

Evaluating Strategy with Grey Theory Utilized in Wireless Sensor Networks

Lun Zhang, *Member ,IAENG* , Yan Lu, Lan Chen, Decun Dong

Abstract—This paper designs evaluation method based on the grey theory which aims at evaluate the usage of Wireless Sensor Networks in rail traffic. With the features of the facts that: The requested data sizes for assessment via grey theory method are not big, distribution rules of the data samples are not obliged ,and the evaluation workload is small , we use grey theory to verify each index mark in current evaluation system. Hereafter, the detailed assessment example is taken out to prove that the index mark getting from this method has practical significance and performs efficiently.

Index Terms— Evaluating Strategy, Grey Theory, Wireless Sensor Networks.

I. INTRODUCTION

The application of Wireless Sensor Networks (WSN) system includes all kinds of factors. It is the content of factor analysis that which of factors is leading or subordinate; which of them influence big or small; which of them need develop or suppress; which of them is latent or obvious and so on. The basic method of factor analysis used in the past is statistical method such as regression analysis. Regression analysis[1][2] is a universal method, but it is very difficult to deal with the system which is multi-elemental and nonlinear.

Generally speaking, the disadvantages of regression analysis are as follows:

- 1) Regression analysis requires numerous data. It is difficult to find rules of statistics when size of data is small.
- 2) Regression analysis requires that distribution is linear, exponential or logarithmic. Generally speaking, required distribution should be typical and not rambling.
- 3) The workload of calculation is big. The workload of linear regression calculation with one or two factor is not big. But if the number of factors is beyond 2, the workload cannot be finished by manual computation.
- 4) Some abnormal situation maybe appears. Because calculation of Regression analysis mainly is power operation and arithmetic, the results of

This work was supported by the National Science Foundation of China. (Project No.: 50408034)

Lun Zhang is with School of Traffic Engineering, Tongji University, No.4800,Cao An Road, 201804, Shanghai, China (phone: 86-21-69589881; fax: 86-21-65980212; e-mail: Lun_zhang@ mail.tongji.edu.cn).

Yan Lu is with School of Traffic Engineering, Tongji University, No.4800, Cao An Road,201804,Shanghai,China(e-mail: xvela630@hotmail.com).

Lan Chen is with School of Traffic Engineering, Tongji University, No.4800,Cao An Road,201804,Shanghai, China (e-mail: claxning@126.com).

Decun Dong is with School of Traffic Engineering, Tongji University, No.4800,Cao An Road,201804,Shanghai, China (e-mail: ddc58@sohu.com).

Calculation is easy to appear polarity mistake for calculation errors and positive correlation is made to be negative correlation. In the end, the correct phenomenon is distorted and reversed.

In view of former disadvantages and drawback, grey system theory uses the method of grey relation analysis to do system analysis. Grey system theory is put forward by Professor Julong Deng of Central China University of Science and Technology in the late 1970s, early 1980s[3][4]. It is applied in many fields such as society, economics, technology, agriculture, ecology, biology, etc. The major content of grey theory have grey philosophy, grey generation, grey analysis, grey modeling, grey forecasting, grey evaluation, grey decision and grey control. This paper mainly use grey evaluation theory in grey theory to do relation analysis on each index in rail transit WSN. Then do general evaluation of rail transit WSN with the feature of grey theory that small required data size, easy calculation and high precision.

II. THE ASSESSMENT SYSTEM OF WSN

Function evaluation is the most fundamental part in the system evaluation content. The function of the system decides the running state and Quality of Service of the system, and whether meets and finishes the needs of transmission. We design the basic evaluation indexes are energy validity, lifecycle, time delay, apperceive precision, expandability, the power of fault-tolerant, etc with the basic features of WSN.

The adaptability has two types: the one is system adaptability and the other is environment adaptability.

Theory should not be divorced from reality. There are many problems of theory and engineering that need be resolved such as routing, agreement, energy consumption, etc. The practicability evaluation indexes raised by this paper mainly include demand for services, economy, operability, reliability, efficiency, etc.

Qos(Quality of Service) is "the agreement on information transportation and sharing between network and user, as well as between users communicating on the network--to meet user's requirements, sensor network must be able to provide adequate resources to users and work with acceptable performance indexes for users.

The advanced nature is a category of value evaluation. The advanced technology evaluation means social value, economic value and the value of technology of WSN application. This is a general evaluation of new technology and its using in new field.

Based on the above, the contents of the WSN function evaluation are expressed in Table I.

TABLE I

THE WSN EVALUATION ARCHITECTURE		
Evaluation content	Index	Remarks
Function evaluation	Energy Consumption evaluation	Energy validity
		Lifecycle
	Transmission functional evaluation	Time delay
		Apperceive precision
		expandability
		the power of fault-tolerant
Adaptability evaluation	Functional achievement evaluation	
	Systematic adaptability	
	environment adaptability	
Practicability evaluation	demand for services	
	economy	
	operability	
	reliability	
	efficiency	
QOS evaluation	Information real time nature	
	Transmission reliability	
	Transmission data size and efficiency	
	Multidimensional external environmental factors	
	Other	
Advanced technology evaluation	Society value	
	Economic value	
	Technology value	

III. GREY THEORY CONCEPTUAL FRAME AND ASSESSING METHOD

A. Basic Concept

"Grey" is the concept being situated between "white" and "black".

White system is the system which the information is assured and the data is whole.

Black system is the system which the information is not assured and the data is little.

Grey system is the system which some information is assured and some is not; some data is whole and some is not; some information is know and some is not. Since the establishment of grey theory, it has used well in social facets. This paper tries to apply the grey statistical evaluation method to analysis the application of WSN rail transit evaluation. Then it can increase handling efficiency of expert evaluation data.

The Grey evaluation is that for project, object, plan, etc, conduct evaluation by qualitative grey cluster based on different condition, demand and interest to get sequence and situation of project, object, plan, etc.

Grey evaluation has four types:

- Grey cluster evaluation
- Grey statistics evaluation
- Grey situation evaluation
- Grey connection pattern evaluation

Because the expert evaluation data can be simplified, this paper sets grey statistics evaluation as the evaluation method

Grey statistics evaluation: if $d_{1j}, d_{2j}, \dots, d_{wj}$ is the sample column of the decision units $i=1, 2, \dots, w$ based on item j , using exchange can make j sample column change to the evaluation value of grey cluster $k=1, 2, \dots, k=n$ based on item j , which is called the grey statistics evaluation of grey cluster that item j belongs to.

B. Grey Statistics Formula

➤ Definition1: set d_{ij} as the samples of unit i to item j and set d as sample matrix

$$d = \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1m} \\ d_{21} & d_{22} & \cdots & d_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ d_{w1} & d_{w2} & \cdots & d_{wm} \end{bmatrix} \quad (1)$$

Then:

1) Call $(d_{1j}, d_{2j}, \dots, d_{wj})$, $j=1, 2, \dots, m$ the sample columns of item j .

2) Set f_k as whitenization function of grey cluster k .

Then call $\sum_{i=1}^w f_k(d_{ij})$ the unit global summation of k 's grey cluster whitenization value based on j .

3) Call $\sum_{k=1}^n \sum_{i=1}^w f_k(d_{ij})$ the unit global summation of j .

➤ Definition2: set $f_k(d_{ij})$ as k 's grey cluster whitenization

value based on d_{ij} and set σ_{jk} as real number from 0 to 1. Set M_s as exchange. If

$$M_s : \{f_k(d_{ij})\} \rightarrow \sigma_{jk},$$

$$M_s(\{f_k(d_{ij})\}) \rightarrow \sigma_{jk},$$

$$\sigma_{jk} = \frac{\sum_{i=1}^w f_k(d_{ij})}{\sum_{k=1}^n \sum_{i=1}^w f_k(d_{ij})},$$

$$k \in K = \{1, 2, \dots, n\},$$

Then:

1) σ_{jk} is the grey evaluation value of grey cluster k based on item j .

2) M_s is the grey statistics (exchange).

3) Call sequence σ_j grey cluster 1, grey cluster to grey cluster n

$$\sigma_j = (\sigma_{j1}, \sigma_{j2}, \dots, \sigma_{jn})$$

4) If

$$\sigma_{jk^*} = \max\{\sigma_{j1}, \sigma_{j2}, \dots, \sigma_{jn}\}$$

item j belongs to grey cluster k^* and record as $j \in k^*$.

➤ Definition3: set d_{ij} as sample and as f_k whitenization function of grey cluster k . Then

1) While $0 \leq f_k(d_{ij}) \leq 1$; $f_k(d_{ij})$ is k 's whitenized value based on d_{ij} .

2) $\sum f_k(d_{ij})$ is integrative whitenization.

➤ Theorem1: set K as grey cluster set, $K = \{1, 2, \dots, n\}$ and f_k as whitenization function of grey cluster k , $k \in K$. Then

1) Upper whitenization function is f_1 and

$$f_1 \Rightarrow f_1(c_1, \infty)$$

2) Middle whitenization function is f_k and

$$f_k \Rightarrow f_k(-, c_k, +)$$

3) Lower whitenization function is f_n and

$$f_n \Rightarrow f_n(0, c_n)$$

➤ Theorem2: set K as grey cluster set, $K = \{1, 2, \dots, n\}$ and f_k as whitenization function of grey cluster k , $k \in K$. And moreover f_1 is start whitenization function; f_n is end whitenization function; $f_i, i \neq 1, i \neq n$ is middle whitenization function. Then whitenization function is as follows:

1) Start whitenization function f_1

$$f_1 = f_1(c_1, \infty) \Rightarrow f_1(d_{ij}) = \begin{cases} \frac{1}{c_1} d_{ij}, d_{ij} \in [0, c_1] \\ 1, d_{ij} \in [c_1, \infty] \end{cases} \quad (2)$$

2) Middle whitenized function $f_i (i \in K, i \neq 1, i \neq n)$

$$f_i = f_i(-, c_i, +) \Rightarrow f_i(d_{ij}) = \begin{cases} 1, d_{ij} \in [0, c_n] \\ -\frac{1}{c_n} d_{ij} + 2, d_{ij} \in [c_n, c_{2n}] \end{cases} \quad (3)$$

3) End whitenization function f_n

$$f_n = f_n(0, c_n) \Rightarrow f_n(d_{ij}) = \begin{cases} 1, d_{ij} \in [0, c_n] \\ -\frac{1}{c_n} d_{ij} + 2, d_{ij} \in [c_n, c_{2n}] \end{cases} \quad (4)$$

➤ Theorem2: set σ_{jk} as grey cluster K 's evaluation value based on item j . Then:

1) σ_{jk} 's form dimension is i, j, k ;

2) The form set of σ_{jk} 's form dimension is $\{I, J, K\}$,

$I = \{1, 2, \dots, w\}$, $J = \{1, 2, \dots, m\}$, $K = \{1, 2, \dots, n\}$;

3) The form set of σ_{jk} 's denominator

$$\sum_{k=1}^n \sum_{i=1}^w f_k(d_{ij})$$

is $\{I, J, K\}$ and its manifest dimension subset is J , its disappearing subset are I and K and I and k are disappearing dimension;

4) The form set of σ_{jk} 's numerator $\sum_{i=1}^w f_k(d_{ij})$

is $\{I, J, K\}$ and its manifest dimension subset is J , its disappearing subset are I and K and I and k are disappearing dimension;

5) The manifest dimension of σ_{jk} are j and k .

Detailed algorithm flow is as Fig.1.

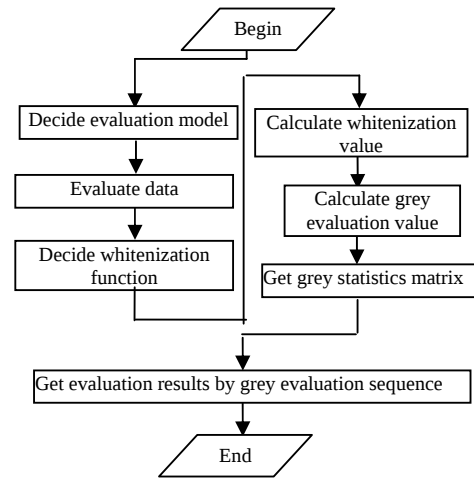


Fig.1 The algorithm flow of grey theory

IV. APPLICATION EXAMPLE

Order that evaluation index j represents j kinds of indexes. There are 5 kinds of evaluation sub-indexes and an overall evaluation index at the present time. Namely $j \in J = \{1, 2, \dots, 6\}$, $m = 6$. Order that decision unit i represents i experts. Namely $i \in I = \{1, 2, \dots, 20\}$, $w = 20$. Order that grey cluster K represents one sort of evaluation results.

$K = \{1, 2, 3, 4, 5\} = \{\text{"type 1"}, \text{"type 2"}, \text{"type 3"}, \text{"type 4"}, \text{"type 5"}\}$

Set d_{ij} as evaluation results of j indexes by i^{th} experts and d as matrix of d_{ij} . The sample matrix is as follows.

$$d = \begin{matrix} & \begin{matrix} \text{index1} & \text{index2} & \dots & \text{index6} \end{matrix} \\ \begin{matrix} \text{expert1} \\ \text{expert2} \\ \dots \\ \text{expert20} \end{matrix} & \begin{bmatrix} d_{11} & d_{12} & \dots & d_{1m} \\ d_{21} & d_{22} & \dots & d_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ d_{w1} & d_{w2} & \dots & d_{wm} \end{bmatrix} \end{matrix}$$

Each evaluation data is expressed in Table II. Each expert makes centesimal grade score of each index and each score is the sum of each index's score.

TABLE II
THE VALUE OF MATRIX D

d =	Om	Tf	Ad	Pr	Qos	At
S1	53	64	74	20	44	63
S2	50.4	58	54	20	55	65
S3	52.4	71	61	38	47	45
S4	46.8	60	53	13	44	64
S5	49	68	63	16	59	39
S6	53.2	81	63	19	61	42
S7	57.4	72	69	35	54	57
S8	53.2	74	66	26	61	39
S9	50.4	80	54	13	60	45
S10	50.4	81	53	37	36	45
S11	43.2	70	56	13	39	38
S12	56.6	79	77	23	44	60
S13	54.2	72	75	16	54	54
S14	51.8	57	63	36	41	62
S15	54.8	74	71	14	55	60
S16	46.8	52	73	21	45	43
S17	46	64	70	12	43	41
S18	41	60	57	11	32	45
S19	46	57	50	13	53	57
S20	50	62	63	34	42	49

Notes: **Om**--Overall merit **Tf**--Transmission function **Ad**--adaptability **Pr**--practicability **Qos**--Quality of service **At**--Advanced technology
We use the follow XY scatter gram to express the evaluation data by experts directly.

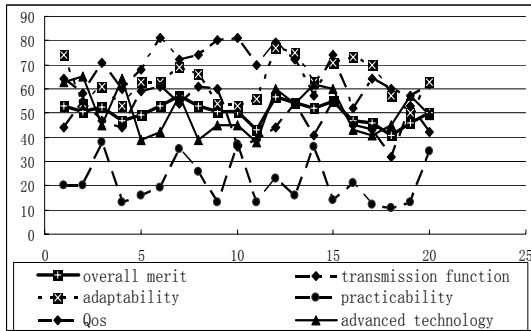


Fig.2 The XY scatter gram

A. Whitenized Function

1) The first grey cluster f_1

$c_1 = 80$ (when score is higher than 80, the grade is 1)

$$f_1 \Rightarrow f_1(c_1, \infty) = f_1(80, \infty)$$

2) The second grey cluster f_2

$c_2 = 70$ (when score is about 70, the grade is 2)

$$f_2 \Rightarrow f_2(-, c_2, +) = f_2(-, 70, +)$$

3) The third grey cluster f_3

$c_3 = 50$ (when score is about 50, the grade is 3)

$$f_3 \Rightarrow f_3(-, c_3, +) = f_3(-, 50, +)$$

4) The forth grey cluster f_4

$c_4 = 30$ (when score is about 30, the grade is 4)

$$f_4 \Rightarrow f_4(-, c_4, +) = f_4(-, 30, +)$$

5) The fifth grey cluster f_5

$c_5 = 20$ (when score is lower than 20, the grade is 5)

$$f_5 \Rightarrow f_5(0, c_5) = f_5(0, 20)$$

B. K kinds of whitenization value calculation

$$\sum_{i=1}^w f_k(d_{ij}) = \sum_{i=1}^{20} f_k(d_{ij}) \quad (5)$$

1) $j=1$

As $k=1$,

$$\begin{aligned} \sum_{i=1}^{20} f_k(d_{ij}) &= \sum_{i=1}^{20} f_1(d_{i1}) = f_1(53) + f_1(50.4) + \dots + f_1(50) \\ &= \frac{53}{80} + \frac{50.4}{80} + \frac{52.4}{80} + \dots + \frac{50}{80} = 12.5825 \end{aligned}$$

As $k=2$,

$$\begin{aligned} \sum_{i=1}^{20} f_k(d_{ij}) &= \sum_{i=1}^{20} f_2(d_{i1}) = f_2(53) + f_2(50.4) + \dots + f_2(50) \\ &= \frac{53}{70} + \frac{50.4}{70} + \frac{52.4}{70} + \dots + \frac{50}{70} = 14.38 \end{aligned}$$

We can get data in Table.3 by former formula when $j=1, 2, 3, 4, 5, 6$ and $k=1, 2, 3, 4, 5$.

C. 4.3 Calculation of General Whitenized Value

$$\sum_{k=1}^5 \sum_{i=1}^{20} f_k(d_{ij}) \quad (6)$$

1) $j=1$

This item is the general whitenized value of overall merit.

$$\begin{aligned} \sum_{k=1}^5 \sum_{i=1}^{20} f_k(d_{ij}) &= \sum_{k=1}^5 \sum_{i=1}^{20} f_k(d_{i1}) \\ &= \sum_{i=1}^{20} f_1(d_{i1}) + \sum_{i=1}^{20} f_2(d_{i1}) + \dots + \sum_{i=1}^{20} f_5(d_{i1}) \\ &= 12.5825 + 14.38 + 18.62 + 6.447 + 0 = 52.0292 \end{aligned}$$

2) $j=2$

$$\begin{aligned} \sum_{k=1}^5 \sum_{i=1}^{20} f_k(d_{ij}) &= \sum_{k=1}^5 \sum_{i=1}^{20} f_k(d_{i2}) \\ &= \sum_{i=1}^{20} f_1(d_{i2}) + \sum_{i=1}^{20} f_2(d_{i2}) + \dots + \sum_{i=1}^{20} f_5(d_{i2}) \\ &= 16.925 + 17.8286 + 0.5333 + 0 = 48.1669 \end{aligned}$$

We can get the second line data in Table III by using the same method to calculate other evaluation item.

TABLE III

THE VALUE OF σ_{jk}

		$\sum_{i=1}^{20} f_k(d_{ij})$	$\sum_{k=1}^5 \sum_{i=1}^{20} f_k(d_{ij})$	σ_{jk}
$j=1$	$k=1$	12.5825	52.0292	0.2418
	$k=2$	14.38		0.2764
	$k=3$	18.62		0.3579
	$k=4$	6.447		0.1239
	$k=5$	0		0
$j=2$	$k=1$	16.925	48.1669	0.3514
	$k=2$	17.8286		0.3701
	$k=3$	12.88		0.2674
	$k=4$	0.5333		0.0111
	$k=5$	0		0
$j=3$	$k=1$	15.8125	49.4458	0.3198
	$k=2$	17.5		0.3539
	$k=3$	14.7		0.2973
	$k=4$	1.4333		0.029
	$k=5$	0		0
$j=4$	$k=1$	5.375	47.951	0.1121
	$k=2$	6.1429		0.1281
	$k=3$	8.6		0.1793
	$k=4$	12.333		0.2572
	$k=5$	15.5		0.3232
$j=5$	$k=1$	12.1125	51.272	0.2362
	$k=2$	13.8429		0.27
	$k=3$	16.9		0.3296
	$k=4$	7.7667		0.1515
	$k=5$	0.65		0.0127
$j=6$	$k=1$	12.6625	50.6139	0.2502
	$k=2$	14.4714		0.2859
	$k=3$	16.58		0.3276
	$k=4$	6.7		0.1324
	$k=5$	0.2		0.004

D. Grey Evaluation Value

$$\sigma_{jk} = \sum_{i=1}^{20} f_k(d_{ij}) / \sum_{k=1}^5 \sum_{i=1}^{20} f_k(d_{ij}) \quad (7)$$

Based on the first line data and the second line data in Table.2 we can get the value of σ_{jk} as follows:

When $i=1$ and $k=1$,

$$\sigma_{11} = \sum_{i=1}^{20} f_1(d_{i1}) / \sum_{k=1}^5 \sum_{i=1}^{20} f_k(d_{i1}) = \frac{12.5825}{14.38} = 0.2418$$

We can get the third line data in Table.2 by using the same method.

E. Grey Statistics Matrix

Set the third line data in Table.2 as matrix which is expressed as follows:

$$\sigma = \begin{bmatrix} 0.2418 & 0.2764 & 0.3579 & 0.1239 & 0 \\ 0.3514 & 0.3701 & 0.2674 & 0.0111 & 0 \\ 0.3198 & 0.3539 & 0.2973 & 0.0290 & 0 \\ 0.1121 & 0.1281 & 0.1793 & 0.2572 & 0.3232 \\ 0.2362 & 0.2700 & 0.3296 & 0.1515 & 0.0127 \\ 0.2502 & 0.2859 & 0.3276 & 0.1324 & 0.004 \end{bmatrix}$$

F. Grey Evaluation Sequence & Grey Statistics Evaluation

When $j=1$, this data is based on overall merit.

$$\sigma_1 = (\sigma_{11}, \sigma_{12}, \sigma_{13}, \sigma_{14}, \sigma_{15})$$

$$= (0.2418, 0.2764, 0.3579, 0.1239, 0)$$

$$\sigma_{1k^*} = \max\{\sigma_{11}, \sigma_{12}, \sigma_{13}, \sigma_{14}, \sigma_{15}\} = 0.3579 = \sigma_{13}$$

The former calculation shows that $k^* = 3$. Namely the general evaluation of this item is "Grade 3" and the corresponding description is "Middle". We can get the final result of grey evaluation of each item by using the same method which is expressed in Table IV.

TABLE IV
THE FINAL RESULT OF GREY EVALUATION

j=	Item	General evaluation	corresponding description
1	Overall merit	Grade 3	Middle
2	Transmission function	Grade 4	Qualified
3	Adaptability	Grade 4	Qualified
4	Practicability	Grade 5	Unqualified
5	Quality of service	Grade 3	Middle
6	Advanced technology	Grade 3	Middle

The final evaluation result of this project is Grade 3—Middle. Using grey evaluation in rail transit WSN evaluation has the major advantages that evaluation method is simple, calculation is easy and result is precise.

V. CONCLUSION AND FOREGROUND

The advantages of using grey theory to evaluate rail transit are as follows:

1) The requested data sizes of grey theory are not big. Namely we can use less evaluation data by experts to get the correct evaluation of the characteristics of Wireless Sensor Networks.

2) The distribution of the data is not required. The detailed expert evaluation data can be rambling.

3) Evaluation workload is small. Compared to traditional regression analysis we find the algorithm of grey evaluation is simple and effective, also saving numerous time and manpower.

4) The final evaluation data is true and reliable. Because the data of expert evaluation is intuitive, the usage of grey theory in Wireless Sensor Networks for evaluation has widely potential application.

Using less data to get the final reliable evaluation result is the advantage and reason for grey theory. Its detailed expected applications are as follows:

1) The grey theory evaluation method can apply to verifying the score of each index in current evaluation system. It is a very important job to guarantee having practical meaning of each index score in system. It is helpful to a certain extent that grey theory evaluation can do comparative analysis by quantification of dynamic process. This can verify current score and strengthen persuasion.

2) Now most of evaluation systems make score on the final situation of project and the evaluation result is only a "mark". From this "mark" we cannot find the weakness of project and the crux of causing problem, also it cannot give the direction of the project to improve further guidance. But we can get the information as above mentioned provided by grey theory evaluation.

ACKNOWLEDGMENT

We would like to thank Professor Zhong-ren Peng from University of Florida of U.S., Professor Yulei Rao from Central Southern University of China, and Professor Xiao-guang Yang from Tongji University of China for their comments on this paper.

REFERENCES

- [1] Jeffrey Wooldridge, *Introductory Econometrics: A Modern Approach*, South-Western Educational Publishing, 2002
- [2] William H. Greene, *Econometric Analysis (4th Edition)*, Prentice Hall, 2007
- [3] Deng Julong. *The grey theory*. Press of Huazhong University of Science and Technology. 2003.9.1.
- [4] Deng Julong. *The grey theoretical basis method*. Press of Huazhong University of Science and Technology. 2005.
- [5] K.Sohrabi, J. Gao, V. Ailawadhi, and G.J. Pot-tie, Protocols for self-organization of a wireless sensor network, *IEEE Personal Communications Magazine*, Oct 2000, 7(5):16~27.
- [6] Liu Sifeng. *The grey theory and application* (Version 3). Science Press, 2004.11.
- [7] Lun Zhang, Jun Qu, Yan Lu, Urban Mass Transit based Wireless Sensor Networks Applying Model. *Urban Maas Transit*, Vol.19, No12, 200, pp, 28-32
- [8] Santi P and Blough D. An evaluation of connectivity in mobile wireless ad hoc networks. *Proc .IEEE DSN 2002*, Bethesda, MD, 2002, pp.89-98
- [9] Younis O, Fahmy S and Santi P. Robust communications for sensor networks in hostile environments. *Proe. IEEE International Workshop on Quality of Service (IWQoS)*, Montreal, 2004, pp. 10-19.