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Control of Marangoni Convection in a Variable-Viscosity Fluid Layer with Deformable Surface

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Abstract—**The effectiveness of a proportional feedback control to suppress the Marangoni instability in a variable-viscosity fluid layer with a deformable free upper surface is investigated. Viscosity variation and deformable free surface have destabilizing effects on the stability limit. The stability thresholds for the short-scale mode are strongly dependent on viscosity variation and controller gain while the stability thresholds for the long-scale mode are greatly influenced by gravity and surface deformation. The feedback control strategy through thermal perturbation in the boundary data is shown effective in suppressing the Marangoni convection and delaying the onset of instability.**

Keywords: ***Marangoni convection, feedback control, variable viscosity, deformable surface, instability***

1 Introduction

Surface-tension-driven and buoyancy-driven convective flows have long been studied since the pioneering works of Benard [1], Rayleigh [2] and Pearson [3]. Convective flows are of practical importance in material processing technology in industrial applications. The industrial need has motivated extensive theoretical, experimental and numerical investigations to clarify the onset mechanism of the instability. Since convective flows are undesirable, it is beneficial to have a means to control the convective motions and achieve the preferable flow characteristics. Tang and Bau [4, 5] successfully applied the feedback control strategy to suppress the Rayleigh-Bénard convection. Bau [6] demonstrated that a proportional feedback control was effective in delaying the onset of convection in Marangoni-Bénard problems of Pearson [3] and Takashima [13, 14]. Arifin et al. [7] investigated the effect of a feedback control on Marangoni-Bénard convection for a free-slip bottom.

The stabilising effects of magnetic field and rotation on Marangoni convection have been analysed by Hashim and Arifin [8], Hashim and Sarma [9, 10] and Sarma and Hashim [11]. The aforementioned studies only dealt with fluids with invariant viscosity. However, viscosity of most fluids is known to decrease with temperature [15] and has a destabilising effect on convection [16, 17, 18, 19, 20]. Slavtchev and Ouzounov [19] studied the destabilising effect of temperature-dependent viscosity in the Marangoni problem in microgravity. The effects of viscosity variation, gravity waves and surface deformation on Marangoni instability has been analysed by Kalitzova-Kurteva et al. [20].

In this paper, we will demonstrate the possibility to alter the stability characteristics in Marangoni problem of a temperature-dependent-viscosity fluid layer. The thermal proportional feedback control is employed to suppress the intensity of Marangoni convection. We will show that the critical Marangoni number can be increased to delay the onset of convection and appreciably to stabilise the liquid layer.

2 Problem Formulation

2.1 Governing equations

Consider a horizontal layer of quiescent fluid of depth d on a rigid heat-conducting wall with a free upper surface. Variations of the dynamic viscosity μ and the surface tension σ of the fluid with temperature are assumed exponential and linear, respectively,

$$\mu = \mu_0 \exp\left[-\gamma\left(T - T_0\right)\right], \quad (1)$$
$$\sigma = \sigma_0 - \varepsilon\left(T - T_0\right), \quad (2)$$

where T is the temperature of the liquid, μ_0 and σ_0 correspond to values at a reference temperature T_0, γ and ε, which are positive for most fluids, correspond to the rate of change of the dynamic viscosity and the surface tension with temperature, respectively. Other physical properties of the liquid are assumed constant. The surface of the horizontal wall coincides with the xy-plane and the z-coordinate measures the vertical distance from the wall.

In the reference state, the fluid is at rest with the pressure

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and the liquid temperature are

$$p_{st} = p_g + \rho g (d - z), \quad (3)$$
$$T_{st} = T_w - \beta z, \quad (4)$$

where p_g is the gas pressure, ρ the density, g the acceleration due to gravity, T_w the temperature at the wall and $\beta > 0$. When motion sets in, the velocity $\mathbf{v} = (u, v, w)$, pressure p and temperature T fields obey the usual balance equations of mass, momentum and energy [20],

$$\nabla \cdot \mathbf{v} = 0, \quad (5)$$
$$\rho \left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right] = -\nabla p + \nabla \cdot (2\mu \mathbf{D} \mathbf{v}) + \rho \mathbf{g}, \quad (6)$$
$$\frac{\partial T}{\partial t} + (\mathbf{v} \cdot \nabla) T = \chi \nabla^2 T, \quad (7)$$

where χ is the thermal diffusivity, $\mathbf{g}(0, 0, -g)$ the gravitational acceleration, and $\mathbf{D}$ the deformation rate tensor.

2.2 Linearised controlled problem

We wish to extend the work of Kalitzova-Kurteva et al. [20] by applying a simple control mechanism of Bau [6] to suppress the intensity of convection and subsequently delay the onset of convection. The stability of the liquid system under controlled environment will be studied by applying a very simple linear active control of proportional feedback. Thermal control strategy can be easily applied and thus simplify the problem of mathematical formulation to a large extent where a slight modification in the temperature field does not alter the internal mechanism in the system. The temperature perturbation field is measured by a continuous distribution of sensors embedded in a plane parallel to the xy-plane at a chosen level. Each sensor emits signals to a thermal actuator positioned directly beneath it on the heated surface. By the proportional feedback rule, the actuator modifies the heated surface temperature using a proportional relationship between the temperature at the upper surface, z=1, and the lower surface, z=0, [6]

$$T(x, y, 0, t) = T(x, y, 0) - K \left[T(x, y, 1, t) - T(x, y, 1) \right], \quad (8)$$

or equivalently

$$T'(x, y, 0, t) = -K T'(x, y, 1, t), \quad (9)$$

where K is the controller gain and T' denotes the deviation of the fluid's temperature from its conductive value.

The stability of the problem will be investigated by performing a linear stability analysis. In formulation of the dynamic conditions in the liquid system, the governing equations and boundary conditions are linearised. We consider a small disturbance,

$$(w', T', \zeta) = \left[-W(z), \Theta(z), Z \right] \exp\left[i \left(\alpha_x x + \alpha_y y \right) + \omega t \right], \quad (10)$$

where $\zeta = \zeta(x, y, t)$ is the deviation from the flat free surface, $W(z)$, $\Theta(z)$ and Z the amplitudes, $\alpha = \left(\alpha_x^2 + \alpha_y^2 \right)^{1/2}$ the wave number, and ω the time constant.

Substituting (10) into the linearised equations from (5)–(7) and introducing the quantities d, d^2/χ, χ/d, $\mu_0 \chi/d^2$, and βd as the scales for distance, time, velocity, pressure, and temperature, respectively, yield [20]

$$f(z) \left[\left(D^2 - \alpha^2 + N^2 + 2ND \right) \left(D^2 - \alpha^2 \right) + 2N^2 \alpha^2 \right] W = \mathrm{Pr}^{-1} \omega \left(D^2 - \alpha^2 \right) W, \quad (11)$$
$$\left[\omega - \left(D^2 - \alpha^2 \right) \right] \Theta = -W, \quad (12)$$

where $D = \mathrm{d}/\mathrm{d}z$.

The boundary conditions at both surface boundaries, $z = 0$ and $z = 1$, comprise of,

$$W(0) = DW(0) = 0, \quad (13)$$
$$W(1) + \omega Z = 0, \quad (14)$$
$$f(1) \left[\left(D^2 - 3\alpha^2 \right) DW(1) + N \left(D^2 + \alpha^2 \right) W(1) \right] + \frac{\alpha^2 \left(\mathrm{Bo} + \alpha^2 \right) Z}{Cr} = \mathrm{Pr}^{-1} \omega DW(1), \quad (15)$$
$$f(1) \left(D^2 + \alpha^2 \right) W(1) - \alpha^2 Ma \left[\theta(1) - Z \right] = 0, \quad (16)$$
$$D\Theta(1) + \mathrm{Bi} \left[\Theta(1) - Z \right] = 0, \quad (17)$$

while the uniform temperature boundary at the wall surface, $z = 0$, is reinstated to include a controller rule with gain K,

$$\Theta(0) + K\Theta(1) = 0. \quad (18)$$

The dimensionless parameters are $Ma = \varepsilon \beta d^2 / \chi \mu_0$ the Marangoni number, $Cr = \mu_0 \chi / \sigma d$ the Crispation number, $\mathrm{Bo} = \rho g d^2 / \sigma_0$ the Bond number, $\mathrm{Bi} = hd/\lambda$ the Biot number, $\mathrm{Pr} = \mu_0 / \rho \chi$ the Prandtl number and $N = \gamma \beta d$ the viscosity parameter where λ is the thermal conductivity of the fluid and h is the heat transfer coefficient between the liquid and the gas phase at the upper free surface. The function $f(z)$ is given by

$$f(z) = \exp \left[N \left(z - 1 + \frac{T_0 - T_s}{\beta d} \right) \right]. \quad (19)$$

In relation with some previous works of controlled and uncontrolled systems, when $N = 0$, the system (11)–(18) reduces to a system of a constant-viscosity liquid with the application of a feedback control considered by Bau [6]. For $K = 0$ and $N = 0$ the system coincides with a constant-viscosity liquid of Marangoni problem of Takashima [13]. When $K = 0$ and $Cr = 0$ we recover the variable-viscosity Marangoni problem of Slavtchev and Ouzounov [19] with the nondeformable free surface and setting $K = 0$, $N = 0$ and $Cr = 0$, we recover the classical Marangoni problem of Pearson [3].

Since the dynamic viscosity of the fluid varies with temperature, the reference temperature for a variable-viscosity fluid can be taken as temperature at the bottom boundary μ_w, temperature at the upper free surface μ_s or mean value of viscosities at both boundaries $\overline{\mu} = (\mu_w + \mu_s)/2$ [19, 20]. The system (11)–(18) will be solved for Ma_s, the Marangoni number corresponds to μ_s. Then, Ma_s will be used to determine the mean Marangoni number $\overline{Ma}$ given by

$$\overline{Ma} = \frac{2\varepsilon\beta d^2}{\chi(\mu_s + \mu_w)} = \frac{2Ma_s}{1 - \exp(-N)}, \tag{20}$$

in relation to the modified Crispation number for the mean value of viscosities,

$$\overline{Cr} = \frac{\chi(\mu_s + \mu_w)}{2\sigma d} = \frac{[1 - \exp(-N)]\, Cr}{2}. \tag{21}$$

Thus, our conclusions of the Marangoni instability will be based on the marginal stability curves $\overline{Ma}$.

We restrict to the case of a deformable surface $\overline{Cr} \neq 0$ and consider the influences of no gravity Bo $= 0$, gravity waves Bo $= 0.1$ and the heat transfer mechanism at the free upper surface Bi $= 0$ and Bi $= 0.1$. Bo $= 0.1$ is representative for thin layers of some oils used in experiments on earth [20]. Bi $= 0$ represents a thermally perfectly insulated free surface and is considered as the most unstable situation since the whole thermal energy communicated in the system remains inside the liquid layer. We also include Bi $= 0.1$ since the Biot number Bi is at most 0.1 for a thin layer of liquid.

3 Results and discussion

We seek a closed form solution for the marginal stability curves of the steady ($\omega = 0$) Marangoni convection and by setting $\omega = 0$ in (11), the solution for $W(z)$ which satisfies the boundary conditions (13) and (14) is,

$$\begin{aligned} W(z) &= A_1\{[\exp(k_1 z) - \exp(k_2 z)]\cos(k_3 z) \\ &\quad - [A_2\exp(k_1 z) - A_3\exp(k_2 z)]\sin(k_3 z)\}, \end{aligned} \tag{22}$$

where A_1 is an arbitrary constant and k_1, k_2, k_3, k, A_2 and A_3 are given by,

$$k_1 = -\frac{N}{2} + \frac{1}{\sqrt{2}}\left(k^2 + \alpha^2 + \frac{N^2}{4}\right)^{1/2}, \tag{23}$$

$$k_2 = -\frac{N}{2} - \frac{1}{\sqrt{2}}\left(k^2 + \alpha^2 + \frac{N^2}{4}\right)^{1/2}, \tag{24}$$

$$k_3 = \frac{1}{\sqrt{2}}\left[k^2 - \left(\alpha^2 + \frac{N^2}{4}\right)\right]^{1/2}, \tag{25}$$

$$k = \left[\left(\alpha^2 + \frac{N^2}{4}\right)^2 + \alpha^2 N^2\right]^{1/4}, \tag{26}$$

$$A_2 = \cot k_3 + \frac{(k_2 - k_1)\exp(k_2 - k_1)}{k_3[1 - \exp(k_2 - k_1)]}, \tag{27}$$

$$A_3 = \cot k_3 + \frac{k_2 - k_1}{k_3[1 - \exp(k_2 - k_1)]}. \tag{28}$$

Substituting $W(z)$ in (12), the complete solution for the temperature is

$$\Theta(z) = F_1\sinh(\alpha z) + F_2\cosh(\alpha z) + \Theta_p(z), \tag{29}$$

where F_1 and F_2 are to be determined from the boundary conditions (17) and (18). $\Theta_p(z)$ denotes the particular solution corresponding to the nonhomogeneous equation involving $W(z)$. Thus,

$$\begin{aligned} \Theta_p(z) &= A_1\Big\{[C_1\exp(k_1 z) + C_2\exp(k_2 z)]\cos(k_3 z) \\ &\quad + [C_3\exp(k_1 z) + C_4\exp(k_2 z)]\sin(k_3 z)\Big\}, \end{aligned} \tag{30}$$

where

$$C_1 = \frac{2A_2k_1k_3 + \left(k_1^2 - k_3^2 - \alpha^2\right)}{4k_1^2k_3^2 + \left(k_1^2 - k_3^2 - \alpha^2\right)^2}, \tag{31}$$

$$C_2 = -\frac{2A_3k_2k_3 + \left(k_2^2 - k_3^2 - \alpha^2\right)}{4k_2^2k_3^2 + \left(k_2^2 - k_3^2 - \alpha^2\right)^2}, \tag{32}$$

$$C_3 = \frac{2k_1k_3 - A_2\left(k_1^2 - k_3^2 - \alpha^2\right)}{4k_1^2k_3^2 + \left(k_1^2 - k_3^2 - \alpha^2\right)^2}, \tag{33}$$

$$C_4 = -\frac{2k_2k_3 - A_3\left(k_2^2 - k_3^2 - \alpha^2\right)}{4k_2^2k_3^2 + \left(k_2^2 - k_3^2 - \alpha^2\right)^2}. \tag{34}$$

The expressions for $W(z)$ and the derivatives of $W(z)$ and $\Theta_p(z)$ at $z = 1$ for the determination of the remaining unknown quantities are listed as follows,

$$\begin{aligned} W(1) &= A_1\Big[(\exp k_1 - \exp k_2)\cos k_3 \\ &\quad - (A_2\exp k_1 - A_3\exp k_2)\sin k_3\Big], \end{aligned} \tag{35}$$

$$\begin{aligned} D^2W(1) &= A_1\Big\{\Big[\left(k_1^2 - k_3^2 - 2A_2k_1k_3\right)\exp k_1 \\ &\quad + \left(2A_3k_2k_3 - k_2^2 + k_3^2\right)\exp k_2\Big]\cos k_3 \\ &\quad + \Big[\left(A_2k_3^2 - A_2k_1^2 - 2k_1k_3\right)\exp k_1 \\ &\quad + \left(2k_2k_3 + A_3k_2^2 - A_3k_3^2\right)\exp k_2\Big]\sin k_3\Big\}, \end{aligned} \tag{36}$$

$$\begin{aligned} D^3W(1) &= A_1\Big\{\Big[\big(k_1^3 - 3k_3^2k_1 - 3A_2k_3k_1^2 \\ &\quad + A_2k_3^3\big)\exp k_1 + \big(3k_3^2k_2 + 3A_3k_3k_2^2 \\ &\quad - k_2^3 - A_3k_3^3\big)\exp k_2\Big]\cos k_3 \\ &\quad + \Big[\left(k_3^3 + 3A_2k_3^2k_1 - 3k_3k_1^2 - A_2k_1^3\right)\exp k_1 \\ &\quad + \big(3k_3k_2^2 - k_3^3 + A_3k_2^3 \\ &\quad - 3A_3k_3^2k_2\big)\exp k_2\Big]\sin k_3\Big\} \end{aligned} \tag{37}$$

$$\Theta_p(1) = A_1\Big[(C_1 \exp k_1 + C_2 \exp k_2)\cos k_3 + (C_3 \exp k_1 + C_4 \exp k_2)\sin k_3\Big], \quad (38)$$

$$D\Theta_p(1) = A_1\Big\{\big[(C_1 k_1 + C_3 k_3)\exp k_1 + (C_2 k_2 + C_4 k_3)\exp k_2\big]\cos k_3 + \big[(C_3 k_1 - C_1 k_3)\exp k_1 + (C_4 k_2 - C_2 k_3)\exp k_2\big]\sin k_3\Big\}. \quad (39)$$

Therefore, we obtain the coefficients F_1 and F_2,

$$F_1 = \frac{1}{R_1}\Big\{\alpha \sinh\alpha\,[A_1(C_1+C_2) + K\Theta_p(1)] - D\Theta_p(1) - \mathrm{Bi}\Theta_p(1) + \cosh\alpha\big[\mathrm{Bi}A_1(C_1+C_2) - KD\Theta_p(1)\big]\Big\} + \frac{\mathrm{Bi}CrR_2(1+K\cosh\alpha)}{R_1\alpha^2(\mathrm{Bo}+\alpha^2)}, \quad (40)$$

$$F_2 = -\frac{A(C_1+C_2) + KF_1\sinh\alpha + K\Theta_p(1)}{1+K\cosh\alpha}, \quad (41)$$

where

$$R_1 = \alpha\cosh\alpha + \mathrm{Bi}\sinh\alpha + \alpha K, \quad (42)$$

$$R_2 = -D^3W(1) + 3\alpha^2 DW(1) - ND^2W(1) - N\alpha^2 W(1). \quad (43)$$

The magnitude of the surface deflection Z can be calculated from (15). From boundary condition (16), we obtain an expression for Ma_s in terms of $\alpha, K, N, Cr, \mathrm{Bi}$ and Bo which can be conveniently written in the form

$$Ma_s = -\frac{R_1(\mathrm{Bo}+\alpha^2)\left[\alpha^2 W(1) + D^2W(1)\right]}{R_3}, \quad (44)$$

where

$$R_3 = \alpha\cosh\alpha\Big\{\alpha^2\Theta_p(1)(\mathrm{Bo}+\alpha^2) - \Big[3\alpha^2 DW(1) - D^3W(1) - ND^2W(1) - N\alpha^2W(1)\Big]Cr\Big\} - \alpha^2\sinh\alpha D\Theta_p(1)(\mathrm{Bo}+\alpha^2) - \alpha^3 A_1(\mathrm{Bo}+\alpha^2)(C_1+C_2) + \alpha KCr\Big[D^3W(1) - 3\alpha^2DW(1) + ND^2W(1) + N\alpha^2W(1)\Big]. \quad (45)$$

Fig. 1 shows the marginal stability curves for a case of a deformable surface $\overline{Cr} = 0.001$ and $\mathrm{Bi} = 0$ for some parameters values of Bo, N and K. Each curve has two local minima, one at $\alpha = 0$ of long-scale mode and the other one at $\alpha > 0$ of the short-scale mode. In Fig. 1(a), when $\mathrm{Bo} = 0$ the global minima are at $\alpha = 0$ indicating that only the long-scale mode dominates where the controller gain K is not effective. When the gravity waves are considered $\mathrm{Bo} = 0.1$, as shown in Fig. 1(b), the local minimum at $\alpha = 0$ has a nonzero mean Marangoni number $\overline{Ma}$. As the value of the controller gain K increases, the marginal stability profile increases but as the viscosity parameter increases, the marginal profile decreases. The global minima for constant-viscosity fluid are at $\alpha = 0$ while the global minima for a variable-viscosity fluid of $N = 8$ are at $\alpha > 0$.

Figs. 2 and 3 show the effects of viscosity variation N and controller gain K on $\overline{Ma}_c$ and α_c for $\mathrm{Bo} = 0.1$ and $\overline{Cr} = 0.001$. In Fig. 2, there exists a critical value of viscosity parameter, say N^* where when $N < N^*$, $\overline{Ma}_c$ increases and when $N > N^*$, $\overline{Ma}_c$ decreases. $\overline{Ma}_c$ for $\mathrm{Bi} = 0.1$ is slightly higher than $\overline{Ma}_c$ for $\mathrm{Bi} = 0$. The long-scale mode occurs when $N < N^*$ while the short-scale dominates when $N > N^*$ and α_c decreases as N increases. When $N = N^*$, both modes co-exist marked by discontinuous jumps (vertical lines) of α_c from zeros to nonzero values. As K increases, the effect of Bi on α_c weakens. In Fig. 3, there exists a critical value of controller gain, say K^* where when $K < K^*$, $\overline{Ma}_c$ increases in a short-scale mode but when $K > K^*$, $\overline{Ma}_c$ is insensitive of K in a long-scale mode. When $K = K*$, both modes occur with transitions from an increasing $\overline{Ma}_c$ to a stable $\overline{Ma}_c$ and from a short-scale mode to a long-scale mode.

The effect of deformable surface on the marginal curves, $\overline{Ma}_c$ and α_c are depicted in Fig. 4 and 5. The curve for a deformable surface $\overline{Cr} \neq 0$ differs fundamentally from the curve for a nondeformable surface $\overline{Cr} = 0$. There exist two local minima for $\overline{Cr} \neq 0$ instead of one minimum for the case $\overline{Cr} = 0$. When the surface is increasingly deformed, the minimum at $\alpha > 0$ is invariant but the minimum at $\alpha = 0$ decreases. There exists a value of $\overline{Cr}^*$ to mark the transition from invariant $\overline{Ma}_c$ to a decreasing $\overline{Ma}_c$ as well as the transition from the short-scale mode to a long scale mode. When $\overline{Cr} < \overline{Cr}^*$, there is a weak effect of $\overline{Cr}$ on $\overline{Ma}_c$ but strong effects of K and N on $\overline{Ma}_c$. When $\overline{Cr} > \overline{Cr}^*$ and increases, $\overline{Ma}_c$ decreases, long-scale mode dominates and the effects of K and N weaken.

Viscosity variation and deformable surface inhibit convective motion and have destabilizing effects on the stability thresholds. The use of a proportional feedback control is effective in increasing the critical $\overline{Ma}_c$ and stabilising the liquid layer.

4 Conclusions

Proportional feedback control has been used to investigate and suppress the Marangoni instability in a temperature-dependent-viscosity fluid layer with a deformable upper surface. Viscosity variation and deformable surface have destabilising effects on the stability thresholds and the use of feedback control is shown effective in suppressing the Marangoni convection in a

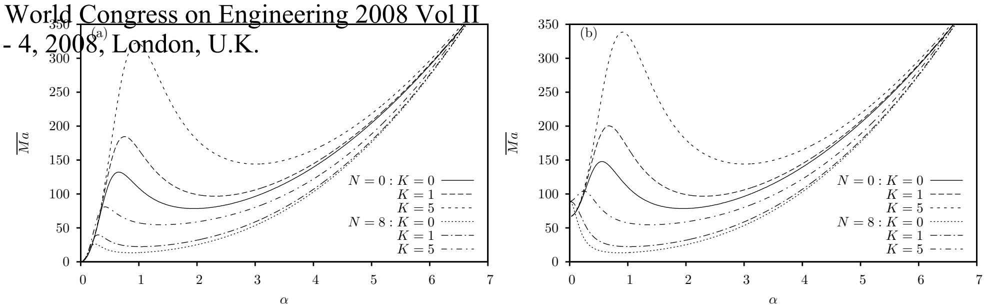

Figure 1: Marginal curves (a) Bo = 0 (b) Bo = 0.1 for Bi = 0, $\overline{Cr} = 0.001$ and various N and K.

temperature-dependent-viscosity liquid layer.

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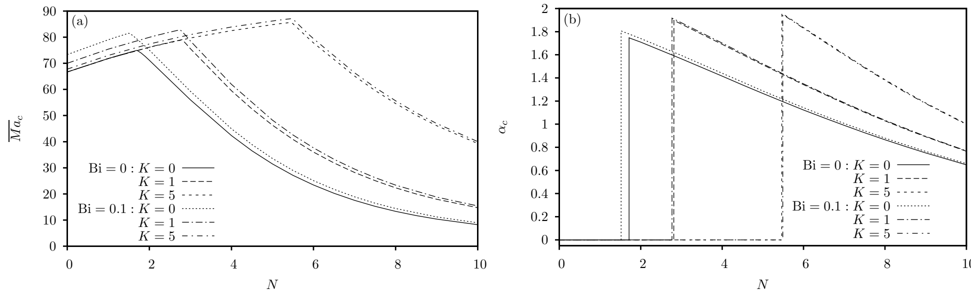

Figure 2: Effect of N on (a) $\overline{Ma}_c$ (b) α_c for Bo $= 0.1$, $\overline{Cr} = 0.001$, Bi $= 0, 0.1$ and $K = 0, 1, 5$.

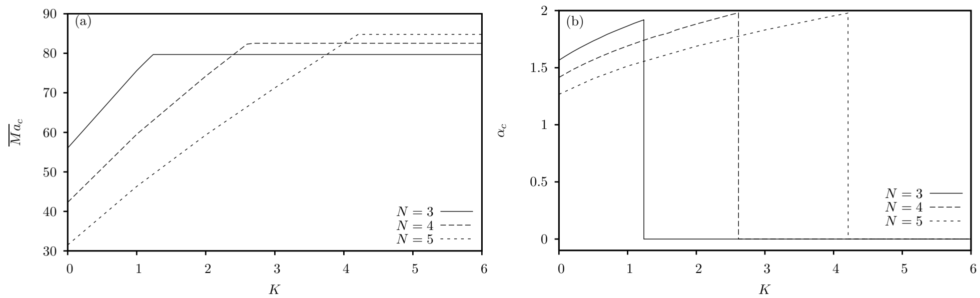

Figure 3: Effect of K on (a) $\overline{Ma}_c$ (b) α_c for Bo $= 0.1$, $\overline{Cr} = 0.001$, Bi $= 0$ and various N.

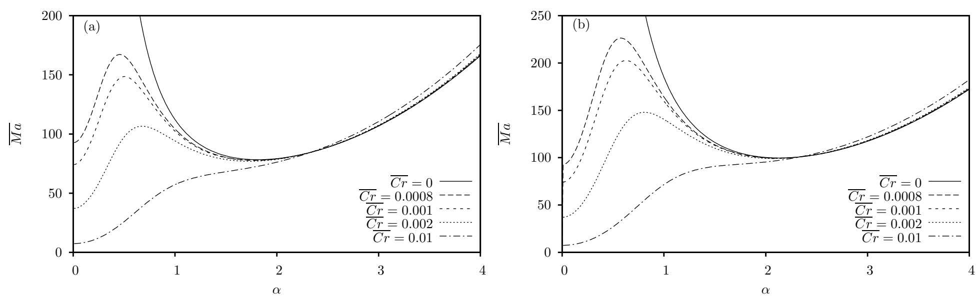

Figure 4: Marginal curves (a) $K = 0$ (b) $K = 1$ for Bi $= 0$, Bo $= 0.1$, $N = 1.5$ and various $\overline{Cr}$.

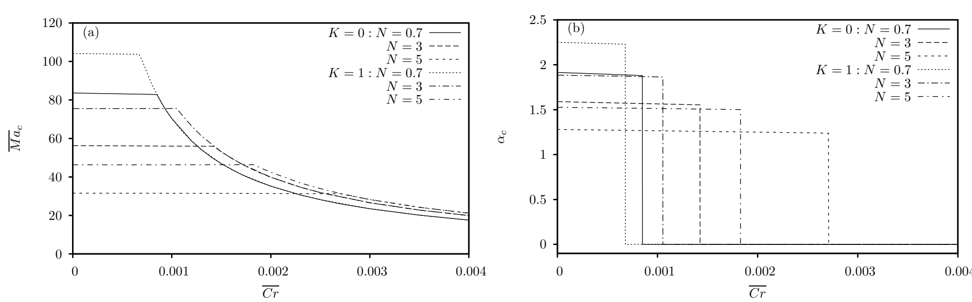

Figure 5: Effect of $\overline{Cr}$ on (a) $\overline{Ma}_c$ (b) α_c for Bi $= 0$, Bo $= 0.1$ and various N and K.