Fuzzy Approach for Restoring Color Images Corrupted with Additive Noise

M.Wilscy and Madhu S. Nair

Abstract—**A fuzzy approach is proposed here for restoring color images that are corrupted with additive noise. The proposed fuzzy approach consists of two sub-filters, where the first fuzzy sub-filter computes the fuzzy distances between the color components of the central pixel and its neighborhood using Gaussian combination membership function, and the second sub-filter corrects the pixels where color component differences are corrupted so much. The performance of the proposed approach is compared with conventional filters, both visually and quantitatively, and experimental results show the feasibility of the new approach.**

Keywords—**Additive Noise, filter, fuzzy, distance measure, color images.**

I. Introduction

IMAGES are often degraded by random noise. Noise can occur during image capture, transmission or processing, and may be dependent on or independent of image content. Noise is usually described by its probabilistic characteristics. Gaussian noise is a very good approximation of noise that occurs in many practical cases [1]. The ultimate goal of restoration techniques is to improve an image in some pre-defined sense. Although there are areas of overlap, image enhancement is largely a subjective process, while image restoration is for the most part an objective process. Restoration attempts to reconstruct or recover an image that has been degraded by using a priori knowledge of the degradation phenomenon [2]. Thus restoration techniques are oriented toward modeling the degradation and applying the inverse process in order to recover the original image. This approach usually involves formulating a criterion of goodness that will yield an optimal estimate of the desired result. By contrast, enhancement techniques basically are heuristic procedures designed to manipulate an image in order to take advantage of the psychophysical aspects of human visual system. For example, histogram equalization is considered an enhancement technique because it is primarily on the pleasing aspects it might present to the viewer, whereas removal of image blur by applying a deblurring function is considered a restoration technique.

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The degradation process is usually modeled as a degradation function that, together with an additive noise term, operates on an input image $f(x, y)$ to produce a degraded image $g(x, y)$. Given $g(x, y)$, some knowledge about the degradation function H, and some knowledge about the additive noise term $\eta(x, y)$, the objective of restoration is to obtain an estimate $\hat{f}(x, y)$ of the original image. The estimate needs to be as close as possible to the original input image and, in general, the more about H and η is known, the closer $\hat{f}(x, y)$ will be to $f(x, y)$. If H is a linear, position-invariant process, then the degraded image is given in the spatial domain by

$$g(x, y) = h(x, y) * f(x, y) + \eta(x, y)$$

where $h(x, y)$ is the spatial representation of the degradation function and the symbol "*" indicates convolution [2].

During image transmission, noise which is usually independent of the image signal occurs. Noise may be additive, where noise and image signal g are independent.

$$f(x, y) = g(x, y) + v(x, y)$$

where $f(x, y)$ is the noisy image signal, $g(x, y)$ is the original image signal and $v(x, y)$ is the noise signal which is independent of g [3]. The additive noise image v models an undesirable, unpredictable corruption of g. The process v is called a two-dimensional random process or a random field. The goal of restoration is to recover an image h that resembles g as closely as possible by reducing v. If there is an adequate model for the noise, then the problem of finding h can be posed as the image estimation problem, where h is found as the solution to a statistical optimization problem. The detailed statistics of the noise process v may are unknown. In such cases, a simple linear filter approach can yield acceptable results, if the noise satisfies certain simple assumptions such as zero-mean additive white noise model [9]. Noise may be impulse noise, which is usually characterized by some portion of image pixels that are corrupted, leaving the remaining pixels unchanged. The most common example of the impulse noise is the salt-and-pepper noise removal.

II. Noise Removal Methods

Noise reduction is the process of removing noise from a signal. Noise reduction techniques are conceptually very similar regardless of the signal being processed, however a priori knowledge of the characteristics of an expected signal can mean the implementations of these techniques vary greatly depending on the type of signal. Although linear image enhancement tools are often adequate in many

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applications, significant advantages in image enhancement can be attained if non-linear techniques are applied [3]. Non-linear methods effectively preserve edges and details of images, whereas methods using linear operators tend to blur and distort them. Additionally, non-linear image enhancement tools are less susceptible to noise. Some common image noise removal methods are:

Gaussian filters: One method to remove noise is to use linear filters by convolving the original image with a mask. The Gaussian mask comprises elements determined by a Gaussian function. This brings the value of each pixel into closer harmony with the value of its neighbors. Gaussian filtering works relatively well, but the blurring of edges can cause problems, particularly if the output is being fed into edge detection algorithms for computer vision applications.

Averaging: Averaging sets each pixel to the average value of itself and its nearby neighbors. Averaging tends to blur an image, because pixel intensity values which are significantly higher or lower than the surrounding neighbourhood would smear across the area. Like the Gaussian filter, averaging is an effective noise suppression technique against Gaussian noise as the deviations are normally distributed, and have intensities relatively close to the original value.

Conservative smoothing: It is explicitly designed to remove isolated pixels of exceptionally low or high pixel intensity (*e.g.* salt and pepper noise) and is, therefore, less effective at removing *additive noise* (*e.g.* Gaussian noise) from an image. Conservative smoothing operates on the assumption that noise has a high spatial frequency and can be attenuated by a local operation which makes each pixel's intensity roughly consistent with those of its nearest neighbors [5].

A huge amount of wavelet based methods [6] are available to achieve a good noise reduction (for the additive noise type), while preserving the significant image details. Due to the linearity of the wavelet transform, additive noise in the image domain remains additive in the transform domain, as well. The wavelet denoising procedure usually consists of shrinking the wavelet coefficients, that is, the coefficients that contain primarily noise should be reduced to negligible values, while the ones containing a significant noise-free component should be reduced less. A common shrinkage approach is the application of simple thresholding nonlinearities to the empirical wavelet coefficients [7], [8]. If the coefficient's magnitude is below the threshold, it is reduced to zero; otherwise, it is kept or modified. Shrinkage estimators can also result from a Bayesian approach, in which a prior distribution of the noise-free data (e.g., Laplacian [9], generalized Gaussian [10], [11], [12], [13], [14]) is integrated in the denoising scheme. Recently, non-Gaussian bivariate distributions capturing the inter-scale dependency were proposed [15] and corresponding nonlinear shrinking functions were derived from these distributions using Bayesian estimation theory [16] [17] [18].

Several fuzzy filters for noise reduction have already been developed, e.g., the iterative fuzzy control based filters from [19], the GOA filter [20], [21], and so on. Most of these state-of-the-art methods are mainly developed for the reduction of fat-tailed noise like impulse noise. Nevertheless, most of the current fuzzy techniques do not produce convincing results for additive noise [22], [23]. Another shortcoming of the current methods is that most of these filters are especially developed for grayscale images. It is, of course, possible to extend these filters to color images by applying them on each color component separately, independent of the other components. However, this introduces many artifacts, especially on edge or texture elements. Some of the existing fuzzy based filters used for color image noise reduction are Fuzzy Median Filter, Gaussian fuzzy filter with median center and moving average center, symmetrical and asymmetrical triangle fuzzy filter with moving average center, iterative fuzzy control based filter (IFCF), fuzzy random impulse noise reduction method and fuzzy bilateral filtering. A new fuzzy method proposed by Stefan Schulte et.al, is a simple fuzzy technique [24] for filtering color images corrupted with narrow-tailed and medium narrow-tailed noise (e.g., Gaussian noise) without introducing the above mentioned artifacts. Their proposed fuzzy method outperforms the conventional filter as well as other fuzzy noise filters. In this paper, we are presenting a modified version of the fuzzy approach proposed by Stefan Schulte, et.al, [24], which uses a Gaussian combination membership function to yield a better result, compared to the conventional filters as well as the recently developed advanced fuzzy filters.

III. PROPOSED FUZZY APPROACH

A digital color image C can be represented in different color space such as RGB, HSV, L*a*b etc. In the proposed method, RGB space is used as the basic color space. Different proportions of red, green and blue light gives a wide range of colors. Colors in RGB space are represented by a 3-D vector with first element being red, the second being green and third being blue, respectively. These three primary color components are quantized in the range 0 to 2^m-1, where m=8. A color image C can be represented by a 2-D array of vectors where (i, j) defines a position in C called pixel and $C_{i,j,1}$, $C_{i,j,2}$, and $C_{i,j,3}$, denotes the red, green and blue components, respectively.

A. Fuzzy Sub-Filter I

The general idea in this method is to take into account the fine details of the image such as edges and color component distances, which will be preserved by the filter. The goal of the first filter is to distinguish between local variations due to image structures such as edges. The goal is accomplished by using Euclidean distances between color components instead of differences between the components as done in most of the existing filters. The proposed method uses 2-D distances instead of 3-D distances (distance between three color components red, green and blue), that is, the distance between red-green (RG) and red-blue (RB) of the neighbourhood centered at (i, j) is used to filter the red component [9]. Similarly, the distance between RG and green-blue (GB) is used to filter the green component and the distance between RB and GB is used to filter the blue component, respectively. The method uses three fuzzy rules to calculate weights for the Takagi-Sugeno fuzzy model [11].

The current image pixel at position (i, j) is processed using a window size of (2K+1)×(2K+1) to obtain the modified color components. To each of the pixels in the window certain weights are then assigned namely $W_{k,l}$, where k, l Є {-1, 0, 1}. $W_{i+k,j+l,1}$, $W_{i+k,j+l,2}$, and $W_{i+k,j+l,3}$ denotes the weights for the red, green and blue component at position (i + k, j + l), respectively. These weights are assigned according to the following three fuzzy rules. Let DIST(a, b) represents the distance between the parameters a and b, and NEIGH(y) represents the neighbourhood of the parameter y. In this case, y represents a pixel with the neighbourhood given by a 3×3 window. The three fuzzy rules can be represented as follows:

1) IF DIST(RG, NEIGH(RG)) is *SMALL* AND DIST(RB, NEIGH(RB)) is *SMALL* THEN the weight $W_{k,l,1}$ is *LARGE*.
2) IF DIST(RG, NEIGH(RG)) is *SMALL* AND DIST(GB, NEIGH(GB)) is *SMALL* THEN the weight $W_{k,l,2}$ is *LARGE*.
3) IF DIST(RB, NEIGH(RB)) is *SMALL* AND DIST(GB, NEIGH(GB)) is *SMALL* THEN the weight $W_{k,l,3}$ is *LARGE*.

In the fuzzy rules DIST represents the Euclidean distance.

$\mathrm{DIST(RG, NEIGH(RG))}=[(C_{i+k,j+l,1} - C_{i,j,1})^2 + (C_{i+k,j+l,2} - C_{i,j,2})^2]^{1/2}$

$\mathrm{DIST(RB, NEIGH(RB))}=[(C_{i+k,j+l,1} - C_{i,j,1})^2 + (C_{i+k,j+l,3} - C_{i,j,3})^2]^{1/2}$

$\mathrm{DIST(GB, NEIGH(GB))}=[(C_{i+k,j+l,2} - C_{i,j,2})^2 + (C_{i+k,j+l,3} - C_{i,j,3})^2]^{1/2}$

Fuzzy sets are commonly represented by membership functions from which the corresponding membership degrees are derived. Membership degrees between zero and one indicate the uncertainty that whether the distance is small or not. In the proposed approach, the membership function SMALL has been modified which incorporates a two-sided composite of two different Gaussian curves. The Gaussian function depends on two parameters σ and c as given by

$$f(x;\sigma,c) = e^{\frac{-(x-c)^2}{2\sigma^2}}$$

The membership function *gauss2mf* (supported by MATLAB) is a combination of two of these two parameters. The first function, specified by σ1 and c1, determines the shape of the leftmost curve. The second function specified by σ2 and c2 determines the shape of the right-most curve. Whenever c1 < c2, the gauss2mf function reaches a maximum value of 1. Otherwise, the maximum value is less than one. The membership function SMALL is defined as

$$\mu_{SMALL}(x) = \text{gauss2mf}(x, [\sigma_x, C_x, \sigma_x, 0])$$

where σ_x is the standard deviation of the distance measure and C_x is the mean of the distance measure, respectively.

In the above fuzzy rules, the intersection of two fuzzy sets is involved. The intersection of two fuzzy sets A and B is generally specified by a binary mapping T, which aggregates two membership functions as follows:
$\mu_{A\cap B}(y) = T(\mu_A(y), \mu_B(y))$, where μ_A and μ_B are the membership functions for the two fuzzy sets A and B, respectively. The fuzzy intersection operator, known as triangular norms (T-norms), used in this paper is the algebraic product T-norms. For example, the antecedent of Fuzzy rule 1 is:

$$\mu_{SMALL}(\mathrm{DIST(RG, NEIGH(RG))}) \,.\, \mu_{SMALL}(\mathrm{DIST(RB, NEIGH(RB))})$$

The above obtained value, called the activation degree of the fuzzy rule 1, is used to obtain the corresponding weight. So the weights $W_{i+k,j+l,1}$, $W_{i+k,j+l,2}$, and $W_{i+k,j+l,3}$ are calculated as follows:

$$W_{i+k,j+l,1} = \mu_{SMALL}(\mathrm{DIST(RG, NEIGH(RG))}) \,.\, \mu_{SMALL}(\mathrm{DIST(RB, NEIGH(RB))})$$

$$W_{i+k,j+l,2} = \mu_{SMALL}(\mathrm{DIST(RG, NEIGH(RG))}) \,.\, \mu_{SMALL}(\mathrm{DIST(GB, NEIGH(GB))})$$

$$W_{i+k,j+l,3} = \mu_{SMALL}(\mathrm{DIST(RB, NEIGH(RB))}) \,.\, \mu_{SMALL}(\mathrm{DIST(GB, NEIGH(GB))})$$

The output of the Fuzzy Sub-filter I, denoted as FS1, is then given by:

$$FS1_{i,j,1} = \frac{\sum_{k=-K}^{+K}\sum_{l=-K}^{+K} W_{i+k,j+l,1} . C_{i+k,j+l,1}}{\sum_{k=-K}^{+K}\sum_{l=-K}^{+K} W_{i+k,j+l,1}}$$

$$FS1_{i,j,2} = \frac{\sum_{k=-K}^{+K}\sum_{l=-K}^{+K} W_{i+k,j+l,2} . C_{i+k,j+l,2}}{\sum_{k=-K}^{+K}\sum_{l=-K}^{+K} W_{i+k,j+l,2}}$$

$$FS1_{i,j,3} = \frac{\sum_{k=-K}^{+K}\sum_{l=-K}^{+K} W_{i+k,j+l,3} . C_{i+k,j+l,3}}{\sum_{k=-K}^{+K}\sum_{l=-K}^{+K} W_{i+k,j+l,3}}$$

where $FS1_{i,j,1}$, $FS1_{i,j,2}$ and $FS1_{i,j,3}$ denotes the red, green and blue components of the FS1 output image respectively.

B. Fuzzy Sub-Filter II

The second sub-filter is used as a complementary filter to the first one. The goal of this sub-filter is to improve the first method by reducing the noise in the color components differences without destroying the fine details of the image. In this step, the local differences in the red, green and blue environment are calculated separately. These differences are then combined to calculate the local estimation of the central pixel. In this step also, a window of size (2L+1)×(2L+1) is used centered at (i, j) to filter the current image pixel at that position. The local differences for each element of the window for the three color components are calculated as follows:

$DR_{k,l} = FS1_{i+k,j+l,1} - FS1_{i,j,1}$, $DG_{k,l} = FS1_{i+k,j+l,2} - FS1_{i,j,2}$

$DB_{k,l} = FS1_{i+k,j+l,3} - FS1_{i,j,3}$, where k, l Є {-1, 0, +1}.

The correction term **ε** is calculated as follows:

$$\varepsilon_{k,l} = (1/3).(DR_{k,l} + DG_{k,l} + DB_{k,l}) \quad \text{for } k, l \in \{-L, \ldots, 0, \ldots, +L\}.$$

The output of the Fuzzy sub-filter 2, is then given by

$$FS2_{i,j,1} = \frac{\sum_{k=-L}^{+L}\sum_{l=-L}^{+L}(FS1_{i+k,j+1,1} - \varepsilon_{k,l})}{(2L+1)^2}$$

$$FS2_{i,j,2} = \frac{\sum_{k=-L}^{+L}\sum_{l=-L}^{+L}(FS1_{i+k,j+1,2} - \varepsilon_{k,l})}{(2L+1)^2}$$

$$FS2_{i,j,3} = \frac{\sum_{k=-L}^{+L}\sum_{l=-L}^{+L}(FS1_{i+k,j+1,3} - \varepsilon_{k,l})}{(2L+1)^2}$$

where $FS2_{i,j,1}$, $FS2_{i,j,2}$ and $FS2_{i,j,3}$ denotes the red, green and blue components of the output image respectively.

IV. Results and Discussion

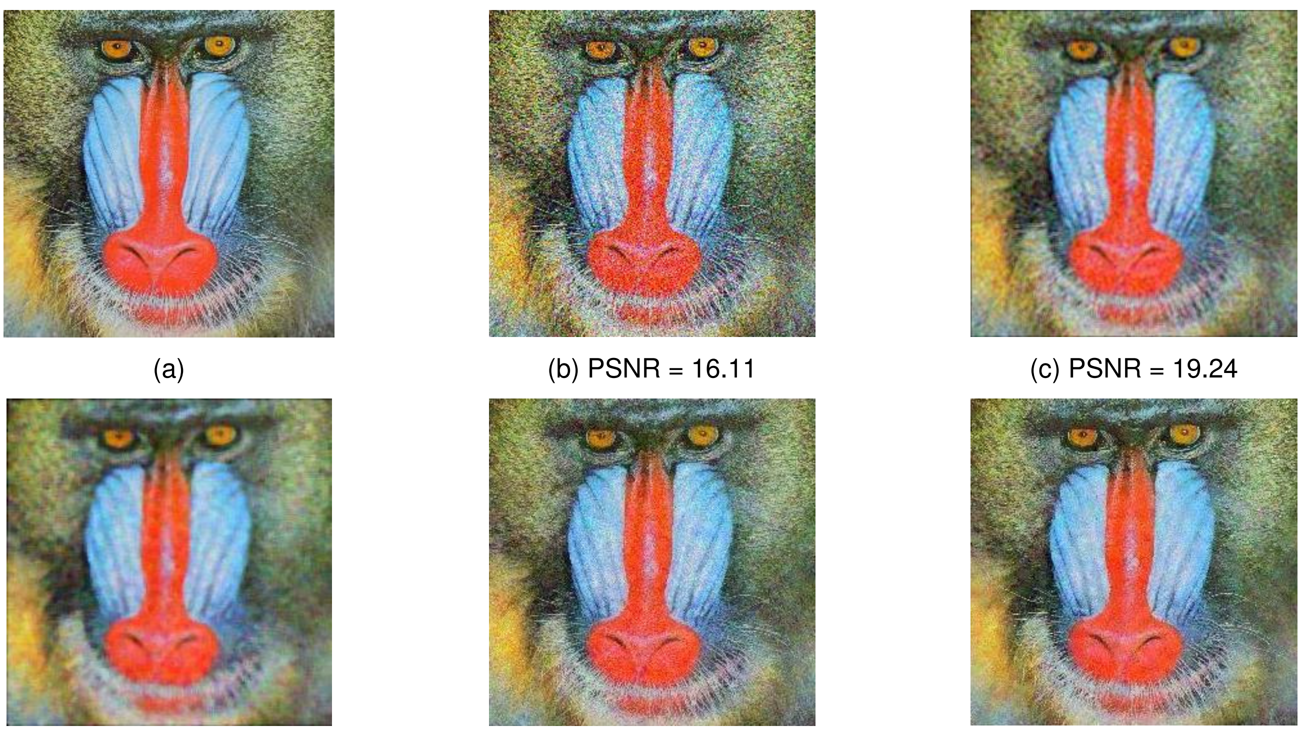
The performance of the discussed filter has been evaluated and compared with conventional filters dealing with additive noise, using MATLAB tool. As a measure of objective similarity between a filtered image and the original one, we use the peak signal-to-noise ratio (PSNR) in decibels (dB).

$$\text{PSNR}(img, org) = 10 \log_{10}(S^2/\text{MSE}(img, org))$$

This similarity measure is based on another measure, namely the mean-square error (MSE).

$$\text{MSE}(img, org) = \frac{\sum_{c=1}^{3}\sum_{i=1}^{N}\sum_{j=1}^{M}[org(i,j,c) - img(i,j,c)]^2}{3.N.M}$$

where *org* is the original color image, *img* is the filtered color image of size *N.M*, and *S* is the maximum possible intensity value (with *m*-bit integer values, *S* will be 2^m-1). The standard color images used in this paper are Baboon, Lena and House images of size 256×256. The original image, the noisy image (original image corrupted with Gaussian noise with a selected σ value) and restored images using mean filter, median filter, fuzzy method of [24] and the modified fuzzy method of the above mentioned standard color images along with their corresponding PSNR values are shown in figures 1, 2 and 3. From experimental results, it has been found that our proposed method receives the best numerical and visual performance for low levels and higher levels of additive noise, by appropriately selecting window size for the two fuzzy sub-filters. Numerical results that illustrate the denoising capability of the proposed method, modified method and conventional methods are pictured in Table I. Table I shows the PSNRs for the colored House image that were corrupted with Gaussian noise for σ = 5, 10, 20, 30 and 40. The window size for different filters is appropriately chosen to give better PSNR value. The PSNR value of the noisy image and the best performing filter were shown bold.



(a) (b) PSNR = 16.11 (c) PSNR = 19.24

(d) PSNR = 18.90 (e) PSNR =21.61 (f) PSNR = 21.83

Fig. 1. (a) Original Baboon image (256×256) (b) Noisy image (Gaussian noise, σ = 40) (c) After applying Mean filter (3×3 window) (d) After applying Median filter (5×5 window) (e) After applying Fuzzy filter of [24] with K=3 (7×7 window) and L=2 (5×5 window) (f) After applying Proposed Fuzzy filter with K=3 (7×7 window) and L=2 (5×5 window)

Fig. 2. (a) Original Lena image (256×256) (b) Noisy image (Gaussian noise, σ = 30) (c) After applying Mean filter (3×3 window) (d) After applying Median filter (5×5 window) (e) After applying Fuzzy filter of [24] with K=3 (7×7 window) and L=2 (5×5 window) (f) After applying Proposed Fuzzy filter with K=3 (7×7 window) and L=2 (5×5 window)

Fig. 3. (a) Original House image (256×256) (b) Noisy image (Gaussian noise, σ = 20) (c) After applying Mean filter (3×3 window) (d) After applying Median filter (3×3 window) (e) After applying Fuzzy filter of [24] with K=3 (7×7 window) and L=2 (5×5 window) (f) After applying Proposed Fuzzy filter with K=3 (7×7 window) and L=2 (5×5 window)

TABLE I

COMPARATIVE RESULTS IN PSNR OF DIFFERENT FILTERING METHODS FOR VARIOUS DISTORTIONS OF GAUSSIAN NOISE FOR THE (256×256) COLORED HOUSE IMAGE

	PSNR (dB)				
	σ = 5	σ = 10	σ = 20	σ = 30	σ = 40
Noisy	**34.13**	**28.12**	**22.10**	**18.57**	**16.08**
Mean	28.05	27.70	26.57	25.13	23.72
Median	32.31	30.81	27.66	25.02	22.95
Proposed Fuzzy Method	34.12	31.79	28.38	25.85	23.76
Modified Fuzzy Method	**34.22**	**32.77**	**29.33**	**26.51**	**24.18**

The main advantages of this new and simple filter are the denoising capability and the reconstruction capability of the destroyed color component differences. A numerical measure, such as the PSNR, and visual observation have shown convincing results. Future research will be focused on the construction of fuzzy filtering methods for color images to suppress multiplicative noise such as speckle noise.

V. Conclusion

A fuzzy filter for restoring color images corrupted with additive noise is proposed in this paper. The proposed filter is efficient and produces better restoration of the color images compared to other filters. Numerical measures such as PSNR and visual observation have shown convincing results. Also the proposed method outperforms most of the conventional sharpening filters and other fuzzy filters. Further work can be focused on the construction of other fuzzy filtering methods for color images to suppress multiplicative noise such as speckle noise.

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