Different Kinds of Boundary Elements for Solving the Problem of the Compressible Fluid Flow around Bodies-a Comparison Study

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***Abstract*— The paper presents a comparison study between the numerical solutions obtained when using different kinds of boundary elements for solving the problem of the bidimensional compressible fluid flow around obstacles, by applying the boundary element method. The indirect boundary element method with sources distribution applied to this problem offers a singular boundary integral equation which is solved using constant, linear and quadratic boundary elements. For some particular cases exact solutions exist for this problem. Some computer codes are made for each of the considered boundary elements and numerical solutions are obtained for the case of a circular obstacle and an elliptical one. The numerical solutions obtained in these cases are compared with the exact ones and the errors are analyzed. Very good results are obtained, even for small numbers of boundary elements, when quadratic boundary elements are used.**

***Index Terms*— boundary element method, compressible fluid flow, linear boundary element, quadratic boundary element.**

I. INTRODUCTION

The boundary integral method (BEM) is a modern numerical technique used to solve boundary value problems for systems of partial differential equations.

There exist two principal variants of applying this method: the direct method and the indirect one. Both of them offer the principal advantage of the BEM over the other numerical method - the ability to reduce the problem dimension by one. This property is advantageous as it reduces the size of the system the problem is equivalent with, and so improves computational efficiency. To achieve this reduction of dimension it is necessary to formulate the governing equation as a boundary integral equation, which is usually a singular one (see [1], [2]), and for this, both techniques the indirect technique and the direct one can be used.

This paper is focused on solving the singular boundary integral equations obtained when the first variant is applied for the bidimensional problem of an inviscid, compressive subsonic fluid flow around bodies, considering the case of a non-lifting obstacle, by using different types of boundary elements. A comparison study between the numerical solutions obtained in these cases for the same number of nodes chosen on the boundary is also made.

The problem of a uniform, steady, potential motion of an ideal inviscid fluid of subsonic velocity $U_\infty \bar{i}$, pressure p_∞ and density ρ_∞ that is perturbed by the presence of a fixed body of a known boundary, noted *C*, assumed to be smooth and closed is described, using dimensionless variables, by the following mathematical model:

$$\begin{cases} \dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}=0 \\ \dfrac{\partial v}{\partial x}-\dfrac{\partial u}{\partial y}=0 \end{cases}, \tag{1}$$

with the boundary condition:

$$(\beta+u)n_x+\beta^2 v n_y = 0 \text{ on } C, \text{ and } \lim_{\infty}\bar{v}=0, \tag{2}$$

where *u* and *v* are the components along the axes of $\bar{v}$, the dimensionless perturbation velocity, $\bar{n}$ is the normal unit vector outward the fluid, β has the usual signification, $\beta=\sqrt{1-M^2}$ and *M* the Mach number for the unperturbed motion. We want to find out the perturbed motion, and the fluid action on the body.

II. THE BOUNDARY INTEGRAL EQUATION

Applying the indirect method with sources distribution the singular boundary integral equation the problem is reduced at is obtained (see [3]):

Assimilating the boundary with a distribution of sources of unknown intensity, *f*, first there are deduced the components of the perturbation velocity in the fluid domain and then with a limit process their expressions on the boundary. Using the boundary condition a singular boundary integral equation is obtained. For getting this boundaruy integral equation the definition of the Cauchy principal value of an integral is used and the unknown function *f* is assumed to satisfy a hölder condition-essential for the existence of the boundary equation.

The boundary integral equation has the following form:

$$\left(n_x^{0^2}+\beta^2 n_y^{0^2}\right) f(\bar{x}_0)+\frac{1}{\pi}\int_C f(\bar{x})\frac{(x-x_0)n_x^0+\beta^2(y-y_0)n_y^0}{\left|\bar{x}-\bar{x}_0\right|^2}ds = 2\beta n_x^0 \tag{3}$$

where n_x^0, n_y^0 are the components of the normal unit vector outward the fluid evaluated at $\bar{x}^0$ a point situated on *C*.

The boundary integral equation is a singular one. The sign " ' " denotes the Cauchy principal value of the integral.

In order to solve the singular boundary integral equation we use three types of boundary elements: constant boundary elements, linear isoparametric boundary elements and quadratic ones

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A. Case of constant boundary elements

We approximate the boundary by a polygonal line $\{L_j\},\ j=\overline{1,N}$ with the nodes on the real boundary and we consider that the unknown is constant on each segment. We consider that on each L_i the unknown is equal with the value taken in the midpoint of the segment, noted

$$\bar{x}_i^0 = \frac{\bar{x}^i + \bar{x}^{i+1}}{2},\ i \in \{1,2,...,N\},\ \bar{x}^{N+1} = \bar{x}^1. \quad (4)$$

In (3) we consider than $\bar{x}_0 = \bar{x}_i^0$ and we deduce the discrete form of the singular boundary integral equation:

$$\left(n_x^{0^2} + \beta^2 n_y^{0^2}\right) f(\bar{x}_0) + \frac{1}{\pi}\sum_{j=1}^{N} f_j \int_{L_j} \frac{(x-x_0)n_x^0 + \beta^2 (y-y_0) n_y^0}{|\bar{x}-\bar{x}_0|^2} ds = 2\beta n_x^0 \quad (5)$$

Imposing relation (5) to be satisfied on every midpoint, we get (see [4]) the following linear algebraic system which unknowns are the values of the sources intensity for the middle points of the segments:

$$a_i f_i + \sum_{j=1}^{N} A_{ij} f_j = A_i\ ,\ i=\overline{1,N}, \quad (6)$$

where

$$\begin{cases} 2a_i = n_x^2\left(\bar{x}_i^0\right) + \beta^2 n_y^2\left(\bar{x}_i^0\right) \\ A_{ij} = n_x\left(\bar{x}_i^0\right) U_{ij} + \beta^2 n_y\left(\bar{x}_i^0\right) V_{ij} \\ A_i = \beta n_x\left(\bar{x}_i^0\right) \end{cases} \quad (7)$$

The coefficients depend only on the coordinates of the nodes chosen for the boundary discretization. All the coefficients in (6) can be analytically evaluated and no errors appear due to their evaluation. After solving the system (6) the components of the velocity are found and then the local pressure coefficient.

B. Case of linear boundary elements

In order to solve the singular boundary equation we chose now the case of linear isoparametric boundary elements. We approximate the contour C with a polygonal line having the segments L_i , i=1,N and the extremes: $\left(x_i^1, y_i^1\right)$ şi $\left(x_i^2, y_i^2\right)$in a local numbering system. We have relations: $\left(x_i^2, y_i^2\right) = \left(x_{i+1}^1, y_{i+1}^1\right),\quad 1 \le i \le N-1$ and $\left(x_N^2, y_N^2\right) = \left(x_1^1, y_1^1\right)$, contour C being closed.

For describeing the geometry of a boundary element we use a local system of coordinates which has the origin in the first node of an element , and so we have the relations:

$$\begin{cases} x = x_i^1 \varphi^1 + x_i^2 \varphi^2 \\ y = y_i^1 \varphi^1 + y_i^2 \varphi^2 \end{cases},\ t \in [0,1], \quad (8)$$

where φ_1, φ_2 are the form functions given by

$$\varphi^1(t) = 1-t,\ \varphi^2(t) = t\ . \quad (9)$$

Using isoparametric boundary elements we have, for the unknown f, the local representation:

$$f = f_i^1 \varphi^1 + f_i^2 \varphi^2\ , \quad (10)$$

where f_i^1, f_i^2 are the nodal values of the unknown, it means the values of f at the extremes of the boundary element L_i, in the local numbering. These values satisfy the relations: $f_i^2 = f_{i+1}^1, 1 \le i \le N-1$, and $f_N^2 = f_1^1$.

For $\bar{x}_0 = \bar{x}_j^1,\ \forall j = \overline{1,N}$ in (3), we get an algebraic system of N equations each of them of the following form:

$$\left(n_x^{j^2} + \beta^2 n_y^{j^2}\right) f_j^1 + \frac{1}{\pi}\sum_{i=1}^{N} \int_{L_i} \left(f_i^1\varphi^1 + f_i^2\varphi^2\right) \frac{\left(x-x_j^1\right)n_x^j + \beta^2\left(y-y_j^1\right)n_y^j}{\left|\bar{x}-\bar{x}_j^1\right|^2} ds = 2\beta n_x^j \quad (11)$$

For simplifying the writing we shall not use the prim sign to specify that an integral must be understand in its Cauchy sense.

With the notations:

$$a_{ij} = \frac{1}{\pi}\int_{L_i} \varphi^1 \frac{\left(x-x_j^1\right)n_x^j + \beta^2\left(y-y_j^1\right)n_y^j}{\left|\bar{x}-\bar{x}_j^1\right|^2} ds$$

$$b_{ij} = \frac{1}{\pi}\int_{L_i} \varphi^2 \frac{\left(x-x_j^1\right)n_x^j + \beta^2\left(y-y_j^1\right)n_y^j}{\left|\bar{x}-\bar{x}_j^1\right|^2} ds \quad (12)$$

we get the following equivalent form for (11):

$$\sum_{i=1}^{N} f_i^1 a'_{ij} + \sum_{i=1}^{N} f_i^2 b_{ij} = 2\beta n_x^j\ , \quad (13)$$

where $a'_{ij} = a_{ij}\ \ for\ i \ne j$, and

$$a'_{jj} = \left(n_x^{j^2} + \beta^2 n_y^{j^2}\right) + a_{jj},\ i,j = \overline{1,N}\ . \quad (14)$$

After some calculous we obtain for the coefficients of system (13) the following expressions:

$$a_{ij} = \frac{l_i}{\pi}\int_0^1 (1-t)\frac{\left[x_i^1 - x_j^1 + t\left(x_i^2 - x_i^1\right)\right]n_x^j}{at^2 + 2bt + c} dt + \frac{l_i}{\pi}\int_0^1 (1-t)\frac{\beta^2\left[y_i^1 - y_j^1 + t\left(y_i^2 - y_i^1\right)\right]n_y^j}{at^2 + 2bt + c} dt =$$

$$= \frac{l_i n_x^j}{\pi}\left[\left(x_i^1 - x_j^1\right)I_0 + \left(x_i^2 - 2x_i^1 + x_j^1\right)I_1 - \left(x_i^2 - x_i^1\right)I_2\right] + \frac{l_i\beta^2 n_y^j}{\pi}\left[\left(y_i^1 - y_j^1\right)I_0 + \left(y_i^2 - 2y_i^1 + y_j^1\right)I_1\right] - \frac{l_i\beta^2 n_y^j}{\pi}\left(y_i^2 - y_i^1\right)I_2$$

$$b_{ij} = \frac{l_i}{\pi}\int_0^1 t\frac{\left[x_i^1 - x_j^1 + t\left(x_i^2 - x_i^1\right)\right]n_x^j}{at^2 + 2bt + c} dt + \frac{l_i}{\pi}\int_0^1 t\frac{\beta^2\left[y_i^1 - y_j^1 + t\left(y_i^2 - y_i^1\right)\right]n_y^j}{at^2 + 2bt + c} dt =$$

$$= \frac{l_i n_x^j}{\pi}\left[\left(x_i^1 - x_j^1\right)I_1 + \left(x_i^2 - x_i^1\right)I_2\right] + \frac{l_i\beta^2 n_y^j}{\pi}\left(y_i^1 - y_j^1\right)I_1 +$$

$$+\frac{l_i \beta^2 n_y^j}{\pi}\left(y_i^2 - y_i^1\right) I_2$$

where :

$$I_k = \int_0^1 \frac{t^k}{at^2 + 2bt + c} dt, \; k = 0,1,2 \,. \tag{15}$$

For the components of the normal unit vector outward the fluid we have the following formulas:

$$n_x^j = \frac{y_j^2 - y_j^1}{l_j}, n_y^j = \frac{x_j^1 - x_j^2}{l_j}, \; \forall j = \overline{1, N} \,. \tag{16}$$

The nonsingular integrals can be computed analitycally and for the singular ones the definition of the Cauchy principal value can be used (see [5]). In paper [6] their expressions are givven.

Using the same notations as before, the components of the velocity on the boundary (for the node $\overline{x}_j^1$, $j = \overline{1, N}$) can be evaluated with the formulas:

$$u_j^1 = -\frac{1}{2} f_j^1 n_x^j - \frac{1}{2\pi} f_j^1 \frac{x_j^1 - x_j^2}{l_j} - \frac{1}{2\pi} f_j^2 \frac{x_j^2 - x_j^1}{l_j} -$$

$$-\sum_{\substack{i=1 \\ i \neq j}}^{N} \frac{l_i}{2\pi} f_i^1 \left[\left(x_i^1 - x_j^1\right) I_0 + \left(x_i^2 - 2x_i^1 + x_j^1\right) I_1 - \left(x_i^2 - x_i^1\right) I_2 \right] -$$

$$-\sum_{\substack{i=1 \\ i \neq j}}^{N} \frac{l_i}{2\pi} f_i^2 \left[\left(x_i^1 - x_j^1\right) I_1 + \left(x_i^2 - x_i^1\right) I_2 \right]$$

$$v_j^1 = -\frac{1}{2} f_j^1 n_y^j - \frac{1}{2\pi} f_j^1 \frac{y_j^1 - y_j^2}{l_j} - \frac{1}{2\pi} f_j^2 \frac{y_j^2 - y_j^1}{l_j} -$$

$$-\sum_{\substack{i=1 \\ i \neq j}}^{N} \frac{l_i}{2\pi} f_i^1 \left[\left(y_i^1 - y_j^1\right) I_0 + \left(y_i^2 - 2y_i^1 + y_j^1\right) I_1 - \left(y_i^2 - y_i^1\right) I_2 \right] -$$

$$\sum_{\substack{i=1 \\ i \neq j}}^{N} \frac{l_i}{2\pi} f_i^2 \left[\left(y_i^1 - y_j^1\right) I_1 + \left(y_i^2 - y_i^1\right) I_2 \right] \tag{17}$$

So the local pressure coefficient can be obtained.

C. *Case of quadratic boundary elements*

In this paragraph we use quadratic isoparametric boundary elements of Lagrangean type to solve to solve the singular boundary integral equation (3), so the unknown function is approximated by polynomials of second degree, and the boundary by curved arcs. For obtaining the discret equation the boundary is divided into N unidimensional quadratic boundary elements, each of them with three nodes: two extreme nodes and an interior one. For getting this mesh we need $2N$ nodes on the boundary . Considering that the discrete equation is satisfied in every node, we have for $j = \overline{1,2N}$:

$$\left(n_x^{j\,2} + \beta^2 n_y^{j\,2} \right) f\left(\overline{x}_j\right) + \frac{1}{\pi} \sum_{i=1}^{N} \int_{L_i} f\left(\overline{x}\right) \frac{\left(x - x_j\right) n_x^j + \beta^2 \left(y - y_j\right) n_y^j}{\left|\overline{x} - \overline{x}_j\right|^2} ds = 2\beta n_x^j \tag{18}$$

The quadratic isoparametric boundary element uses the same set of basic functions, noted N_1, N_2, N_3 , for describing the geometry and the unknown function. Using the intrinsic system of coordinates, with the origin in the interior node, these functions have the expressinons:

$$N_1(\xi) = \frac{\xi(\xi - 1)}{2}, \, N_2(\xi) = 1 - \xi^2, \, N_3(\xi) = \frac{\xi(\xi + 1)}{2}, \quad \xi \in [-1,1] \tag{19}$$

Using a matricial notation we obtain the following equation:

$$\left(n_x^{j\,2} + \beta^2 n_y^{j\,2} \right) f\left(\overline{x}_j\right) + \frac{1}{\pi} \sum_{i=1}^{N} \left(\sum_{l=1}^{3} a_{ij}^l f_l^i \right) = 2\beta n_x^j \,, \tag{20}$$

where

$$a_{ij}^l = \int_{-1}^{1} N_l \frac{\left([N]\{x^i\} - x_j\right) n_x^j + \beta^2 \left([N]\{y^i\} - y_j\right) n_y^j}{\left|[N]\{\overline{x}\} - \overline{x}_j\right|^2} J(\xi) d\xi$$

$[N] = (N_1 \; N_2 \; N_3)$,

$\{x^i\}, \{y^i\}$, are column matrices made with the global coordinates of the element L_i nodes, and f_l^i i=1,N, l=1,2,3 are the nodal values of the unknown function for the three nodes of the mentioned element (the value of the unknown for the node number l of the element number i).

Returning to the global system of notation we obtain the following linear algebraic system:

$$[A]\{f\} = \{B\}, \;\; A \in M_{2N}(R), \;\; \{f\} \in R^{2N}, \{B\} \in R^{2N}$$

$$f_j = f\left(\overline{x}_j\right) B_j = 2\pi\beta n_x^j, \;\; j = \overline{1,2N} \,. \tag{21}$$

For getting the matrix $[A]$ we need to evaluate the integrals that appeare. One of them are usual integrals, but the other are singular integrals. For the singular integrals that appear there can be used more techniques, some of them being presented in [7]. One of these methods are: the truncation method, the Cauchy principal value method and the regularization method. We have used in this paper the regularization method because the study made in [8] shows that this method leads in case of quadratic boundary elements to the best results.

After solving the system (21), so after we find the values of f for the $2N$ nodes choosen for the discretization of the boundary we may also compute the velocity for these nodes. We deduce (see [9]):

$$u\left(\overline{x}_j\right) = -\frac{1}{2} f_j n_x^j - \frac{1}{2\pi} \sum_{i=1}^{N} \left(f_1^i b_{ij}^1 + f_2^i b_{ij}^2 + f_3^i b_{ij}^3 \right),$$

$$v\left(\overline{x}_j\right) = -\frac{1}{2} f_j n_y^j - \frac{1}{2\pi} \sum_{i=1}^{N} \left(f_1^i c_{ij}^1 + f_2^i c_{ij}^2 + f_3^i c_{ij}^3 \right), \tag{22}$$

The coefficients from the above expressions depend only on the nodes coordinates chosen for the boundary discretization and they can be found in [9].

III. NUMERICAL RESULTS

In some particular cases the considered problem has exact solution. In [10] there is presented the exact solution for the problem of the uniform ideal incompressible subsonic fluid flow around a circular obstacle.

Some computer codes made in MATHCAD, allow us to compare the numerical solutions obtained when using constant, linear and quadratic boundary elements with the analytical one, and to evaluate the errors that appear in each situation.

The comparison is made through the local pressure coefficient, noted c_p, obtained when there are used 10 and 20 nodes for the boundary discretization.

The following graphics show good agreements and they demonstrate the fact that using quadratic boundary elements and an adequate method for evaluating the singularities we get very good results even for a small number of boundary elements.

We consider first the case of 20 nodes on the boundary and we perform the local pressure coefficient, c_p evaluated at these nodes, on the one hand when different kinds of boundary elements are used and on the other hand for the exact solution.

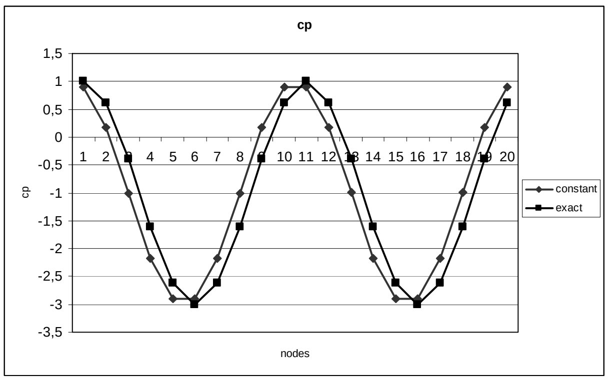

Fig.1. c_p for the exact solution and the numerical one obtained for constant boundary elements.

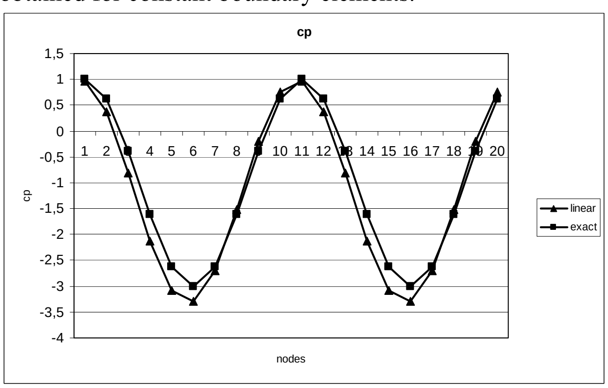

Fig.2. c_p for the exact solution and the numerical one obtained for linear boundary elements.

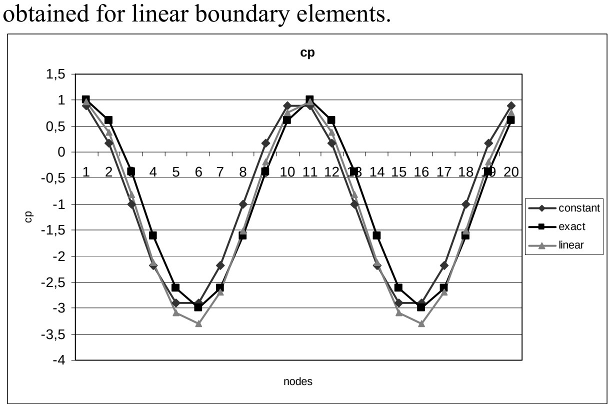

Fig.3. c_p for the exact solution, the numerical ones obtained for constant and linear boundary elements.

Evaluating the errors that appears we get the following graphic.

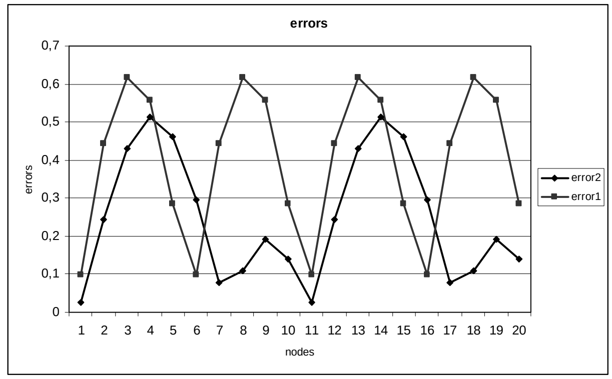

Fig.4. The absolute error between the exact solution and the numerical one obtained: for the case of constant boundary elements (Error1), and linear boundary elements (Error2).

As we notice the error is smaller for the case of linear boundary elements in case of 16 of the 20 nodes.

When using quadratic boundary elements we get very good results as we can see from the next graphic.

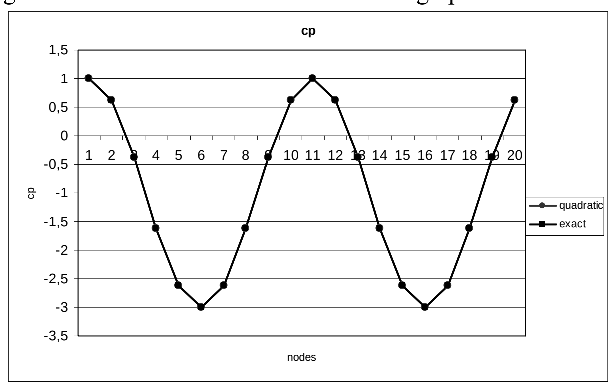

Fig.5. c_p for the exact solution and the numerical one obtained for quadratic boundary elements.

We can see that the values obtained for the local pressure coefficient in case of using quadratic boundary elements are almost equal with the exact values. That is why we can see only one line on the graphic. The absolute error that appears is performed in the following graphic.

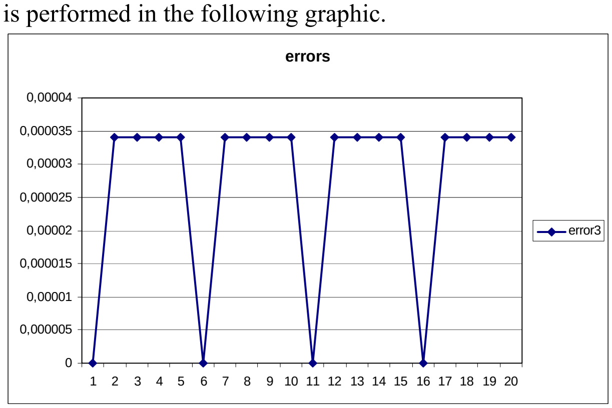

Fig.6. The absolute error between the exact solution and the one obtained for quadratic boundary elements.

The errors are so small not only for the reason of using quadratic boundary elements but also because the singular integrals that appear have been treated with a special attention.

Using a good method for evaluating the singular integrals that appear is a stage of great practical importance because the coefficients given by these singular integrals are dominants and situated near and on the diagonal of the system matrix, and so they play an important role for a well behavior of the system.

All the nodal values obtained for 20 nodes on the boundary are performed in the next figure.

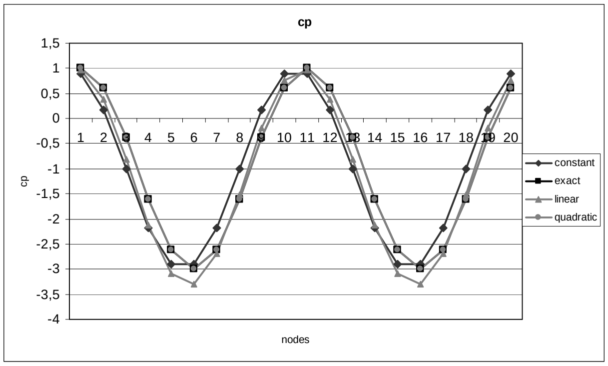

Fig.7. c_p for the exact solution, the numerical one for constant, linear and quadratic boundary elements.

When there are used 10 nodes for the boundary discretization the numerical results are performed in the following graphic and the comparison is also made through the local pressure coefficient.

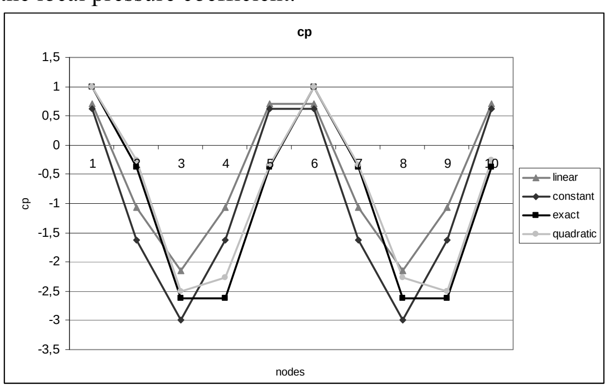

Fig.8. c_p for the exact solution, the numerical one for constant, linear and quadratic boundary elements, case of 10 nodes.

As we can notice the numerical results are not as good as before especially when constant and linear boundary elements are used. The best results, for 10 nodes on the boundary, are obtained as before in case of quadratic boundary elements.

The following graph shows the errors that appear in this case.

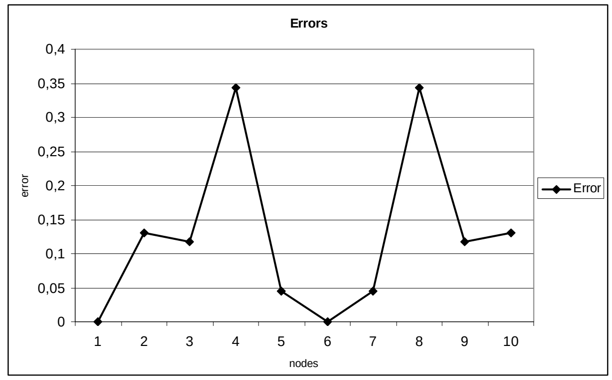

Fig.9. The errors in case of quadratic boundary elements and 10 nodes.

Comparing the errors from Fig.6 and Fig.9 we deduce the fact that the numerical solution obtained when 20 nodes are used for the boundary discretization is well improved.

We can run the computer codes for different numbers of nodes to see which is the optimal number of nodes in each case, a number big enough to lead to a small enough error and also not to big for a justified computational effort.

We can deduce which is the best number of nodes that must be chosen for the boundary discretization for obtaining the best ratio computational efficiency good results.

The computer codes can be used for obstacles with different geometries. In paper [10] the exact solution of the mentioned problem for the case of an elliptical obstacle can be found.

In the following graphic there is made, for an elliptical obstacle, a comparison between the exact solution and the numerical ones obtained for the same cases of boundary elements. There are used 20 nodes for the boundary discretization.

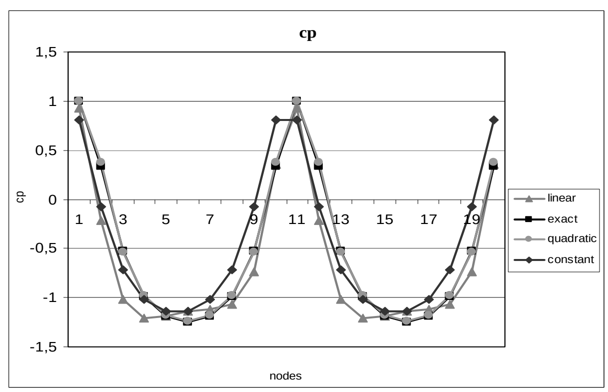

Fig.10. c_p for the exact solution, the numerical one for constant, linear and quadratic boundary element for an elliptical obstacle.

As we see from the above graphic the numerical solution obtained when using quadratic boundary elements is the best and is nearby the exact one even when we choose 20 nodes on the boundary.

The distribution of errors that appear for the chosen types of boundary elements in case of an elliptical obstacle is performed in the following graph.

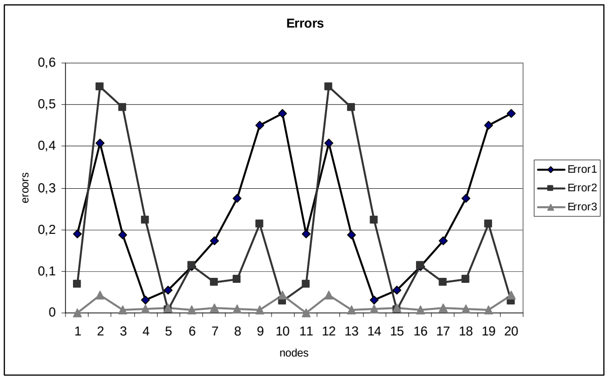

Fig.11. The errors between the exact solution and the numerical one obtained: for the case of constant boundary elements (Error1), linear boundary elements (Error2), and quadratic boundary elements (Error3).

For better seeing the errors in case of quadratic boundary elements we have the following graphic.

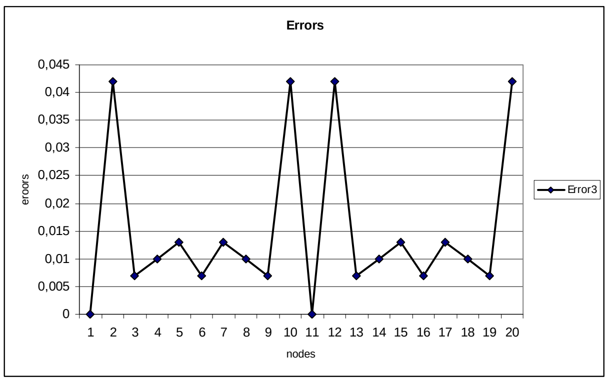

Fig.12 The errors between the exact solution and the numerical one obtained for the case of quadratic boundary elements (Error3).

As it is naturally better results can be obtained by using higher order boundary elements or more nodes for the boundary discretization, but as we see the results are satisfactory when choosing quadratic boundary elements and only 20 nodes on the boundary.

From the above graphics we can observe that the analyzed obstacles are non-lifting ones because of the symmetry of the local pressure coefficient: for corresponding nodes on the upper and the lower boundary it takes the same value. As we know this is a consequence of the fact that the analyzed profiles have smooth boundaries.

With the same computer codes numerical solutions can be obtained for any kind of compressible fluid flows, for different values of Mach number, not only for the ideal case and for other kinds of obstacles with smooth boundaries too.

For profiles with cusped trailing edge using a Kutta-Jukovsky condition and making adequate changes to the computer codes, numerical solutions of the mentioned problem can be found too.

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