaries will be different. Which ever contour is defined with higher dimension will assume the outer boundary. Fig.1 to 8 show the application of this technique with the z-coordinate zero. The different contours used to generate the figure are mentioned at the bottom of each figure. For example, the parametric equations used for generating the fig. 7 are as follows:

```
x= s*a*(2*cos(t)+cos(2*t))+(1-s)*rc*cos(t);
y =s*a*(2*sin(t)-sin(2*t))+(1-s)*rs*sin(t);
z = 0;                                         (16)
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with a = 10, rc = 40 and rs = 40.

This technique leads to the design of washers with any type of cavity irrespective of the outer boundary. This is in contrast to the single contour interpolation technique where the inner and outer boundaries are same [1].

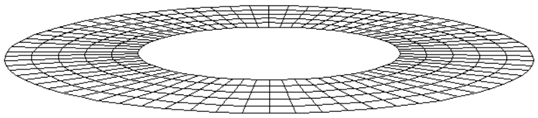

*Fig. 1 Interpolation Between two Circles*

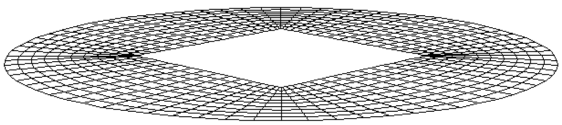

Fig. 2 Interpolation between a Circle and a Square

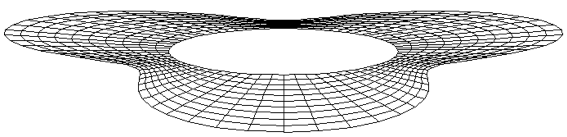

*Fig. 3 Interpolation Between Two Circles one with Radius = A+ B *cos(t)*

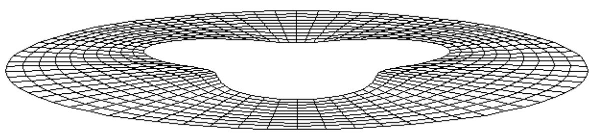

*Fig. 4 Interpolation Between Two Circles one with Radius = A+ B *cos(t)*

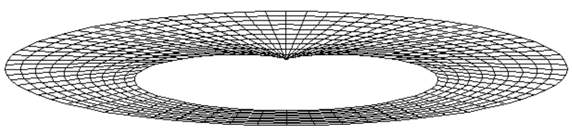

*Fig. 5 Interpolation Between a Circle and a Cardioid with Radius = 1+cos(t)*

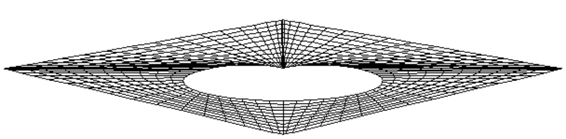

Fig.
6 Interpolation Between a Square and a Cardioid with Radius = 1+cos(t)

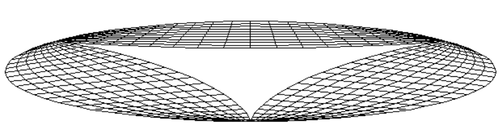

*Fig. 7 Interpolation Between a Circle and a Deltoid*

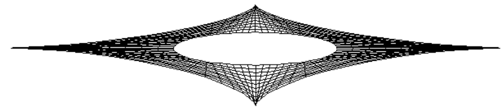

*Fig. 8 Interpolation Between a Circle and an Astroid*

## V. SURFACES WITH THE Z-COMPONENT AS A FUNCTION OF THE INTERPOLATION PARAMETER S:

When the z-component is made as a constant multiple of the interpolation parameter the two contours are connected linearly. It is equivalent to extruding the surface in the z direction. The scalar multiple of s decides the amount of extrusion**.** Fig. 9 to Fig. 13 show how this technique is adopted in generating surfaces or even objects with different bottom and top cross sections.
This results in generation of cylinders of any cross section, pipe connectors for two pipes of different cross sectional areas, flower vases of different caps, etc.

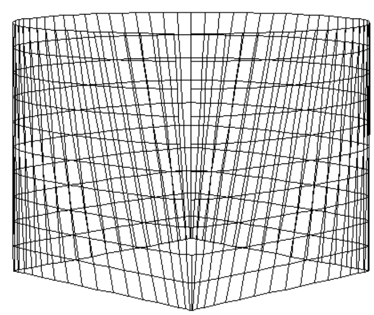

Fig. 9 Interpolation between a Circle and a Square

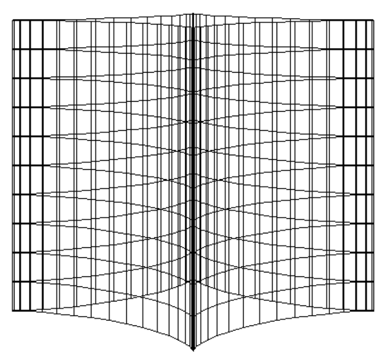

Fig. 10 Interpolation between Two Astroids

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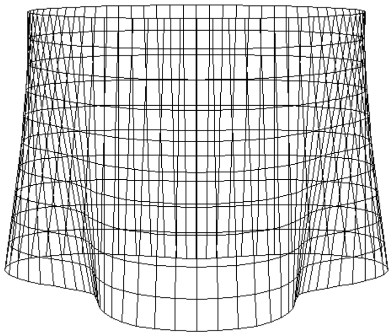

Fig. 11 Interpolation between Two Circles one circle with radius=A+Bcos(n*t)

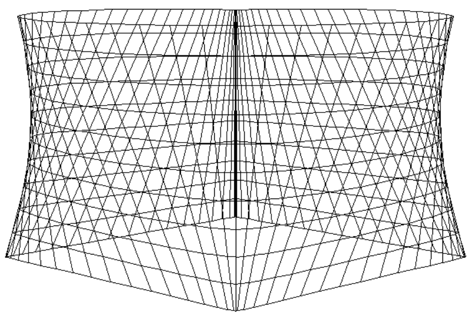

Fig. 12 Interpolation Between a Square and a Cardioid

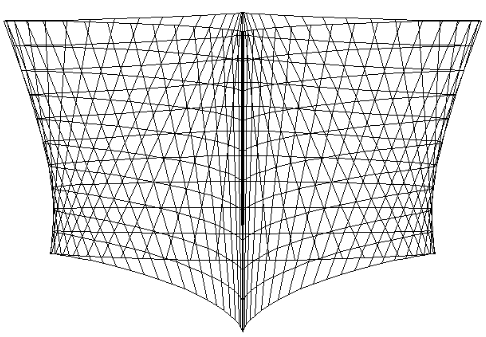

Fig. 13 Interpolation Between an Astroid and a Deltoid

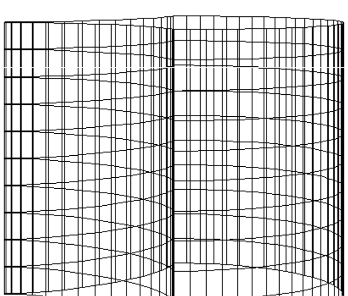

Fig.14 Boundary Control Combining a Circle and an Astroid

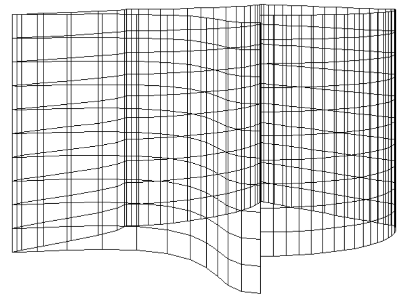

Fig.15 Boundary Control Combining Four Different Contours and Extruded as a Cylinder

It is possible to have boundary control over any one of the contours or over both the contours. Figures 14 and 15 show this effect.

## VI. Non-Linear Interpolation Techniques

In the case of linear interpolation s and (1-s) are used with the two different contours (or even with two similar contours with the same dimension or different dimensions). If either s or (1-s) is replaced by a non-linear function of s then the resulting object will be different from what was obtained in linear interpolation techniques. This is termed as non-linear interpolation between the two contours. Even if two similar contours are interpolated the resulting caps of the object generated will no more be the shape of the original contour. For example, two circles of the same radius are interpolated with (1-s) being substituted with $\sin(2\pi s)$ then the result is shown in Fig.16. Fig.17 to Fig.20 show the results of this technique. The interpolating functions are given at the bottom of each figure.

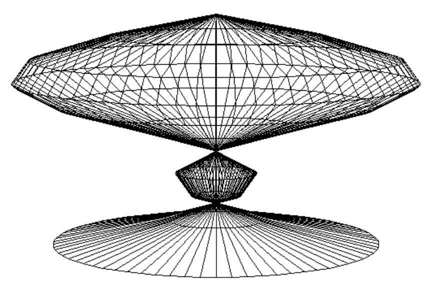

Fig.16 Non-Linear Interpolation between Two Circles with the Interpolating function sin(2*π*s)

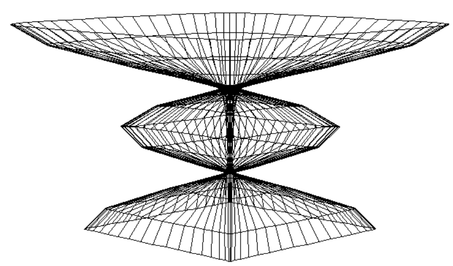

Fig. 17 Non-Linear Interpolation Between a Circle and a Square with the Interpolating function cos(2*π*s)

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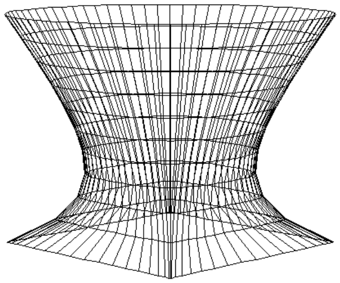

Fig.18 Non-Linear Interpolation Between a Circle and a Square with the Interpolating function exp(-5*s)

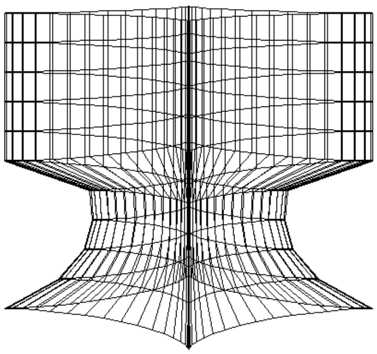

Fig.19 Non-Linear Interpolation Between a Circle and a Square with the Interpolating function (1-s*s)

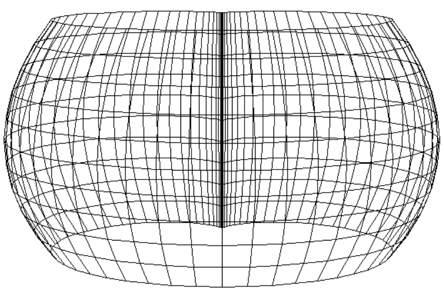

Fig.20 Non-Linear Interpolation Between Two Cardioids with Mixed Interpolating functions

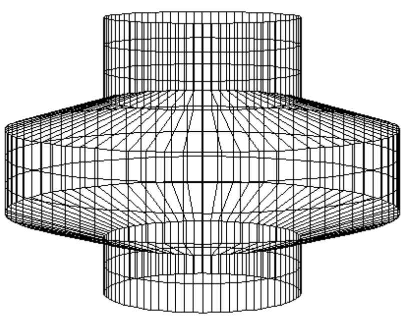

Fig.21 Design of a Shaft

Some of the objects designed using these techniques are shown in from Fig. 21 through Fig. 32.

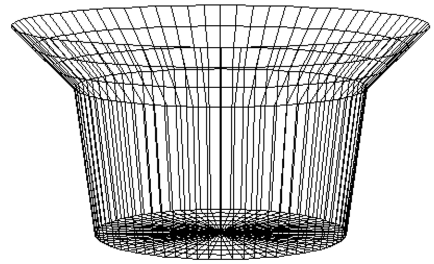

Fig.22 Design of a Basket

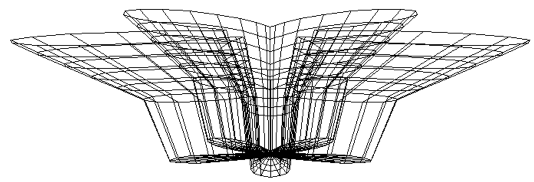

Fig.23 Design of a Basket

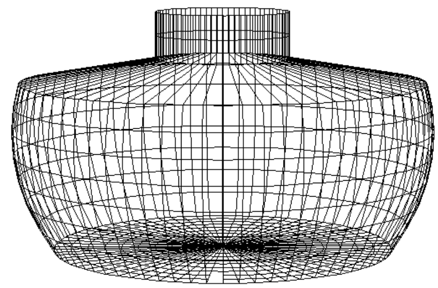

Fig.24 Design of a Bottle

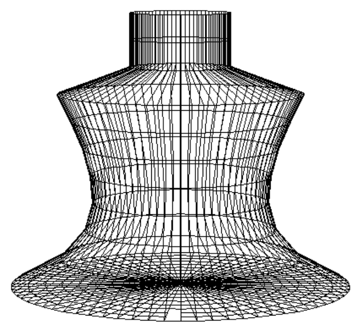

Fig.25 Design of a Bottle

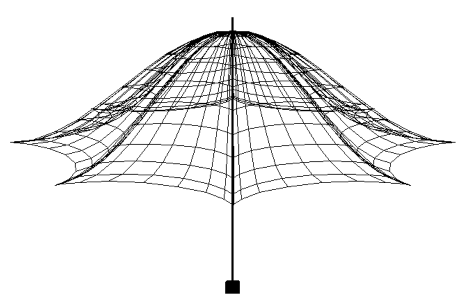

Fig. 26 Design of an Umbrella

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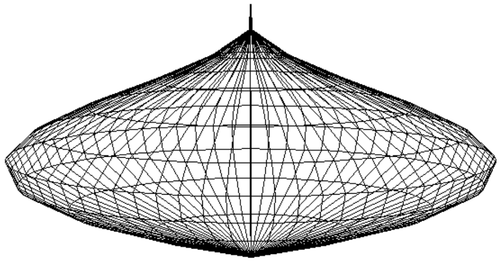

Fig. 27 Generation of a Deformed Surface

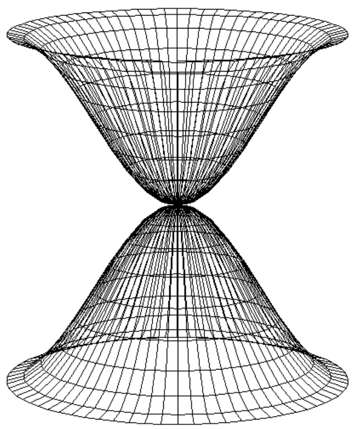

Fig.28 Design of a Bell Shaped Surface

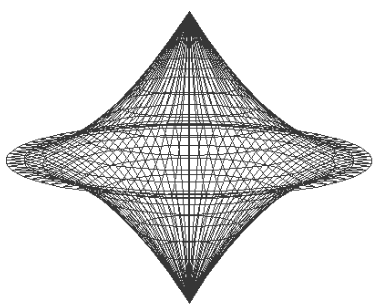

Fig.29 Generation a deformed surface

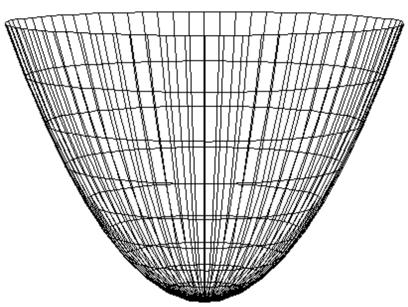

Fig.30 Design of a Bowl

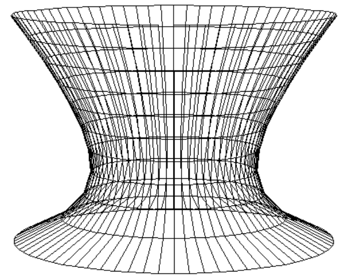

Fig.31 An Object Generated Using Deformation

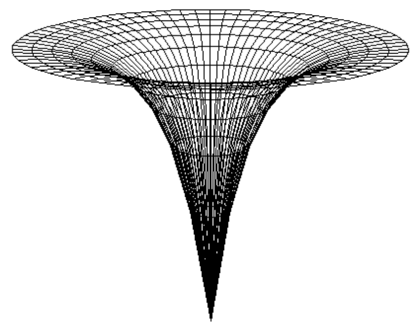

Fig.32 A Surface Resembling a PDE Surface

## VII. CONCLUSION

It is shown that the interpolation technique is a versatile technique in generating 3D surfaces and objects. Many surfaces and objects have been generated and the output shown. It is possible to generate cones of different bases, cylinders of desired cross section, flower vases with different top and bottom cross sections etc. It is possible to have numerous variations of this technique, which will result in different objects. Depending upon the applications, if one has the intuition of deciding which interpolating function is to be used, it is easier to generate the desired objects. With the use of mixed interpolating functions numerous variations of object generation is possible. As the value of interpolating parameter s is to be controlled in relation to the specification of control points object generation is made easy with this approach. For example, at any value of the parameter s, the dimension of the contours may be changed or the normal to the surface can be changed by setting the Z co-ordinate to a different value. The entire work was implemented in **Java.**

## REFERENCES

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