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Solving the Multiresponse Problem in Taguchi Method by Aggressive Formulation in DEA

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Abstract— **This research proposes an efficient approach for solving the multiresponse problem in Taguchi method by aggressive formulation in data envelopment analysis. Each experiment in Taguchi's orthogonal array is treated as a decision making unit (DMU) with multiresponses set as inputs and/or outputs. The efficiency of each DMU is then evaluated by aggressive formulation. Finally, the ordinal value of the DUM's efficiency is used to decide the optimal combination of factor levels for multiresponse problem. Two case studies, which were investigated in previous literature, are provided for illustration. The computational results show that the proposed approach provides the largest anticipated improvement. In conclusion, the proposed approach may provide a great assistant to practitioners for solving the multiresponse problem in the manufacturing applications on Taguchi method.**

Index Terms— **Aggressive formulation, DEA, Multiresponse problem, Taguchi method.**

I. Introduction

Parameter design is a method, popularized by Taguchi [1], for designing products and manufacturing processes that are robust to uncontrollable variations. Taguchi method adopts a fractional factorial experimental design, called an orthogonal array (OA), which reduces the number of experiments under permissive reliability. Typically, the quality response of a process or a product can be divided into three main types: the smaller-the-better (STB); the nominal-the-best (NTB); and the larger-the-better (LTB) type response. Taguchi method has been found only efficient for optimizing a single response problem [2-3].

In today's high competitive markets, most industries manufacture products with more than one quality response of main interest. Recently, optimizing multiresponse problem has received a considerable research attention. Therefore, several approaches [4-8] have been proposed to solve the multiresponse problem in Taguchi method. However, few approaches were reported efficient.

Data envelopment analysis (DEA) has been widely used for evaluating performance for a set of DMUs with multiple inputs and multiple outputs at organizational level, such as banks, hospitals, and universities [9]. DEA combines various inputs and various outputs for a DMU into one performance measure, called relative efficiency. Therefore, this research proposes an approach for solving the multiresponse problem in Taguchi method utilizing DEA techniques. DEA is introduced in the section II. The proposed approach is outlined in section III. Illustrations are provided in section IV. Finally, conclusions are summarized in section V.

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II. Data Envelopment Analysis

DEA is a fractional mathematical programming technique for evaluating the relative efficiency of homogeneous DMUs with multiple inputs and multiple outputs. The most popular DEA technique is the CCR model, developed by Charnes, Cooper, and Rhodes [10]. The CCR model measures the relative efficiency of each DMU once by comparing it to a group of the other DMUs that have the same set of inputs and outputs. Assuming there are n DMUs each with m inputs and s outputs to be evaluated. Let the DMU to be individually evaluated on any trial be designated as DMU$_o$. The relative efficiency, E_{oo}, of DMU$_o$ with inputs of x_{io} (i = 1, ... , m) and outputs of y_{ro} (r = 1, ... , s) is evaluated by solving

$$\begin{aligned} E_{oo} = \text{Max } \theta = (\sum_{r=1}^{s} u_r y_{ro}) / (\sum_{i=1}^{m} v_i x_{io}) & \\ \text{subject to } \quad (\sum_{r=1}^{s} u_r y_{rj}) / (\sum_{i=1}^{m} v_i x_{ij}) \le 1 & \qquad j = 1, \dots, n \\ u_1, u_2, \dots, u_s \ge 0 & \\ v_1, v_2, \dots, v_m \ge 0 & \end{aligned}$$

where u_r and v_i are the virtual weights for the rth output and ith input, respectively, and θ is a scalar. Obviously, the CCR model is nonlinear, which can be transformed into a linear model by setting the sum of the weighted inputs equal to one. The resulting model is called the "input-oriented" CCR model, which is expressed as

$$\begin{aligned} E_{oo} = \text{Max } \theta = \sum_{r=1}^{s} u_r y_{ro} & \\ \text{subject to } \quad \sum_{i=1}^{m} v_i x_{io} = 1 & \\ \sum_{r=1}^{s} u_r y_{rj} \le \sum_{i=1}^{m} v_i x_{ij} & \qquad j = 1, \dots, n \\ u_1, u_2, \dots, u_s \ge 0 & \\ v_1, v_2, \dots, v_m \ge 0 & \end{aligned}$$

The objective function is the ratio of the sum of the weighted outputs. The first constraint ensures the sum of the weighted inputs is equal to one. Using the above model, DMU$_o$ is identified as CCR-efficient if the relative efficiency E_{oo} equals one. Baker and Talluri [11] showed that CCR model may provide misleading efficiency scores through identifying a DMU with an unrealistic weighing scheme to be efficient. Moreover, the E_{oo} may equal to one for more than one DMU. As a result, the CCR-model fails to discriminate among efficient DMUs. In contrast, the aggressive formulation increases discrimination among efficient by allowing efficiency takes a value greater than one and allows for DMU's peer-evaluation instead of self-evaluation [12]. A peer-evaluation means that DMU_o is evaluated according to the optimal weighting scheme of other DMUs. The main idea of aggressive formulation is to obtain a weighing scheme of DMU_o that would be optimal in the CCR model, but have, as a secondary objective, minimization of the cross-efficiencies of the other DMUs. The model of this technique given by

$$\begin{aligned}
\text{Min} \quad & \sum_{r=1}^{s}(u_{ro}\cdot\sum_{j\neq o} y_{rj}) - \sum_{i=1}^{m}(v_{io}\cdot\sum_{j\neq o} x_{ij}) \\
\text{subject to} \quad & \sum_{i=1}^{m}(v_{io}\cdot\sum_{j\neq o} x_{ij}) = 1 \\
& \sum_{r=1}^{s} u_{ro} y_{rj} - \sum_{i=1}^{m} v_{io} x_{ij} \leq \delta, \qquad \forall j \neq o \\
& \sum_{r=1}^{s} u_{ro} y_{ro} - E_{oo}\cdot\sum_{i=1}^{m} v_{io} x_{io} = 0 \\
& u_{ro}, v_{io} \geq 0
\end{aligned}$$

where δ is a scalar, which is very close to zero. Utilizing the optimal u_{ro} and v_{io} values, u^*_{ro} and v^*_{io}, respectively, the cross-efficiencies of DMU_o are then calculated. Let E_{oj} be the cross-efficiency of DMU_j calculated according to the optimal weights of DMU_o. The E_{oj} is calculated as

$$E_{oj} = \sum_{r=1}^{s} u_{ro} y_{rj} / \sum_{i=1}^{m} v_{io} x_{ij} \qquad j \neq o \tag{1}$$

Let e_j be the mean of cross-efficiencies for DMU_j. The e_j is estimated as

$$e_j = \sum_{o\neq j} E_{oj} / (n-1) \qquad j = 1, \ldots, n \tag{2}$$

Once the E_{oj} and e_j values are obtained, a matrix called the "cross-efficiencies matrix" is constructed and used for comparing the performance of n DMUs. In this research, the aggressive formulation will be utilized for solving the multiresponse problem in Taguchi method.

III. PROPOSED APPROACH

The proposed approach for solving the multiresponse problem in Taguchi method is outlined in the following steps:

Step 1: Assume n experiments in Taguchi's OA are conducted. Treat each experiment as a DMU. Typically, the efficiency is enhanced if the sum of the weighted outputs is increased and/or the sum of the weighted inputs is decreased. Therefore, set the multi-responses for each DMU based on the following:

i. If all responses are STB type, then set all responses as inputs, whereas set a unit (one) as the output. Conversely, if all responses are LTB type, then set all of them as the outputs, while set one as input.

ii. If all responses are NTB type, then calculate the estimate of quality loss, L_j, for DMU_j as follows [13]:

$$L_j = c(s_j^2 / \bar{y}_j^2) \qquad j = 1, \ldots, n \tag{3}$$

where c is the quality loss coefficient, while $\bar{y}_j$ and s_j are the average and standard deviation of response replicates for each DMU_j, respectively. Set the L_j values as the inputs and one as the output for all DMUs.

iii. If responses are a mix of the three types, set STB type response and L_j value of the NTB type response as inputs, whereas set LTB type response(s) as the output.

Step 2: Obtain the E_{oo} value by solving the input-oriented CCR model for each DMU.

Step 3: Estimate the u^*_{ro} and v^*_{io} values for each DMU by solving aggressive formulation. Then, calculate the E_{oj} and e_j values using Eqs. (1) and (2), respectively. Finally, construct the cross-efficiencies matrix.

Step 4: Decide the ordinal value of e_j. The ordinal value is to rank the e_j values such that the smallest e_j value receives an ordinal value of one, whereas the largest e_j value takes an ordinal value of n. Let AOV_{fl} be the average of the ordinal values at level l of factor f. Calculate the AOV_{fl} value for each factor level. Typically, higher AOV_{fl} implies better performance. Therefore, the optimal factor level is identified as the level that maximizes the value of AOV_{fl}. If a tie occurs in selecting the optimal level for a factor, then choose the factor level that provides the largest anticipated improvement as the optimal level for that factor.

Step 5: Calculate the anticipated improvement due to setting controllable factors at optimal levels obtained by aggressive formation.

IV. ILLUSTRATIONS

Two frequently-investigated case studies are provided to illustrate the proposed approach.

A- Optimization of Polysilicon Process

Taguchi method was used to improve the quality of polysilicon process [14] by optimizing concurrently three responses; the surface defects (STB), thickness (NTB, target is 3600 Å) and deposition rate (LTB). Six process factors were investigated simultaneously including: (A) deposition temperature, (B) deposition pressure, (C) Nitrogen flow, (D) silane flow, (E) settling time, and (F) cleaning method, utilizing L_{18} $(2^1\times3^7)$ array shown in Table 1. The proposed approach was adopted to optimize the three responses concurrently as follows:

Step 1: Each experiment in L_{18} $(2^1\times3^7)$ array is treated as a DMU. The quality loss of thickness, calculated using Eq. (3), and surface defects are set the inputs. Whereas, the deposition rate is set as the output for all DMUs.

Step 2: Each DMU is evaluated by solving the input-oriented CCR model. The E_{oo} (o = 1, ... , 18) is displayed in Table 2. Note that all the E_{oo} values lie between zero and one, while the E_{oo} value for each of DMU_1, DMU_4, DMU_{10}, DMU_{11}, and DMU_{14} is equal to one. Thus, these DMUs are equally identified as CCR-efficient, which shows the weakness of the CCR model in discriminating efficient DMUs.

Step 3: Aggressive formulation model is adopted to evaluate the values of v_{1j}^*, v_{2j}^*, and u_{1j}^* for each DMU$_j$. The results are also shown in Table 2. For illustration, the values of v_{11}^*, v_{21}^*, and u_{11}^* equal to 0.0, 0.00006892, and 0.00000318, respectively, for DMU$_1$ are obtained by solving

$$\text{Min} \quad u_{11}\sum_{j=2}^{18}(y_{rj}) - \sum_{i=1}^{2}(v_{i1}\cdot\sum_{j=2}^{18}x_{ij})$$

$$\text{subject to} \quad \sum_{i=1}^{2}(v_{i1}\cdot\sum_{j=2}^{18}x_{ij}) = 1$$

$$u_{11}y_{rj} - \sum_{i=1}^{2}v_{i1}x_{ij} \le \delta, \qquad j = 2,\ldots,18$$

$$u_{11}y_{r1} - E_{11}\sum_{i=1}^{2}v_{i1}x_{i1} = 0$$

$$u_{r1}, v_{i1} \ge 0$$

The values of v_{1j}^*, v_{2j}^*, and u_{1j}^* for other DMUs are obtained similarly. The E_{oj} and e_j values are then calculated using Eqs. (1) and (2) for each DMU. Finally, the cross-efficiencies matrix is constructed in Table 3. Note in Table 3, the DMUs identified as CCR-efficient have unequal e_j values and hence are no more equally efficient. This shows that the efficiency of the aggressive formulation technique in increasing the discrimination among efficient DMUs.

Step 4: The ordinal values for all e_j values are decided and also listed in Table 3. The AOV_{fl} values are then calculated for all factor levels and plotted in Fig. 1. For illustration, the AOV_{A1}, the efficiency of level 1 for factor A, is calculated as the average of the ordinal values for DMU$_1$, DMU$_2$, DMU$_3$, DMU$_{10}$, DMU$_{11}$, and DMU$_{12}$, then divided by six. From Fig. 1, it is clear that $A_1B_1C_1D_2E_2F_2$ is the combination of factor levels that optimizes the three responses concurrently.

Table 1. Experimental data of polysilicon process.

DMU$_j$	Control factors*							Inputs		Outputs
	e	A	B	C	D	E	F	thickness Loss (x_{1j})	Surface defects (x_{2j})	Deposition rate (y_{1j})
DMU$_1$	1	1	1	1	1	1	1	0.00030	0.67	14.5
DMU$_2$	1	1	2	2	2	2	2	0.00027	36.22	36.6
DMU$_3$	1	1	3	3	3	3	3	0.00025	135.78	41.4
DMU$_4$	1	2	1	1	2	2	3	0.00006	17.00	36.1
DMU$_5$	1	2	2	2	3	3	1	0.00719	1087.78	73.0
DMU$_6$	1	2	3	3	1	1	2	0.00051	839.89	49.5
DMU$_7$	1	3	1	2	1	3	2	0.00726	776.33	76.6
DMU$_8$	1	3	2	3	2	1	3	0.00520	2065.33	105.4
DMU$_9$	1	3	3	1	3	2	1	0.00087	2200	115.0
DMU$_{10}$	2	1	1	3	3	2	2	0.00206	0.89	24.8
DMU$_{11}$	2	1	2	1	1	3	3	0.00013	1.00	20.0
DMU$_{12}$	2	1	3	2	2	1	1	0.00016	246.56	39.0
DMU$_{13}$	2	2	1	2	3	1	3	0.00062	150.11	53.1
DMU$_{14}$	2	2	2	3	1	2	1	0.00005	44.44	45.7
DMU$_{15}$	2	2	3	1	2	3	2	0.00018	1359.44	54.8
DMU$_{16}$	2	3	1	3	2	3	1	0.00065	14.33	76.8
DMU$_{17}$	2	3	2	1	3	1	2	0.00629	2201.22	105.3
DMU$_{18}$	2	3	3	2	1	2	3	0.01438	3333.33	91.4

* e indicates empty column.

Table 2. The results of aggressive formulation.

DMU$_j$	CCR-Model	Aggressive Formulation			
	E_{jj}	δ	v_{1j}^*	v_{2j}^*	u_{1j}^*
DMU$_1$	1.00000	0.000018	0.00000000	0.00006892	0.00000318
DMU$_2$	0.38025	0.001007	0.00000000	0.00006909	0.00002600
DMU$_3$	0.22037	0.000234	21.65440000	0.00000000	0.00002882
DMU$_4$	1.00000	0.001507	0.00000000	0.00006900	0.00003249
DMU$_5$	0.02626	0.001171	0.00000000	0.00007450	0.00002915
DMU$_6$	0.09788	0.000000	21.77700000	0.00000000	0.00002196
DMU$_7$	0.03359	0.000860	0.00000000	0.00007281	0.00002479
DMU$_8$	0.03032	0.000445	24.25418000	0.00000000	0.00003628
DMU$_9$	0.13343	0.000000	21.94908000	0.00000000	0.00002216
DMU$_{10}$	1.00000	0.000000	0.00000000	0.00006892	0.00000247
DMU$_{11}$	1.00000	0.000024	0.00000000	0.00006892	0.00000345
DMU$_{12}$	0.25200	0.000000	21.61228000	0.00000000	0.00002234
DMU$_{13}$	0.16001	0.001421	0.00000000	0.00006964	0.00003150
DMU$_{14}$	1.00000	0.000000	21.56102000	0.00000000	0.00002359
DMU$_{15}$	0.30722	0.000000	21.62162000	0.00000000	0.00002182
DMU$_{16}$	0.67157	0.000153	0.00000000	0.00006898	0.00000864
DMU$_{17}$	0.02609	0.000529	24.91281000	0.00000000	0.00003883
DMU$_{18}$	0.01227	0.001793	0.00000000	0.00008947	0.00004004

Table 3. The cross-efficiencies matrix by aggressive formulation for polysilicon process.

DMU$_o$ \ DMU$_j$	DMU$_1$	DMU$_2$	DMU$_3$	DMU$_4$	DMU$_5$	DMU$_6$	DMU$_7$	DMU$_8$	DMU$_9$	DMU$_{10}$	DMU$_{11}$	DMU$_{12}$	DMU$_{13}$	DMU$_{14}$	DMU$_{15}$	DMU$_{16}$	DMU$_{17}$	DMU$_{18}$
DMU$_1$		0.0467	0.0141	0.0981	0.003101	0.0027	0.0046	0.0024	0.0024	1.28756	0.9241	0.0073	0.0163	0.0475	0.0019	0.2476	0.0022	0.0013
DMU$_2$	8.1438		0.1147	0.7991	0.025253	0.0222	0.0371	0.0192	0.0197	10.48572	7.5260	0.0595	0.1331	0.3870	0.0152	2.0167	0.0180	0.0103
DMU$_3$	0.0643	0.1832		0.8049	0.013519	0.1288	0.0141	0.0270	0.1756	0.01598	0.2021	0.3317	0.1138	1.3162	0.4043	0.1567	0.0223	0.0085
DMU$_4$	10.1914	0.4759	0.1436		0.031603	0.0278	0.0465	0.0240	0.0246	13.12210	9.4183	0.0745	0.1666	0.4843	0.0190	2.5238	0.0225	0.0129
DMU$_5$	8.4685	0.3954	0.1193	0.8309		0.0231	0.0386	0.0200	0.0205	10.90372	7.8261	0.0619	0.1384	0.4024	0.0158	2.0971	0.0187	0.0107
DMU$_6$	0.0487	0.1388	0.1670	0.6100	0.010245		0.0106	0.0204	0.1331	0.01211	0.1532	0.2513	0.0862	0.9974	0.3064	0.1188	0.0169	0.0064
DMU$_7$	7.3675	0.3440	0.1038	0.7229	0.022846	0.0201		0.0174	0.0178	9.48613	6.8086	0.0538	0.1204	0.3501	0.0137	1.8245	0.0163	0.0093
DMU$_8$	0.0722	0.2060	0.2477	0.9048	0.015196	0.1448	0.0158		0.1974	0.01797	0.2272	0.3728	0.1279	1.4795	0.4545	0.1762	0.0250	0.0095
DMU$_9$	0.0487	0.1390	0.1672	0.6105	0.010255	0.0977	0.0107	0.0205		0.01212	0.1533	0.2516	0.0863	0.9984	0.3067	0.1189	0.0169	0.0064
DMU$_{10}$	0.7767	0.0363	0.0109	0.0762	0.002408	0.0021	0.0035	0.0018	0.0019		0.7177	0.0057	0.0127	0.0369	0.0014	0.1923	0.0017	0.0010
DMU$_{11}$	1.0821	0.0505	0.0152	0.1062	0.003355	0.0029	0.0049	0.0026	0.0026	1.39326		0.0079	0.0177	0.0514	0.0020	0.2680	0.0024	0.0014
DMU$_{12}$	0.0499	0.1423	0.1712	0.6253	0.010503	0.1001	0.0109	0.0210	0.1364	0.01242	0.1570		0.0884	1.0225	0.3141	0.1218	0.0173	0.0066
DMU$_{13}$	9.7894	0.4571	0.1379	0.9606	0.030356	0.0267	0.0446	0.0231	0.0236	12.60445	9.0467	0.0715		0.4652	0.0182	2.4242	0.0216	0.0124
DMU$_{14}$	0.0528	0.1506	0.1812	0.6618	0.011115	0.1059	0.0116	0.0222	0.1444	0.01314	0.1662	0.2727	0.0936		0.3324	0.1289	0.0183	0.0070
DMU$_{15}$	0.0487	0.1389	0.1671	0.6104	0.010251	0.0977	0.0107	0.0205	0.1332	0.01212	0.1533	0.2515	0.0863	0.9981		0.1188	0.0169	0.0064
DMU$_{16}$	2.7119	0.1266	0.0382	0.2661	0.008409	0.0074	0.0124	0.0064	0.0066	3.49171	2.5061	0.0198	0.0443	0.1289	0.0051		0.0060	0.0034
DMU$_{17}$	0.0753	0.2146	0.2581	0.9426	0.015832	0.1509	0.0165	0.0316	0.2057	0.01872	0.2367	0.3884	0.1333	1.5414	0.4735	0.1835		0.0099
DMU$_{18}$	9.6843	0.4522	0.1364	0.9502	0.03003	0.0264	0.0442	0.0228	0.0234	12.46919	8.9497	0.0708	0.1583	0.4602	0.0180	2.3982	0.0214	
e_j	3.451546	0.217537	0.1290407	0.622386	0.014957	0.058069	0.01983	0.017806	0.0746	4.4328488	3.24542	0.15017	0.09551	0.65689	0.15897	0.88919	0.01556	0.0072595
Ordinal value	**17**	**12**	**9**	**13**	**2**	**6**	**5**	**4**	**7**	**18**	**16**	**10**	**8**	**14**	**11**	**15**	**3**	**1**

Step 5: The anticipated improvement in each response due to setting factors at $A_1B_1C_1D_2E_2F_2$ and the anticipated improvements gained by other approaches in previous studies, including engineering judgment [14] the sum of the weighted normalized quality losses [13], PCA [5], and DEA based ranking (DEAR) [6], are displayed in Table 4. Clearly in Table 4, the largest anticipated improvements in thickness (= 14.84 dB) and surface defects (= 63.72 dB) correspond to the proposed approach. However, the largest anticipated improvement in deposition rate (= -9.34 dB) corresponds to the sum of the weighted of normalized quality losses. Nevertheless, among all techniques, the proposed approach provides the largest total anticipated improvement (= 69.22 dB).

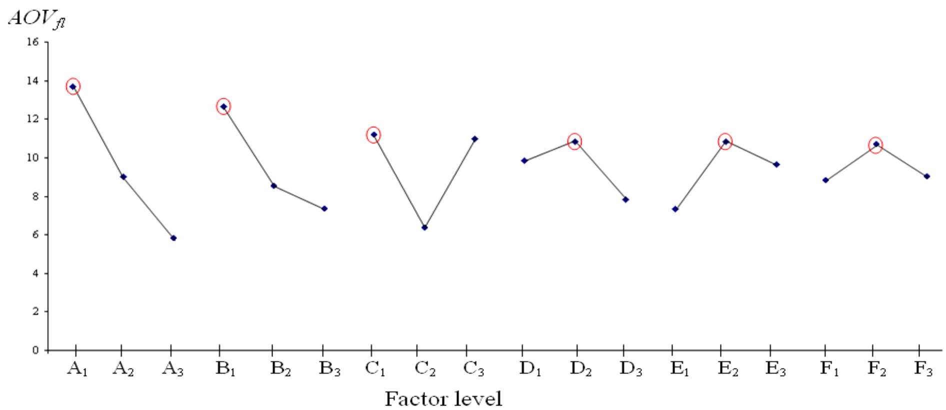

Fig. 1. Optimal factor levels for polycilicon process (optimal level is identified by circle).

Table 4. The anticipated improvement for polysilicon process

Quality response (dB)	Starting condition (I)	Optimal condition (II)					Anticipated improvement (II-I)				
		Engineering judgment [14]	Sum of weighted quality loss [13]	PCA [5]	DEAR [6]	Proposed approach	Engineering judgment [14]	Sum of weighted quality loss [13]	PCA [5]	DEAR [6]	Proposed approach
Thickness	29.95	36.79	40.24	41.23	41.32	44.79	6.84	10.29	11.28	11.37	14.84
Surface defects	-56.69	-19.84	-24.22	-2.29	1.20	7.03	36.85	32.47	54.40	57.89	63.72
Deposition rate	34.97	29.60	32.44	27.21	27.21	25.64	-5.37	-2.53	-7.76	-7.76	-9.34
Total anticipated improvement (dB)							38.32	40.23	57.92	61.5	69.22

B- Optimization of Gear Hobbing Operation

Genetic algorithm was employed to optimize four STB type responses of gear hobbing operation involving: left profile (LP) error, right profile (RP) error, left helix (LH) error, and right helix (RH) error [7]. Six controllable factors were investigated including: (A) direction of hobbing, (B) number of passes, (C) source of hob, (D) feed, (E) speed, and (F) job run out. The L_{18} $(2^1\times3^7)$ array was used for providing the layout of experimental work. Each experiment is treated as a DMU with LP error, RP error, LH error, and RH are set as the inputs, whereas a unit (one) is set as the output for all DMUs as shown in Table 5. The proposed approach to optimize the four responses concurrently is described briefly as follows. First, the E_{oo} values are obtained by solving CCR model then displayed in Table 6. Then, the aggressive formulation is applied to calculate the optimal input and output weights of each DMU. The E_{oj} values are computed for each DMU. Then, the e_j values with their corresponding ordinal values are obtained and listed in the last two columns of Table 6. Finally, the AOV_{fl} values are calculated and plotted in Fig. 2. In this figure, it is noted that $A_2B_1C_1D_3E_2F_2$ is the optimal combination of factor levels. Table 7 displays the anticipated improvement in each response at $A_2B_1C_1D_3E_2F_2$. The anticipated improvement gained by genetic approach [7] is also displayed in Table 7. The total anticipated improvement (= 11.2506 dB) due to setting factor levels at $A_2B_1C_1D_3E_2F_2$ larger than the anticipated improvement by genetic algorithm (= 4.1498 dB). Based on the above, it is concluded that the proposed approach is effective for solving the multiresponse problem in Taguchi method for gear hobbing operation.

Table 5. Experimental data of gear hobbing operation.

DMU_j	Control factor								Inputs				Outputs
	A	**BC**	**D**	**E**	**F**	*Empty*			LP error (x_{1j})	RP error (x_{2j})	LH error (x_{3j})	RH error (x_{4j})	Output (y_{1j})
DMU_1	1	1	1	1	1	1	1	1	72.53	73.97	47.37	42.90	1
DMU_2	1	1	2	2	2	2	2	2	75.67	74.23	32.43	39.10	1
DMU_3	1	1	3	3	3	3	3	3	74.20	73.10	51.93	51.10	1
DMU_4	1	2	1	1	2	2	3	3	74.80	77.03	61.27	55.03	1
DMU_5	1	2	2	2	3	3	1	1	75.37	75.93	82.97	59.80	1
DMU_6	1	2	3	3	1	1	2	2	71.83	73.93	35.83	42.30	1
DMU_7	1	3	1	2	1	3	2	3	75.10	71.97	54.47	60.07	1
DMU_8	1	3	2	3	2	1	3	1	77.03	74.80	56.17	44.90	1
DMU_9	1	3	3	1	3	2	1	2	77.63	72.27	57.87	59.83	1
DMU_{10}	2	1	1	3	3	2	2	1	73.67	76.80	42.33	47.10	1
DMU_{11}	2	1	2	1	1	3	3	2	74.23	79.03	48.83	34.20	1
DMU_{12}	2	1	3	2	2	1	1	3	71.97	75.37	42.03	30.77	1
DMU_{13}	2	2	1	2	3	1	3	2	75.10	74.53	34.17	34.73	1
DMU_{14}	2	2	2	3	1	2	1	3	76.50	74.50	40.33	37.83	1
DMU_{15}	2	2	3	1	2	3	2	1	72.83	74.77	42.33	40.37	1
DMU_{16}	2	3	1	3	2	3	1	2	75.63	78.73	45.17	35.27	1
DMU_{17}	2	3	2	1	3	1	2	3	75.40	77.07	42.93	39.27	1
DMU_{18}	2	3	3	2	1	2	3	1	75.90	72.00	50.90	47.40	1

Table 6. The results of aggressive formulation.

DMU_j	CCR-Model (E_{jj})	Aggressive formulation (weights)						
		Inputs				Output	e_j	Ordinal values
		v^*_{1j}	v^*_{2j}	v^*_{3j}	v^*_{4j}	u^*_{1j}		
DMU_1	0.996769	0.000000	0.000000	0.000000	0.001317	0.056334	0.870804	8
DMU_2	1.000000	0.000000	0.000000	0.001195	0.000000	0.038750	1.017562	17
DMU_3	0.995628	0.000000	0.000783	0.000000	0.000000	0.056996	0.807809	6
DMU_4	0.960339	0.000787	0.000000	0.000000	0.000000	0.056535	0.740510	2
DMU_5	0.965977	0.000787	0.000000	0.000000	0.000000	0.057326	0.671432	1
DMU_6	1.000000	0.000000	0.000000	0.001200	0.000000	0.042987	0.965613	15
DMU_7	1.000000	0.000000	0.000782	0.000000	0.000000	0.056312	0.768230	4
DMU_8	0.972930	0.000000	0.000784	0.000000	0.000000	0.057068	0.807354	5
DMU_9	0.995866	0.000000	0.000783	0.000000	0.000000	0.056326	0.749967	3
DMU_{10}	0.975113	0.000000	0.000000	0.001209	0.000000	0.049911	0.871228	9
DMU_{11}	0.969096	0.000000	0.000000	0.000000	0.001302	0.043168	0.905022	10
DMU_{12}	1.000000	0.000000	0.000000	0.000000	0.001297	0.039899	0.998115	16
DMU_{13}	1.000000	0.000000	0.000000	0.001197	0.000000	0.040914	1.030143	18
DMU_{14}	0.992851	0.000000	0.000000	0.001206	0.000000	0.048301	0.944495	14
DMU_{15}	0.991241	0.000000	0.000000	0.001209	0.000000	0.050737	0.916241	12
DMU_{16}	0.952392	0.000000	0.000000	0.000000	0.001304	0.043812	0.917718	13
DMU_{17}	0.963499	0.000000	0.000000	0.000000	0.001311	0.049609	0.908288	11
DMU_{18}	1.000000	0.000000	0.000782	0.000000	0.000000	0.056337	0.830599	7

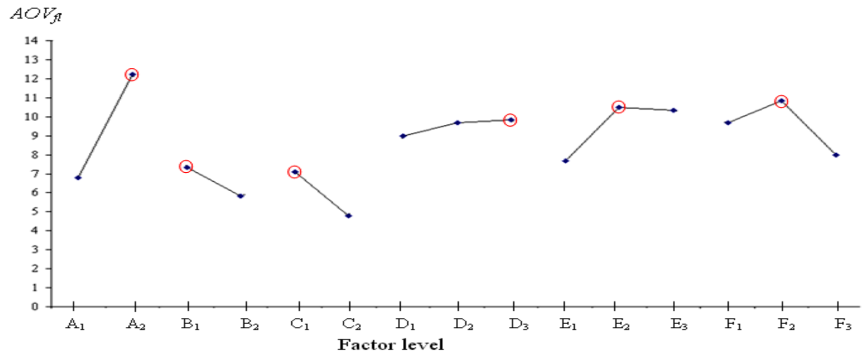

Fig. 2. Optimal factor levels for gear hobbing operation (optimal level is identified by circle).

Table 7. The anticipated improvement for gear hobbing operation.

Quality response (dB)	Initial condition (I)	Optimal condition (II)		Anticipated improvement =(II) – (I)	
		Genetic algorithm [7]	Proposed approach	Genetic algorithm [7]	Proposed approach
LP error	-37.8581	-37.4917	-37.1800	0.3664	0.6781
RP error	-37.4952	-37.4045	-37.4984	0.0907	-0.0032
LH error	-36.6009	-34.4082	-31.4320	2.1927	5.1688
RH error	-35.7397	-34.2396	-30.3328	1.5001	5.4069
Total anticipated improvement (dB)				4.1498	11.2506

V. Conclusions

An effective approach for solving the multiresponse problem in Taguchi method is proposed in this research. Two case studies were presented for illustration. In conclusion, DEA techniques are not only efficient at organizational level, but also effective in manufacturing at operational level.

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