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General Formulae for Frequency Shifts Using a Parallel Transport of Wave 4-Vector in Flat Minkowski Space

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Abstract—**In this paper we present general formulae which unify all known frequency redshifts/blueshifts. The obtained formulae for frequency shifts are based on parallel transport of a wave 4-vector in a flat Minkowskian space. This transport is performed by an antisymmetric tensor field which consists of a 3-vector of acceleration and 3-vector of angular velocity. It leads to the well known Doppler effect from the Special Relativity and the gravitational redshift. Accepting the assumption of time dependent gravitational potential in the Universe and its (almost)linear change by time, the cosmological Hubble redshift and the blueshift detected for the spacecrafts Pioneer 10 and 11 are explained. We also give a possible explanation of the anomaly of apparent increasing of the velocities of many spacecrafts after their launching from the Earth, detected by considering the Doppler effect.**

Keywords: frequency redshift, Doppler effect, cosmological redshift, gravitational redshift, Pioneer anomaly.

1 Introduction

The recent research in gravitation showed that it is possible to construct a theory of gravitation with the following properties: i) it is built in a flat Minkowskian space, i.e. a small step ahead from the Special Relativity, ii) it is very close and similar to the theory of electrodynamics, and iii) it is in accordance to all gravitational phenomena which are experimentally verified until now. The first steps were made in 1993 [11] and recently it was studied in many details [12]. That theory uses orthonormal coordinate frames and hence it is independent of the General Relativity (GR). But, if someone wants to see the relatinship between that theory and the GR we can also start from the standard metric from GR and apply the process of orthonormalization. That process is shown in the appendix in special case of time dependent metric. This orthonormalization can not be done globally, but only along a chosen curve, because in general case it leads to a Pfaffian system and hence to nonholonomic coordinates with respect to the coordinates from the GR. Thus, this theory can not be considered a part of GR for a special choice of the coordinate system. Analogously to the electrodynamics, it is built on an antisymmetric tensor denoted by ϕ_{ij}, which is analogous to the tensor of electromagnetic field and which is introduced in section 2. The equations of motion are introduced and are applied in case of gravitation [11, 12] and also in case of inertial forces [13]. In this paper it is not necessary to present the corresponding post-Newtonian equations of motion, because the Newtonian order of the acceleration is sufficient to deduce the frequency shifts up to c^{-2}. So, we can use the well known gravitational potential from the Newtonian mechanics which is analogous to the electromagnetic potential.

The orthonormal coordinates are "ideal" for calculating angles and precessions and they are the same as in GR [11, 12], but in orthonormal coordinates the effect of frequency shift seems to disappear. In this paper (case B in section 3) we show that in orthonormal coordinates we come to the same formula for gravitational frequency shift, where the Newton acceleration on the trajectory of the photons appears essential for this frequency shift. The goal of this paper is not to complete the mentioned theory for frequency shifts, but the goal is to deduce some global formulae for frequency shifts and to apply them in various special cases. This is presented in section 2. The methods in this paper are independent from the theory presented in [11, 12], because only the facts that are mentioned in section 2 are necessary to be given.

In section 3 we will discuss the Doppler red/blue shift, the gravitational redshift, the cosmological redshift and the Pioneer blueshift (Pioneer anomaly) as special cases of the obtained formulae. The consideration of the cosmological redshift and the Pioneer anomaly are based on the time-dependent gravitational potential [14, 15], which is presented in the Appendix of the paper. We give also a possible explanation of the anomalous amount of surplus velocity of spacecrafts after their launching from the Earth, detected by the Doppler effect techniques.

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For the sake of simplicity, we will use the notation $x^4 = ict$ for the time coordinate instead of $x^4 = ct$, and hence the Minkowskian metric is $g_{ij} = \delta_{ij}$, so there is no difference between the upper and lower indices.

2 General formulae

The 4-vector of light is a null-like vector (with zero module) given by

$$(C_j) = (\frac{\cos\alpha}{i}, \frac{\cos\beta}{i}, \frac{\cos\gamma}{i}, 1), \quad (2.1)$$

where $(\cos\alpha, \cos\beta, \cos\gamma)$ determines the 3-direction of motion of the light ray. This vector is analogous to the wave 4-vector ([6]). While (2.1) refers to the coordinate system in which the emitter of the light ray rests, the general form of this vector can be considered in the following way. Let us denote the initial frequency, i.e. the frequency in the coordinate system where the light rays are emitted, by ν_0, and denote by ν the frequency observed in another (moving) coordinate system. Then the general form of the vector (2.1) is given by

$$(C_j) = \frac{\nu}{\nu_0}(-i\vec{n}, 1), \quad (2.2)$$

where $\vec{n} = (n_x, n_y, n_z)$ is a unit 3-vector of the position of the light ray at the moment when light ray is emitted toward the receiver of the light ray, and ν is the frequency of the received signal. If the positions of the emitter and receiver are the same, then the 3-vector $\vec{n}$ is not determined, and thus at that point may have discontinuity in the received frequency, from redshift into blueshift or vice versa. Observed from the coordinate system where the light ray is emitted, the 3-vector $\vec{n}$ coincides with the unit vector $\frac{\vec{c}}{c}$, where $\vec{c}$ is the 3-vector of velocity of the light ray. Notice that for any two space-time points X and Y with coordinates (x_1, x_2, x_3, x_4) and (y_1, y_2, y_3, y_4), the 4-components $(y_1 - x_1, y_2 - x_2, y_3 - x_3, y_4 - x_4)$ transform as 4-vector in the space of Minkowski. Now, if Y and X are the emitter and the receiver of the light ray in the space of Minkowski, then the vector $(y_1 - x_1, y_2 - x_2, y_3 - x_3, y_4 - x_4)$ is proportional with $(-i\vec{n}, 1)$ from (2.2). Since the vector (2.2) refers to different pair (X, Y) of space-time points, the function $\frac{\nu}{\nu_0}$ appears as a coefficient of proportionality. Thus, we may treat the vector (2.2) as a 4-vector in the space of Minkowski.

Let us consider the frequency of a light signal emitted by the emitter at a point A and the frequency received by the receiver at the point B. We denote by $(P)_A$ and $(P)_B$ the Lorentz transformations at the points A and B with respect to us (as an observer) respectively. For the sake of simplicity we will consider a special case assuming that the initial point A rests with respect to us, i.e. $(P)_A = I$, and, assume also that there is no difference in the gravitational potential between us and the point A. Hence $\nu_A = \nu_0$.

Since we need the derivative of the Lorentz transformation $P = (P)_B$, we use the antisymmetric tensor $\frac{dP}{dt}P^T$ from the Lie algebra of the corresponding orthogonal group, which given in matrix form is represented by

$$(\phi_{ij}) = ic \begin{bmatrix} 0 & -iw_z/c & iw_y/c & -a_x/c^2 \\ iw_z/c & 0 & -iw_x/c & -a_y/c^2 \\ -iw_y/c & iw_x/c & 0 & -a_z/c^2 \\ a_x/c^2 & a_y/c^2 & a_z/c^2 & 0 \end{bmatrix}, \quad (2.3)$$

where (a_x, a_y, a_z) denotes the 3-vector of acceleration and (w_x, w_y, w_z) denotes the 3-vector of angular velocity. So, for arbitrary Lorentz transformation P, $\phi = \frac{dP}{dt}P^{-1}$, i.e. $(\phi_{ij}) = \frac{dP_{ik}}{dt}P^{-1}_{kj}$, and the components of ϕ are given by (2.3). Notice that $P^{-1} = P^T$, because P is a Lorentz transformation (including also space rotation) in the coordinate system with ict as time coordinate.

Now, let us start from the equality $C_i = P_{ij}C^0_j$, where C^0_j are the components of the vector (2.1) at A and they are constants, and C_i are given by (2.2). By differentiation of the previous equality we obtain

$$\frac{dC}{dt} = \frac{dP}{dt}C^0, \quad \text{i.e.} \quad \frac{dC_i}{dt} = \frac{dP_{ik}}{dt}C^0_k, \quad (2.4)$$

$$\frac{dC_i}{dt} = \frac{dP_{ik}}{dt}P^{-1}_{kj}P_{js}C^0_s.$$

Using that $\frac{dP_{ik}}{dt}P^{-1}_{kj} = \phi_{ij}$ and $P_{js}C^0_s = C_j$, finally we obtain

$$\frac{dC_i}{dt} = \phi_{ij}C_j. \quad (2.5)$$

From (2.4) we obtain the following frequency observed at B

$$\nu_0 + \nu_0 \int_A^B \frac{dP_{4j}}{dt}C^0_j dt = \nu_B. \quad (2.6)$$

Also, (2.5) can be written in the following form

$$\Delta C_i = \int_A^B \phi_{ij}C_j dt, \quad (2.7)$$

where C_j are given by (2.2). Hence, we obtained the general formulae (2.6) and (2.7) for the frequency shifts. Notice that the calculations via (2.6) and (2.7) are much easier if we use that the coordinate system in which the point A (emitter) rests. So, we will follow this assumption and find the scalar ν.

3 Special types of frequency shifts

We apply the results from the previous section to the known frequency shifts.

a) Doppler effect from the Special Relativity.

In the Special Relativity ϕ_{ij} is a zero matrix. The formula (2.5) shows that the action of force, and the acceleration of the system, i.e. $(a_x, a_y, a_z) \neq (0, 0, 0)$ yields a change of the frequency. So, we can assume that the system with a velocity $\vec{v}$ has been accelerated previously, and when the force has disappeared, it continued to move with constant velocity. If there is no gravitation, having in mind that C_j from (2.1) is a constant vector, formula (2.6) becomes

$$\Delta C_4 = \int_A^B \frac{d(P_{4j}C_j)}{dt}dt = [(P_{4j})_B - (P_{4j})_A]C_j.$$

Thus the frequency at B takes value $\nu_0 P_{4j} C_j$. For the sake of simplicity, assume that B moves along the x-axis, i.e. $v = v_x \neq 0$, $v_y = v_z = 0$. Then,

$$\nu_B = \nu_0 (P_{4j})_B C_j =$$

$$= \nu_0\Big(\frac{v}{ic\sqrt{1-\frac{v^2}{c^2}}}, 0, 0, \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\Big)\cdot\Big(\frac{\cos\alpha}{i}, \frac{\cos\beta}{i}, \frac{\cos\gamma}{i}, 1\Big) =$$

$$= \nu_0 \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\Big(1 - \frac{v\cos\alpha}{c}\Big).$$

This is the well known formula from the Special Relativity. This formula can be obtained by using only that (2.2) is a 4-vector ([6]).

b) Gravitational redshift.

Assume that the point A emits a light signal with frequency ν_0, and it is received at the point B. We can assume that the velocity of B is 0. Hence, $P_A = P_B = I$ and the frequency at B is given by (see (2.7))

$$\nu_0 + \nu_0 \int_A^B \phi_{4j} C_j dt =$$

$$= \nu_0 + \nu_0 ic \int_A^B \Big(\frac{a_x}{c^2}, \frac{a_y}{c^2}, \frac{a_z}{c^2}\Big)\Big(\frac{dx}{icdt}, \frac{dy}{icdt}, \frac{dz}{icdt}\Big)dt.$$

We denoted here by $(dx/dt, dy/dt, dz/dt)$ the observed 3-vector of velocity of the light. Further, we use that

$$(a_x, a_y, a_z) = \Big(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z}\Big),$$

where V is the gravitational potential (for the sign of V see the Appendix). Hence, for the frequency at B we obtain

$$\nu_0 + \nu_0 \int_A^B \phi_{4j} C_j dt =$$

$$= \nu_0 + \nu_0 \frac{1}{c^2}\int_A^B \Big(\frac{\partial V}{\partial x}\frac{dx}{dt} + \frac{\partial V}{\partial y}\frac{dy}{dt} + \frac{\partial V}{\partial z}\frac{dz}{dt}\Big)dt =$$

$$= \nu_0 + \nu_0 \frac{1}{c^2}\int_A^B \frac{dV}{dt}dt = \nu_0 + \nu_0 \frac{V(B)-V(A)}{c^2} =$$

$$= \nu_0\Big(1 + \frac{V(B)-V(A)}{c^2}\Big).$$

This formula represents just the well known gravitational redshift/blueshift. Notice that in this case the expression which is integrated is a total differential, and hence the integration does not depend on the curve of integration.

c) Cosmological (Hubble) redshift.

In the consideration of the cosmological redshift and the Pioneer anomaly we refer to the results presented in the Appendix. By $(A.5')$ the Hubble redshift is considered as a consequence of the gravitational redshift. It is assumed that the gravitational potential in the universe changes almost linearly. The change of the gravitational potential is such that it explains the Hubble redshift for the closer galaxies. Moreover, type Ia Supernova (SNe Ia) light curves provide convenient standard clocks for testing cosmological time dilation, which means that a process that takes Δt_0 as measured by the emitter appears to be lower compared with the observer when the light emitted by that process reaches the observer. Recent evidence from supernovae [7] showed time dilation for a single SN Ia at the 96.4% confidence level, using the time variation for the spectral features. Analogous measurements are obtained in [8]. This GR time dilation is in agreement with the time dilation studied in [4].

d) Pioneer anomaly.

Since the velocity of photons is c, the acceleration (A.10) leads to $\vec{a} = \vec{c}H$. It is obvious that by replacing $\vec{a}$ with $\vec{c}H$ in (2.3), from (2.7) we obtain $\Delta\nu = \nu_0 tH$, $\nu = \nu_0(1+tH)$, which are the frequency drift formula (15) presented in [1] and the anomalous blueshift respectively. Notice that in (2.7) we used that $P_A = P_B = I$. This is appropriate in this case because the radio signal starts from the Earth and after reaching the spacecraft it returns back to the Earth. After excluding the Doppler effect caused from the motion of the spacecraft, and also some other effects like the solar corona, remains the Pioneer anomaly. Notice that the mentioned acceleration $\vec{a} = \vec{c}H$ refers only to the photons, while the acceleration of the planets are $\vec{v}H$, where $\vec{v}$ is the velocity of the corresponding planet.

The frequency drift is calculated to "about $6\cdot 10^{-6}\mathrm{s}^{-2}$" [1], or 1.5 Hz in 8 years ($\approx 5.94\cdot 10^{-6}\mathrm{s}^{-2}$), using summation of 3156 independent single measurements (in one year) with duration of 1000s each, in 8 year period of measuring. However, we obtained $\frac{\Delta\nu}{t} \approx 5.23\cdot 10^{-6}\mathrm{s}^{-2}$ for the total frequency drift using $H = 70$km/s/Mpc (as of the 2006 data). The slight disagreement is probably a consequence of excluding various influences and averaging by a method that is more suitable for the dilation effect produced by the time-dependent gravitational potential.

However, the anomaly is not due to an anomalous motion of the spacecraft, but it is invoked by some influence on the signal. As it is obvious, this influence is of such nature, that it is negligible on small scales, but it becomes noticeable on larger (than 20 AU) scales.

e) Velocity anomalies of spacecrafts launched

FROM THE EARTH.

Studying the trajectories of many spacecrafts launched from the Earth it is detected that their velocities are larger of order 1 cm/s than expected values [2]. The present knowledge of the Earth's atmosphere, Earth's gravitation and precise data of the initial conditions of a spacecraft launched from the Earth, provides much better precision of the velocity of the spacecraft near the Earth than 1 cm/s. We shall give a possible explanation of this effect, based on the influence of the force which acts on the photons. Since this increasing of the velocity is obtained via the Doppler effect, this effect is apparent, and we should only explain the reason of the unexpected redshift. The Earth's atmosphere determines the spacecraft motion very precisely for a short time after launching from the Earth. But, also, the Earth's atmosphere has influence to the photons on their way between the navigation instruments and the spacecraft and vice versa. Indeed, there is a friction between the atmosphere and the photons. This force acts oppositely to the motion of the photons and thus the redshift is observed. The friction is minimal if the signal comes orthogonal to the Earth's axis of rotation (because the velocity of photons is orthogonal to the velocity of the orbiting atmosphere), i.e. if the spacecraft moves in a plane parallel to the plane of the equator. Otherwise the friction is larger, because the motion of the atmosphere is not orthogonal to the direction of the photones and hence the redshift is also larger.

Remark. Similar global frequency shift in the GR is considered by the formula [18]

$$\frac{\nu_{obs}}{\nu_{em}} = \frac{g_{ij}P^i U^j_{obs}}{g_{rs}P^r U^s_{em}},$$

where U_{obs} (U_{em}) denotes the 4-vector of velocity for observer (emitter) and P is the 4-momentum of the photon. This formula is convenient for the cases A and B. But if we introduce time dependent metric we can not simultaneously explain the cases C and D, because C is a redshift, but D is blueshift. According to this formula the flyby anomaly from case E can not be considered. This shows that the influence of the acceleration (deceleration) of photons is of primary importance, compared with the alternative influence by the metric.

4 Appendix: Time-Dependent Gravitational Potential

The recent papers [14, 15, 16, 17] developed an idea about a linear (or almost linear) change of the gravitational potential in the Universe. It is assumed that the sign of the usual gravitational potential V is such that V is larger near the massive bodies. Suppose that a light signal starts from a star with a frequency ν_0, then according to the General Relativity after time t its frequency is $\nu = \nu_0(1 + \frac{1}{c^2}\Delta V)$. Accepting a time-varying global gravitational potential in the Universe which changes uniformly (or almost uniformly), such that $\partial V/\partial t$ is a very small constant, the frequency of the received signal will be

$$\nu = \nu_0\Big(1 + \frac{t}{c^2}\frac{\partial V}{\partial t}\Big).$$

Specially, on a proper distance $R = ct$ in the rest frame of the observer, its frequency will be

$$\nu = \nu_0\Big(1 + \frac{R}{c^3}\frac{\partial V}{\partial t}\Big).$$

On the other hand, the Hubble law $v = RH$ in the classical Doppler formula gives

$$\nu = \nu_0\Big(1 - \frac{RH}{c}\Big),$$

where H is the Hubble constant, $H \approx 70$ km/s/Mpc. From the last two equalities, we obtain that

$$\frac{\partial V}{\partial t} = -c^2 H \approx -2 \times 10^3 \frac{\text{cm}^2}{\text{s}^3}. \qquad (A.1)$$

Let us denote by X, Y, Z, T the coordinates according to an observer in absence of time-dependent gravitational potential. This is an analogous situation conceptually used in gravitational theories: an observer far from the massive bodies, where the gravitation disappears. Since we assume that the time-dependent gravitational potential is present everywhere in the Universe, such an observer practically does not exist, but theoretically, we adopt its existence in the mentioned Machian-like manner. More precisely, dX, dY, dZ, and dT are the infinitesimal increments in the space-time coordinates at a considered point in presence of the time-dependent gravitational potential, according to the observer where the time-dependent gravitational potential is absent. After norming these 1-forms, according to the 1PN metric from the GR, we obtain the next 1-forms

$$w_x = \Big(1 + \frac{V}{c^2}\Big)^{-1} dX = \Big(1 + tH\Big)dX, \qquad (A.2)$$

$$w_y = \Big(1 + \frac{V}{c^2}\Big)^{-1} dY = \Big(1 + tH\Big)dY, \qquad (A.3)$$

$$w_z = \Big(1 + \frac{V}{c^2}\Big)^{-1} dZ = \Big(1 + tH\Big)dZ, \qquad (A.4)$$

$$w_t = \Big(1 - \frac{V}{c^2}\Big)^{-1} dT = \Big(1 - tH\Big)dT, \qquad (A.5)$$

Since the right sides of (A.2), (A.3), and (A.4) are not total differentials, the equations $dx = w_x$, $dy = w_y$, $dz = w_z$, and $dt = w_t$ are not solvable with respect to x, y, z, i.e. x, y, and z, are not functions of X, Y, Z, and T in general case (non-integrable Pfaffian system). Only t is a function of T and without loss of generality, we may assume that $t = T = 0$ at a chosen moment. Note that a chosen curve in the X, Y, Z, T space-time, corresponds to an unique curve in the x, y, z, t space-time. Thus, we agree to call x, y, z and t "normed coordinates", while dx, dy, dz, and dt make an orthonormal tetrad. So, along a chosen curve, instead of (A.2-5) we can write

$$dx = \Big(1 + tH\Big)dX, \qquad (A.2')$$

$$dy = \Big(1 + tH\Big)dY, \qquad (A.3')$$

$$dz = \Big(1 + tH\Big)dZ, \qquad (A.4')$$

$$dt = \Big(1 - tH\Big)dT, \qquad (A.5')$$

and operate using the differential calculus. The equations $(A.2')$, $(A.3')$, and $(A.4')$ are not equalities between 1-forms, but equalities along a chosen curve.

By accepting linear change of the gravitational potential in the Universe given by (A.1), the first and the most simple application is the explanation of the Hubble redshift directly from $(A.5')$. Moreover, the assumption of presence of the (linearly decreasing) time-dependent gravitational potential in the Universe provides general formula for all kinds of red/blue shifts.

We can continue to work with the last obtained system since it is sufficient to assume that $(A.2')$, $(A.3')$, $(A.4')$ and $(A.5')$ are satisfied (i.e. we can accept them as axioms). Thus, we may use tH instead of $\frac{V}{c^2}$. The coefficients $1+tH$ and $1-tH$ are linear because we work with small time intervals, so $tH \approx 0$. Thus, we can perform an approximation by neglecting H^2 and smaller quantities not losing much of the precision. From $(A.2')$, $(A.3')$, $(A.4')$, and $(A.5')$ we obtain

$$\Big(\frac{dX}{dT}, \frac{dY}{dT}, \frac{dZ}{dT}\Big) = \Big(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\Big)(1 - 2tH) \qquad (A.6)$$

and by differentiating this equality by T we have

$$\Big(\frac{d^2X}{dT^2}, \frac{d^2Y}{dT^2}, \frac{d^2Z}{dT^2}\Big) =$$
$$\Big(\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}, \frac{d^2z}{dt^2}\Big) - 3tH\Big(\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}, \frac{d^2z}{dt^2}\Big) -$$
$$-2H\Big(\frac{dX}{dT}, \frac{dY}{dT}, \frac{dZ}{dT}\Big) =$$
$$\Big(\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}, \frac{d^2z}{dt^2}\Big) - 3tH\Big(\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}, \frac{d^2z}{dt^2}\Big) -$$
$$-2H\Big(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\Big). \qquad (A.7)$$

In the normed coordinates x, y, z, t can not be observed any acceleration caused by the presence of the time-dependent gravitational potential, i.e. we can take there $H = 0$. Thus, according to X, Y, Z, T coordinates, there is an additional slight acceleration

$$-3tH\Big(\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}, \frac{d^2z}{dt^2}\Big) - 2\Big(H\frac{dX}{dT}, H\frac{dY}{dT}, H\frac{dZ}{dT}\Big).$$

Since x, y, z, t are not functions of X, Y, Z, T, we should perform a holonomic simulation, looking for a functional dependence of the form

$$x = (1 + \lambda tH)\bar{X},\ y = (1 + \lambda tH)\bar{Y},$$
$$z = (1 + \lambda tH)\bar{Z},\ dt = (1 - \mu tH)d\bar{T}, \qquad (A.8)$$

where $\lambda = const.$ and $\mu = const.$, and $\bar{X}, \bar{Y}, \bar{Z}, \bar{T}$ are the designations for the corresponding holonomic coordinates, which leads to the same acceleration (A.7). Namely, the simulation is consisted in obtaining the equation (A.7) by change of coordinates classically. This would give approximative (regarding our conditions) but observable and comparable results. From (A.8) we have

$$\Big(\frac{d^2\bar{X}}{d\bar{T}^2}, \frac{d^2\bar{Y}}{d\bar{T}^2}, \frac{d^2\bar{Z}}{d\bar{T}^2}\Big) =$$
$$(1 - (\lambda + 2\mu)TH)\Big(\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}, \frac{d^2z}{dt^2}\Big) -$$
$$-(2\lambda + \mu)H\Big(\frac{d\bar{X}}{d\bar{T}}, \frac{d\bar{Y}}{d\bar{T}}, \frac{d\bar{Z}}{d\bar{T}}\Big) =$$
$$(1 - (\lambda + 2\mu)TH)\Big(\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}, \frac{d^2z}{dt^2}\Big) -$$
$$-(2\lambda + \mu)H\Big(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\Big). \qquad (A.9)$$

Comparing the right sides of (A.7) and (A.9) we obtain $\lambda = \frac{1}{3}$, $\mu = \frac{4}{3}$.

More generally, we shall show now that the effects due to the nonholonomity, i.e. due to the time-dependent gravitational potential, depend only on the PPN parameter γ, which has value 1 in the GR. This value is the same in our framework since to the required approximation we use GR metric. Additionally, this value is confirmed by the experiments that use metric proportions (Shapiro time-delay). So, instead of the equations $(A.2' - 5')$ we have the equations

$$dx = \Big(1 + \gamma tH\Big)dX, \qquad (A.2'')$$

$$dy = \Big(1 + \gamma tH\Big)dY, \qquad (A.3'')$$

$$dz = \Big(1 + \gamma tH\Big)dZ, \qquad (A.4'')$$

$$dt = \Big(1 - ktH\Big)dT, \qquad (A.5'')$$

where γ is the well known PPN parameter, and k is an arbitrary constant. We assume that tH is extremely small, such that during a short period, the value of tH is almost a constant. Now analogously to (A.6) and (A.7) we obtain

$$\Big(\frac{dX}{dT}, \frac{dY}{dT}, \frac{dZ}{dT}\Big) = \Big(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\Big)\Big(1 - (k+\gamma)tH\Big) \qquad (A.6'')$$

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and

$$\Big(\frac{d^2X}{dT^2},\frac{d^2Y}{dT^2},\frac{d^2Z}{dT^2}\Big)=$$

$$\Big(\frac{d^2x}{dt^2},\frac{d^2y}{dt^2},\frac{d^2z}{dt^2}\Big)-(2k+\gamma)tH\Big(\frac{d^2x}{dt^2},\frac{d^2y}{dt^2},\frac{d^2z}{dt^2}\Big)-$$

$$-(k+\gamma)H\Big(\frac{dx}{dt},\frac{dy}{dt},\frac{dz}{dt}\Big). \qquad (A.7'')$$

Analogously to (A.8) we consider an adopted transformation (a "simulation")

$$x=\Big(1+\lambda tH\Big)\bar{X},\ y=\Big(1+\lambda tH\Big)\bar{Y},$$

$$z=\Big(1+\lambda tH\Big)\bar{Z},\ dt=\Big(1-\mu tH\Big)d\bar{T}, \qquad (A.8'')$$

where $\lambda=const.$ and $\mu=const.$ From $(A.8'')$ we obtain

$$\Big(\frac{d^2\bar{X}}{d\bar{T}^2},\frac{d^2\bar{Y}}{d\bar{T}^2},\frac{d^2\bar{Z}}{d\bar{T}^2}\Big)=$$

$$=\Big(1-(\lambda+2\mu)TH\Big)\Big(\frac{d^2x}{dt^2},\frac{d^2y}{dt^2},\frac{d^2z}{dt^2}\Big)-$$

$$-(2\lambda+\mu)H\Big(\frac{dx}{dt},\frac{dy}{dt},\frac{dz}{dt}\Big). \qquad (A.9'')$$

Hence, comparing $(A.7'')$ and $(A.9'')$ we obtain the system

$$2k+\gamma=\lambda+2\mu, \qquad k+\gamma=2\lambda+\mu,$$

whose solution is $\lambda=\frac{\gamma}{3}$ and $\mu=k+\frac{\gamma}{3}$.

The perturbation acceleration (the second and the third summand in $(A.7'')$)

$$-(2k+\gamma)tH\Big(\frac{d^2x}{dt^2},\frac{d^2y}{dt^2},\frac{d^2z}{dt^2}\Big)-(k+\gamma)H\Big(\frac{dx}{dt},\frac{dy}{dt},\frac{dz}{dt}\Big)$$

is observed by the observer in absence of the gravitational potential (or observer where the time-dependent gravitational potential remains constant). But, what is the perturbation acceleration observed by the observer in presence of the potential? To solve this problem, it is sufficient in the perturbation acceleration of $(A.9'')$, i.e.

$$-(\lambda+2\mu)TH\Big(\frac{d^2x}{dt^2},\frac{d^2y}{dt^2},\frac{d^2z}{dt^2}\Big)-(2\lambda+\mu)H\Big(\frac{dx}{dt},\frac{dy}{dt},\frac{dz}{dt}\Big),$$

to replace λ by $\lambda-\gamma=-\frac{2}{3}\gamma$ and μ by $\mu-k=\frac{\gamma}{3}$. Hence, for the required perturbation acceleration we get

$$\gamma H\Big(\frac{dx}{dt},\frac{dy}{dt},\frac{dz}{dt}\Big). \qquad (A.10)$$

If we replace H by $-\frac{1}{c^2}\frac{\partial V}{\partial t}$ we can express the perturbation acceleration via the potential:

$$-\gamma\frac{1}{c^2}\frac{\partial V}{\partial t}\Big(\frac{dx}{dt},\frac{dy}{dt},\frac{dz}{dt}\Big). \qquad (A.11)$$

We can express (A.10) in a simple form, $\vec{a}=\gamma\vec{v}H$. The meaning of this formula affects local Lorentz invariance, i.e. SEP(ii) and EEP(ii) [16].

According to (A.8) it is proved in [14] that the time-dependent gravitational potential has no influence to the perihelion precession phenomenon. Moreover, neglecting the relativistic corrections of the planetary orbits, it is shown in [14] that the planetary orbits are not axially symmetric and the angle from the perihelion to the aphelion is $\pi-\frac{\lambda H\Theta\sqrt{1-e^2}}{e\pi}=\pi-\frac{H\Theta\sqrt{1-e^2}}{3e\pi}$, while the angle from the aphelion to the perihelion is $\pi+\frac{\lambda H\Theta\sqrt{1-e^2}}{e\pi}=\pi+\frac{H\Theta\sqrt{1-e^2}}{3e\pi}$, where Θ is the orbital period and e is the eccentricity of the orbit. Notice that these angles are observed according to the observer in absence of time-dependent gravitational potential. For an observer in presence of time-dependent gravitational potential, these angles are $\pi+\frac{2H\Theta\sqrt{1-e^2}}{3e\pi}$ and $\pi-\frac{2H\Theta\sqrt{1-e^2}}{3e\pi}$. From (A.8) it follows that the quotient $\Theta_2:\Theta_1$ of two consecutive orbital periods is equal to $1+\mu\Theta H=1+\frac{4}{3}\Theta H$. This shows that each next orbit has a prolonged period for a factor $1+\frac{4}{3}\Theta H$. But our time Θ is also prolonged for a factor $1+\Theta H$ according to $(A.5')$. Thus, we measure that each next orbit is prolonged for the factor

$$\Theta_2:\Theta_1=\frac{1+\frac{4}{3}\Theta H}{1+\Theta H}=1+\frac{1}{3}\Theta H. \qquad (A.12)$$

Formula (A.12) can also be applied in case of obtaining the orbital period for arbitrary double stars [15]. From (A.12) we obtain

$$\dot{P}_b=\frac{1}{3}P_bH, \qquad (A.13)$$

where P_b is the orbital period of the considered star. In [15], formula (A.13) is tested for the binary pulsars B1885+09 [5] and B1534+12 [9, 10], which have very stable timings, and the results are satisfactory: The formula (A.13) together with the influence of the orbital period decay caused by the gravitational radiation and a non-gravitational influence of kinematic nature in the galaxy, gives the measured value of $\dot{P}_b$ [14],[15]. Note that if we neglect the influence from the Universe (A.13), then the change of the orbital period caused by the gravitational radiation would not well fit with the data. This should be very obvious in the cases of binaries with low eccentricity and long orbital period - the opposite type of the convenient relativistic subclass of pulsars [3], but since this type of binary systems is characterized as noninteresting for relativistic gravity, there is not substantial data for comparison.

According to the previous discussion it also follows that the distance to any object moving freely on an orbit, for example the distance Earth-Moon, increases by the coefficient $1+(1-\lambda)HT=1+\frac{2}{3}HT$. In [15] are also considered: the increasing of the orbital period of the Moon, the distance to the Moon and the change of the average Earth's angular velocity. These quantities depend on tidal dissipation and also on the time-dependent gravitational potential. The time-dependent gravitational potential decreases the data discrepancies of the previous three quantities [15].

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