# A Comparative Study on the Topological Properties of the Hyper-Mesh Interconnection Network

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*Abstract*- **Recent ad*vances in parallel computing and the increasing need for massive parallelism resulted in the emergence of many attractive interconnection networks. Hypercube and mesh topologies have drawn considerable attention due to many of their attractive properties that are suited for parallel computing. However, both has certain major drawbacks that are affecting their over all performance. This paper presents a comparative study on the topological properties of the hyper-mesh interconnection network, which is a new family of networks that combines hypercubes and meshes. Preliminary results reveal that the hyper-mesh improves on the scalability, degree, and cost for the hypercube; and enhances the diameter and bisection width of the mesh, which is considered major drawbacks for the constituent interconnection networks of the hyper-mesh topology. Also, the hyper-mesh improves on the connectivity property of both networks, which is needed for fault-tolerance.**

*Index Terms*- **Hypercubes, hyper-meshes, interconnection networks, meshes, topological properties.**

## I. INTRODUCTION

Interconnection networks with a massive number of processors have evolved to cope with the continued demand for more computing power using parallelism in many applications. In parallel processing, large number of processors cooperates to solve a given problem. This lead to the solution of many computational problems requiring long computing time, that was impossible to solve in real time by existing sequential machines. The investigation of parallel computing is becoming one of the hot topics in theoretical computer science. Research ranges from the study of theoretical abstract parallel computing models and architectures to the actual developments of parallel algorithms and machines. Recently, we have witnessed tremendous advances in hardware technology that lead to the availability of very fast and inexpensive hardware. This made it possible to improve in the architectural features of parallel machines and provided novel interconnection networks between processors. The study of interconnection networks and their computational properties has been pursued to enhance the performance of large-scale parallel computer systems.

The choice of a network topology for parallel systems is a fundamental design decision that involves trade-offs between performance and cost [13, 14, 24]. A great deal of research has been directed towards studying the characteristics of the interconnection topology, which directly affect the expected performance measures of the global system. The ideal solution, of providing a direct link to connect every pair of nodes, is very expensive as the number of nodes becomes large. Therefore, other cost effective schemes should be proposed and evaluated. The total number of links must be reduced, yet providing low communication overhead, as well as allowing simple routing strategies to keep high operational capabilities in the presence of faulty nodes or links. Consequently, among other important factors, which affect the expected system performance, attention must be paid to the amount of extra delay due to the nonexistence of direct links between any pair of nodes, and to the routing procedures to be executed for message-passing communication schemes. Designing a good topology that has all the attractive features without drawbacks is impossible, as there is a trade-off between the different required characteristics.

The hyper-mesh network was introduced to combine two well known interconnection networks, hypercubes and meshes, to obtain a new network that inherits the attractive features of both [2, 21]. The hypercube topology has drawn considerable attention due to many of its attractive properties such as logarithmic diameter, large bandwidth, maximum fault tolerance, and a recursive structure that naturally suites parallel computing [7, 12, 16, 20]. However, it has a large degree, which is a major drawback when the dimension gets larger. On the other hand, the mesh has a low fixed degree that does not increase as the network size increases [8, 16]. However, the performance of the mesh is not as good as that of the hypercube, especially for large network sizes. This paper presents a comparative study on the topological properties of the hyper-mesh interconnection network, which is a new family of networks that combines hypercubes and meshes. We examine the hyper-mesh from a graph theory point of view and consider those features that make its connectivity so appealing. Our study is based on the most common used criteria for evaluating interconnection networks such as size, degree, diameter,

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bisection width, connectivity, and cost. Preliminary results show that the hyper-mesh exhibits the appealing properties of its constituent interconnection networks and eliminates their drawbacks.

The remainder of the paper is organized as follows. In section 2, some related work in the area is briefly reviewed. Section 3 provides the necessary definitions and notations and presents the topological properties for the hyper-mesh and its constituent networks. Section 4 conducts a comparison and evaluation on the most common topological properties of interconnection networks between hypercubes, meshes, and hyper-meshes. Finally, section 5 concludes the paper and discusses some possible future research in this area.

## II. RELATED WORK

Interconnection networks based on the hypercube topology attracted the attention of many researchers due to many of their attractive features suited for parallel computing [12, 16, 20. One of the major drawbacks of the hypercube network is that for large size network, the number of connections required per node is large which has a direct affect in the implementation phase of a parallel machine. For a network of size $N$, the number of connections required per node is $log_2 N$. Thus, the number of connections per node is not practical for large systems. Many variations of the hypercube topology have been proposed by researchers to improve on its properties and to eliminate its drawbacks, either by making some modifications in the link connectivity or by the cross product with other interconnection networks [1, 2, 7, 11, 15, 17, 19, 22, 23]. Loh *et al.* [17] proposed the exchanged hypercube by removing some links to reduce the cost of the hypercube. Efe [9] introduced the crossed cube that reduces the diameter of the hypercube by a factor of two. This was accomplished by using the pair-related relation to change the linking procedure between nodes. El-Amaway and Latifi [11] proposed the folded hypercube to reduce the diameter and the traffic congestion with little hardware overhead by adding a new complement edge to each node. Another group of researchers tried to eliminate the major drawbacks of the hypercube by the cross product of other topologies, without losing the attractive features of the hypercube. Preparata and Vuillemin [19] proposed the cube-connected cycles to eliminate the major drawback in the hypercube structure by reducing the number of connections per processor to *3*. It is produced by replacing each hypercube super node by a cycle of size $n$, where $n$ is the dimension of the hypercube. Abuelrub [2] introduced the hyper-mesh to eliminate the drawbacks of hypercubes and meshes by combining two dimensional meshes and hypercubes. The major advantage of this structure is reducing the degree that the hypercube suffers from and increasing the diameter that the mesh suffers from, in addition to other gained good properties. Awwad *et al*. [3] introduced the arrangement-star that outperforms both the star and the arrangement graph. Day and Tripathi [7], as well as Zheng *et al*. [23] studied the hyper-star, which is a combination of hypercubes and stars. Youssef and Narahari [22] proposed the banyan-hypercube network to reduce the communication overhead of the hypercube. Abdullah *et al*. [1] introduced the chained-cubic tree, which consists of hypercubes cascaded horizontally and connected vetically using a tree structure. The new structure aimed to eliminate the drawback of hypercubes and trees.

In mesh topologies, the number of connections per node is fixed and does not increase as the network size increases. However, the performance of the mesh network is not as good as that of the hypercube [16]. The mesh topology has a high diameter compared to the hypercube, especially for large network sizes. In general, as the node degree increases, the diameter decreases linearly. Many researchers proposed different variations of the mesh structure to improve on the diameter and connectivity [2, 4, 5, 10]. Leighton [16] proposed the mesh of trees, which has both small diameter and large bisection width, to provide a very fast network when considered solely in terms of speed. Marsden *at el*. [18] proposed the OTIS-mesh, which is a mesh based model of computing that exploits the special features of both electronic and optical technologies. In [16], Leighton summarized the different variations of the mesh architecture to improve on the bisection width and communication speed. Some variants of the mesh model allow wrap-around connections between processors on the edge of the mesh. These connections may connect processors in the same raw or column, or they may be toroidal. Efe and Fernandez [10] introduced the mesh-connected trees as a bridge between grids and meshes of trees to enhance the performance of the mesh of trees to suit grids based applications.

## III. DEFINITIONS AND NOTATIONS

This paper uses undirected graphs to model interconnection networks. Let $G= (V, E)$ be a finite undirected graph, where $V$ and $E$ are the vertex and edge sets of $G$, respectively. Each vertex represents a processor and each edge is a communication link between two processors. A *hypercube* of dimension n, denoted $Q_n$, is an undirected graph consisting of $2^n$ vertices labeled from $0$ to $2^n-1$ and $n*2^{n-1}$ edges, such that there is an edge between any two vertices if the binary representations of their labels differ by exactly one bit position. The degree of $Q_n$ is $n$, diameter is $n$, and there exist $n$ node-disjoint paths between any two nodes. Figure 1 shows a hypercube of dimension *3*.

*A mesh*, denoted $M(r, c)$, is a two dimensional topology consisting of $r$ rows and $c$ columns with a total of $r*c$ vertices labeled $(x, y)$, where $1 \le x \le r$ and $1 \le y \le c$. Each interior vertex has exactly *4* neighbors. The degree of $M(r, c)$ is *4*, diameter is $r+c-2$, and there exist at least *2* node-disjoint paths between any two nodes. For simplicity and without loss of generality, we will assume that $r=c=k$ and $k$ is an even number. Figure 2 shows a two dimensional mesh $M(k)$, where $k=2$.

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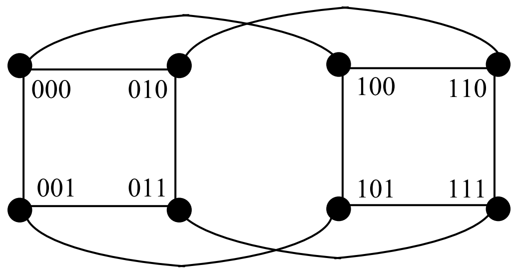


Figure 1: Hypercube of dimension *3*.

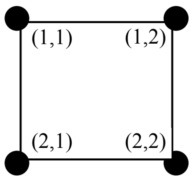


Figure 2: *M (k)*, where *k=2*.

The *cross product* of two graphs is a formal mathematical representation for studying the properties of graphs. This technique is used by many researchers to generate new attractive versions of common basic interconnection networks [6]. Given two interconnection networks $G_1= (V_1, E_1)$ and $G_2= (V_2, E_2)$, where $V_1$ is the set of vertices in $G_1$, $V_2$ is the set of vertices in $G_2$, $E_1$ is the set of edges in $G_1$, and $E_2$ is the set of edges in $G_2$. Then the cross product of $G_1$ and $G_2$ is a new interconnection network $G= G_1 \otimes G_2= (G, V)$, where $V= \{\langle x, y\rangle | x \in V_1$ and $y \in V_2\}$ and $E= \{(\langle x_1, y\rangle, \langle y_1, y\rangle)| (x_1, y_1) \in E_1\} \cup \{(\langle x, x_2\rangle, \langle x, y_2\rangle)| (x_2, y_2) \in E_2\}$. A *hyper-mesh*, denoted *QM (n, k)*, consisting of $2^n k^2$ nodes. It consists of a hypercube $Q_n$ and a mesh *M (k)*. The hyper-mesh is constructed by placing the $2^n$ super-nodes of the hypercube by meshes of size $k*k$. A node labeled *(x, y, z)* consists of the hypercube part labeled *x* and the mesh part labeled *(y, z)*. Figure 3 shows a hyper-mesh, where *n=3* and *k=2*.

The most important topological properties of the hyper-mesh [2] including the following:

1. *Size*: A hyper-mesh *QM (n, k)* consists of $2^n k^2$ nodes.
2. *Degree*: A hyper-mesh of *QM (n, k)* has a degree of *n+4*.
3. *Diameter*: Let *n* be the dimension of the hypercube and let the mesh be of size $k^2$, then the diameter of the hyper-mesh is *n+2k-2*.
4. *Bisection width*: Let *n* be the dimension of the hypercube and let the mesh be of size $k^2$, then the bisection width of the hyper-mesh is $2^{n-1}k^2$.
5. *Connectivity*: Let *u* and *v* be two nodes in *QM (n, r, c)*, then the number of node-disjoint paths is the minimum of (number of neighbors of node *u*, number of neighbors of node *v*), and it is of at most length *n+2k-2*.
6. *Cost*: Let *n* be the dimension of the hypercube and $k^2$ be the size of the mesh, then the cost (number of links) of the hyper-mesh is $2^{n+1}(k^2-k)+nK^2 2^{n-1}$.

Table 1 shows a summary of the topological properties of hypercubes, meshes, and hyper-meshes. The most common properties that are usually used for comparison purposes are the size, the degree, the diameter, the bisection width, the connectivity, and the cost.

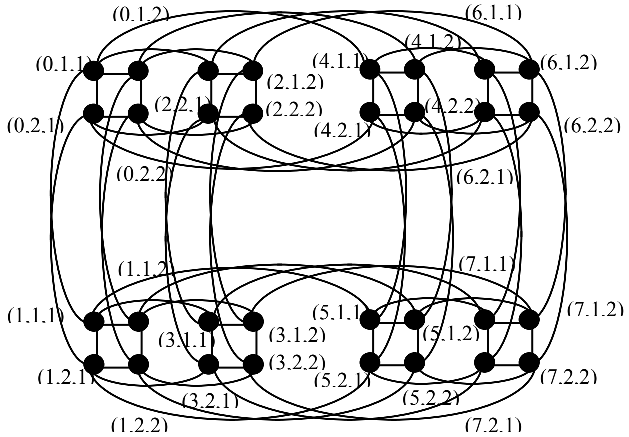


Figure 3: The hyper-meshQM (3, 2).

Table 1: Basic parameters of $Q_n$, M(k), and QM(n, k)

| **Property** | **Hypercube $Q_n$** | **Mesh M (k)** | **Hyper-mesh QM (n, k)** |
|---|---|---|---|
| Size | $2^n$ | $k^2$ | $2^n k^2$ |
| Degree | *n* | *4* | *n+4* |
| Diameter | *n* | *2k-2* | *n+2k-2* |
| Bisection Width | $2^{n-1}$ | *k* | $2^{n-1}k^2$ |
| Connectivity | *n* | *2* | *n+2k-2* |
| Cost | $n2^{n-1}$ | $2k^2-2k$ | $2^{n+1}(k^2-k)+nK^2 2^{n-1}$ |

## IV. COMPARASION AND EVALUATION

In this section, we conduct a comparative study between the three interconnection networks that are under investigation; the hypercube, the mesh, and the hyper-mesh. Our study is based on the most common used criteria for evaluating interconnection networks such as size, degree, diameter, bisection width, connectivity, and cost. The results show that the hyper-mesh exhibits the appealing properties of its constituent interconnection networks and more importantly eliminates their drawbacks.

In reality, we usually have a fixed size network, which might not be equal to the size of the desired network, in terms of the number of processors, in order to solve a specific problem. Therefore, we try to find the smallest network size that has at least as many processors as the desired network, such a network is referred to as the *optimal network*. Furthermore, we use a criterion, which is an indication on the network ability to fit the desired size, denoted as the *degree of accuracy*.

*Degree of accuracy = (Actual size – Desired size) / Actual size x 100%*

This criterion gives an indication of how far the optimal network size from the desired network size. In this measure, the closer the value to 100% is the better the fit to the desired network size. Figure 4 shows the three topologies scaling near to the desired size, where hyper-mesh and mesh networks exhibit almost exact fit to the desired network size. On the other hand, the hypercube network provides fluctuating network size from the desired network. One major draw back of the hypercube is related to its scalability. The size of the hypercube network increases rapidly. The hyper-mesh improves on the scalability of the hypercube with an average improvement of 31.68%, while preserving its desirable characteristics.

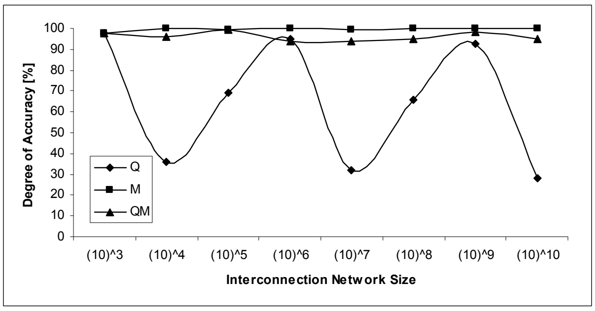


Figure 4: Degree of size accuracy for the three interconnection networks.

An efficient interconnection structure should have a low degree, a small diameter, and a large connectivity for fault-tolerance. Interconnection networks have ranged from the simple to the complex, representing the trade off between speed and cost. At one extreme is the ring, in which each processor is linked to only two other processors, and at the other extreme is the fully connected network, in which each processor has its own private link to every other processor in the network. Between these two extremes, there is a number of networks with intermediate numbers of neighbors. There is a trade off between the diameter and the degree of a network. A network with a low diameter has a large diameter and vice versa. The diameter multiplied by the degree of a network, denoted *DMD*, is a good criterion to measure the efficiency of an interconnection network.

*DMD = Diameter x Degree*

Many researchers who tried to solve the degree problem of the hypercube found themselves involved in other problems such as high diameter, complex designs, and high cost. Therefore, many of the hypercube's topology variations were not practical in reality. Figure 5 shows that the hypercube has the highest degree, which will be costly when the network needs to be scaled up. On the other hand, the mesh has a small constant degree. The figure shows that the hyper-mesh improved on the degree criterion over that of the hypercube with an average improvement of 18.2%. The performance of a parallel machine is influenced directly by the degree of concurrency applied by its interconnection network. To obtain high performance, the diameter must provide a low communication rate between the nodes of the network. Figure 6 shows that the hypercube and the hyper-mesh have a low diameter compared to the mesh. It can be noticed that the hyper-mesh improved on the diameter of the mesh; the diameter of the mesh is linear while the diameter of the hyper-mesh is logarithmic.

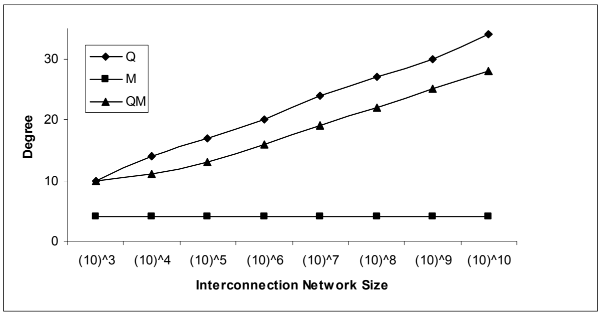


Figure 5: Degree for the three interconnection networks.

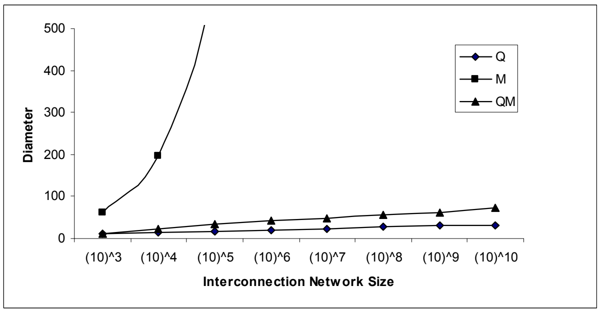


Figure 6: Diameter for the three interconnection networks.

The bisection width, which is the number of links that must be removed to partition the network into two almost equal separate halves, is an important criterion in fault-tolerance. For instance, networks with large bisection width allow faster and reliable communication. However, such networks are difficult to implement using the current technology that is based on a two dimensional layout. Figure 7 shows that both the hypercube and the hyper-mesh exhibit large bisection width, which is more desirable in fault-tolerance. The figure shows that the hyper-mesh improved on the bisection width criterion over that of the mesh by making the bisection width growing exponentially. Another important criterion that is desirable in fault-tolerance is connectivity. In such environment, it is important to have parallel node-disjoint paths in an interconnection network to speed up the transfer of large amount of data and provide alternative routes in cases of node failures. Figure 8 shows that the hyper-mesh has the best connectivity among the three interconnection networks under investigation. It doubles the connectivity for the hypercube and improved it for the mesh from constant to linear.

The number of links that are required by a given network is an important factor that affects its implementation cost. This criterion captures both the real wiring cost and the number of pins required at each node. Figure 9 plots the number of links against network size for the three networks under investigation. It shows that

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the hyper-mesh has a moderate cost compared to the hypercube that has a high cost and the mesh that has a less cost. The figure shows that the hyper-mesh improved on the average cost criterion over that of the hypercube with an average improvement of 48.5%.

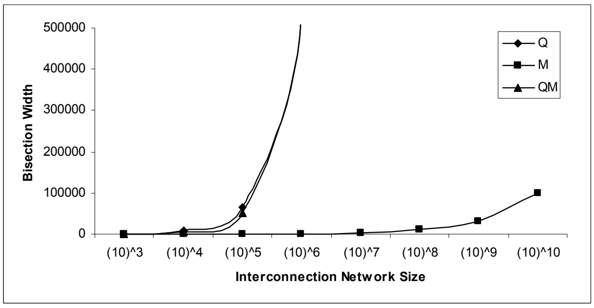


Figure 7: Bisection Width for the three interconnection networks.

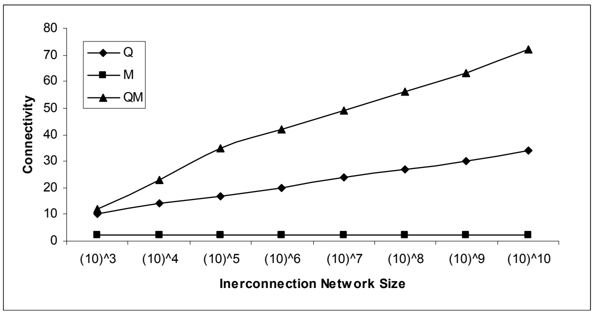


Figure 8: Connectivity for the three interconnection networks.

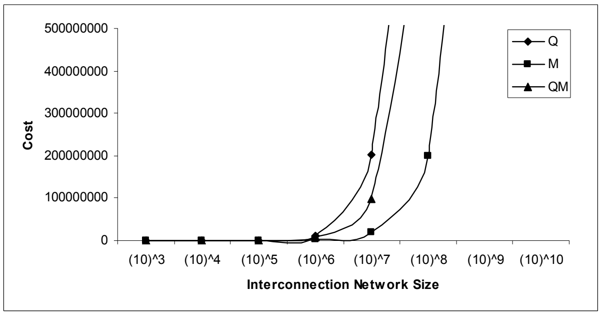


Figure 9: Cost for the three interconnection networks.

## V. CONCLUSIONS AND FUTURE WORK

The last two decades witnessed the introduction of many interconnection networks for parallel computing. Hypercube and mesh interconnection networks have drawn considerable attention due to many of their attractive features. Seeking a good variation of hypercubes and meshes that preserve their attractive properties and reduces their drawbacks led to studying hyper-mesh properties. The study reveals that the hyper-mesh exhibits the appealing properties of its constituent interconnection networks and eliminates their major drawbacks. It improves on the scalability, degree, and cost for the hypercube; and enhances the diameter and bisection width of the mesh, which is considered major drawbacks for the constituent interconnection networks of the hyper-mesh topology. In addition to that, the hyper-mesh improves on the connectivity property of both networks, which is needed for fault-tolerance.

Preliminary investigations show that the hyper-mesh architecture has attractive features that suit parallel computations. A good problem will be to uncover more of the appealing properties of the hyper-mesh. Another interesting problem is to show the ability of this structure to compute, simulate other interconnection networks, and reconfigure itself in the presence of faults.

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