Comparative Strength of Common Structural Shapes Using Genetic Algorithms

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***Abstract*—The motivation for this paper is to develop an approach to optimization of beam design. Under given loading and support conditions, the comparative strength of three (3) common structural shapes was determined. This led to the conclusion that a particular structural shape together with its dimensions will give the optimal solution in beam design in terms of the least cross-sectional area to support the given load, which would then translate to savings in cost and reduction in weight of the structural member. An investigation was also conducted to take into consideration the effect in the dimensions of the structural shapes of uncertainties due to manufacturing limitations and tolerances. This resulted in an assessment of the order of magnitude of this effect on the design variables. In solving the resulting optimization problems, MATLAB's Genetic Algorithm and Direct Search Toolbox was employed.**

***Index Terms*—Beam design, direct search, hybrid genetic algorithm, structural shapes, uncertainty.**

I. Introduction

In the design of a beam to support a particular loading, the structural engineer, after calculating bending moments, selects what he perceives to be the best structural shape and size that will satisfy the allowable stress and then checks the deflection for compliance with the stipulated value. This usually entails an iterative process employing empirical guidelines and trial sections to arrive at a satisfactory solution. Shown in Figure 1 are the structural shapes available to choose from.

Generally, the primary concern of the structural engineer is to satisfy the allowable maximum stress and deflection based on the given loading and support condition. His choice of the structural shape to use is largely discretionary on his part and may be influenced by his personal preference since most of the since most of the above shapes can be used to support the particular loading under design consideration. This paper aims to provide a tool for the structural engineer so that he can make a rational choice of structural shape to use in his design in order to attain the optimal solution of the least cross-sectional area satisfying simultaneously both stress and deflection specifications.

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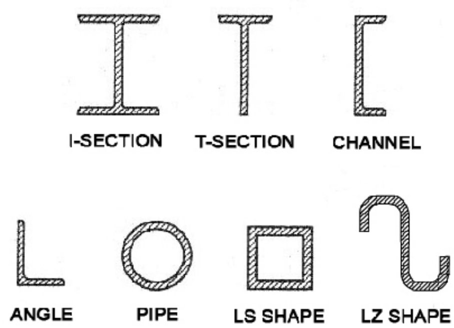

Figure 1. Common structural shapes.

We follow the approach of Hacker and Lewis [3] in formulating an optimization problem that seeks to minimize the cross-sectional area subject to stress and deflection constraints. We apply their approach to two other common shapes (Channel and T-Section) to be able to make a comparative study. We also explain how to handle the problem of uncertainties in the dimensions of the structural shapes due to manufacturing limitation and tolerances, which is a modified version of that presented in [3].

The optimization problems are solved using the Genetic Algorithm and Direct Search Toolbox of MATLAB.

II. Formulations and Results

We will present in detail the formulation of the problem for the particular case of the design of a T-Beam. The computations for the optimal beam design of other structural shapes can be patterned after this.

Figure 2 shows the loading and support conditions as well as the design variables (x_1 through x_4).

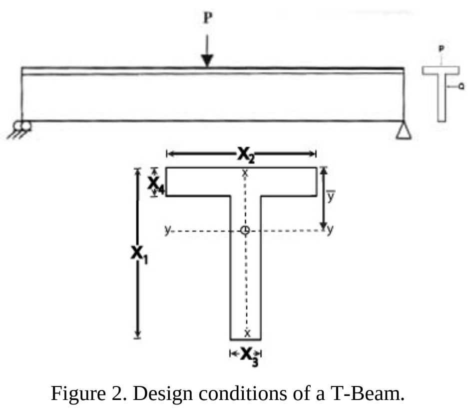

Figure 2. Design conditions of a T-Beam.

The following specifications are stipulated:

- Maximum allowable stress = 16 kN/cm^2
- Maximum allowable deflection = 0.10 cm
- Length (L) of the beam = 200 cm
- Loads: P = 75 kN vertical load and
  Q = 7.5 kN transverse force
- Young's Modulus of Elasticity (E) = 20,000 kN/cm^2
- Simply supported beam

These particular values are specified after taking into consideration the slightly different natures of the three structural shapes that we will compare in this study.

The design elements are calculated as follows:

- Cross sectional area: $A = x_2 x_4 + x_3(x_1 - x_4)$
- Neutral axis from top of section:

$$\bar{y} = \frac{\frac{1}{2}[x_2 x_4^2 + x_3(x_1^2 - x_4^2)]}{A}$$

- Moment of inertia about the horizontal neutral axis of section:

$$I_{yy} = \left[\tfrac{1}{12} x_2 x_4^3 + x_2 x_4 (\bar{y} - \tfrac{1}{2} x_4)^2\right] + \left[\tfrac{1}{12} x_3 (x_1 - x_4)^3 + x_3 (x_1 - x_4)(\tfrac{1}{2}(x_1 + x_4) - \bar{y})^2\right]$$

- Moment of inertia about the vertical neutral axis of section:

$$I_{xx} = \tfrac{1}{12}[(x_1 - x_4)x_3^3 + x_4 x_2^3]$$

- Maximum bending moment at center span due to P:

$$M_P = \tfrac{1}{4} PL = 3750 \text{ kNcm}$$

- Maximum bending moment at center span due to Q:

$$M_Q = \tfrac{1}{4} QL = 375 \text{ kNcm}$$

- Maximum combined stress due to P and Q:

$$\frac{M_P c_P}{I_{yy}} + \frac{M_Q c_Q}{I_{xx}} = \frac{3750\bar{y}}{I_{yy}} + \frac{187.5 x_2}{I_{xx}}$$

Having stated the specifications and computed the design elements, we now state the optimization problems. The first problem is without uncertainty while the second problem takes into consideration dimensional uncertainties arising from, say, manufacturing errors.

A. *The Optimization Problem: No Uncertainty*

We wish to minimize the cross-sectional area of the T-section while making sure that the stress and deflection are below the specified values. The dimensions of the section must also be within prescribed bounds, based on values given in the Steel Handbook [1].

More precisely, we wish to solve the following optimization problem:

Minimize $\quad A(x) = x_2 x_4 + x_3(x_1 - x_4)$,

where

$$5.28 \le x_1 \le 47.50$$
$$10.31 \le x_2 \le 42.49$$
$$0.71 \le x_3 \le 2.84$$
$$0.88 \le x_4 \le 5.11$$

Subject to:

(stress constraint) $\quad g_1(x) = \dfrac{3750\bar{y}}{I_{yy}} + \dfrac{187.5 x_2}{I_{xx}} \le 16$

(deflection constraint) $\quad g_2(x) = \dfrac{PL^3}{48EI_{yy}} = \dfrac{625}{I_{yy}} \le 0.1$

This problem was solved using MATLAB's Genetic Algorithm and Direct Search Toolbox with the following parameters: crossover fraction = 0.8, elite count = 2, generations = 100, mutation function = Gaussian, population size = 200, selection function = uniform, and convergence limit = 1e-6.

Similar optimization problems were also formulated for the Channel and I-Section, and also solved using MATLAB. The obtained results are summarized in this table:

TABLE 1. COMPARATIVE STRENGTH OF COMMON STRUCTURAL SHAPES

Dimensions	Channel	T-Section	I-Section
Area (cm^2)	35.62	41.12	43.61
x_1 (cm)	37.56	37.45	29.81
x_2 (cm)	9.44	17.15	13.40
x_3 (cm)	0.43	0.71	0.71
x_4 (cm)	1.08	0.88	0.88

As can be seen from the above table, the Channel requires the least cross-sectional area to support the loading under the particular support condition. Compared to the I-section, for instance, there is a savings of 22.4% in area which also translates to 22.4 % savings in cost as well as in weight of structural materials for the beam. This is the case when no uncertainty is being considered.

B. *The Optimization Problem: With Uncertainty*

When uncertainty is introduced into the beam dimensions, the optimization problem will have to be modified accordingly. Instead of locating the *point* at which the cross-sectional area is minimum, we will now find the *neighborhood* over which the objective function has (a) minimum weighted mean or (b) minimum variance.

Since evaluating the objective function at all points of a neighborhood is impossible, we will instead identify 33 representative points/vectors in it. We note that at this point, we deviate from the approach of Hacker and Lewis [3]. For instead of *a priori* identifying points in the neighborhood, they made use of Monte Carlo simulation to generate sample points.

The 33 representative points are:

- the center point $x = (x_1, x_2, x_3, x_4)$
- sixteen points in the outer tier of the form

$$(x_1 \pm \Delta_1, x_2 \pm \Delta_1, x_3 \pm \Delta_2, x_4 \pm \Delta_2)$$

- another sixteen points in the inner tier of the form

$$(x_1 \pm \tfrac{1}{2}\Delta_1, x_2 \pm \tfrac{1}{2}\Delta_1, x_3 \pm \tfrac{1}{2}\Delta_2, x_4 \pm \tfrac{1}{2}\Delta_2)$$

Note that the neighborhood is not spherical but rather rectangular in four dimensions. For ease in referencing the points, we denote the center point by P_0 and label the other points P_j, where *j* runs from 1 to 16 for the inner tier points and from 17 to 32 for the outer tier points. Note further that two different increments are being added to the components of the vector due to the big difference in the magnitudes of these dimensions. Based on values for error tolerances in the Steel Handbook [1], suitable values of these increments are $\Delta_1 = 0.5$ cm and $\Delta_2 = 0.05$ cm.

Case 1: The objective function as a weighted mean

In this case, we give weights to the tier points as follows: 6 for the center point, 4 for the inner tier points and 1 for the outer tier points. The 6-4-1 weighting scheme is chosen to approximate the normal distribution.

The objective function now becomes

$$F_{\text{mean}} = \frac{1}{86}\left[6A(P_0) + 4\sum_{j=1}^{16} A(P_j) + \sum_{j=17}^{32} A(P_j)\right].$$

Case 2: The objective function as variance

We give the same weights to the tier points as in the weighted mean case. However the objective function is now computed as the variance of the set of cross-sectional area values at the points P_0 to P_{33}, counting the weights (i.e., the area at P_0 is counted 6 times, the area at P_1 is counted 4 times, and so on).

In both cases, the constraints and the bounds are the same. Since we want the constraints to be satisfied even when there are slight deviations from the specified measurements, the stress and deflection constraints and will now have to be evaluated at all the 33 points. This gives rise to a total of 66 nonlinear constraints (32 for each tier and 2 for the center points), namely, for j = 0, 1, …, 33:

(stress constraint) $g_1(P_j) \le 16$,

(deflection constraint) $g_2(P_j) \le 0.1$.

The bounds are modified as follows:

$$5.28 + \Delta_1 \le x_1 \le 47.50 - \Delta_1$$

$$10.31 + \Delta_1 \le x_2 \le 42.49 - \Delta_1$$

$$0.71 + \Delta_2 \le x_3 \le 2.84 - \Delta_2$$

$$0.88 + \Delta_2 \le x_4 \le 5.11 - \Delta_2$$

Note that with these bounds, the obtained center and tier points after minimization would still satisfy the original bounds set for the dimensions of the T-section.

The optimization problem with uncertainty for the T-beam, and subsequently for the Channel and I-section, are then solved using MATLAB. The results are shown in the following tables:

TABLE 2. RESULTS OF MINIMIZING THE WEIGHTED MEAN OF THE CROSS-SECTIONAL AREA.

Dimensions	Channel	T-Section	I-Section
Area (cm^2)	41.52	44.60	47.66
x_1 (cm)	35.78	37.94	30.36
x_2 (cm)	8.93	17.55	13.83
x_3 (cm)	0.48	0.76	0.76
x_4 (cm)	1.44	0.94	0.94

TABLE 3. RESULTS OF MINIMIZING THE VARIANCE OF THE CROSS-SECTIONAL AREA.

Dimensions	Channel	T-Section	I-Section
Area (cm^2)	63.82	67.02	60.74
x_1 (cm)	29.98	31.02	28.53
x_2 (cm)	8.94	15.56	12.57
x_3 (cm)	1.44	1.61	1.30
x_4 (cm)	1.38	1.22	1.06

Note that compared to the values given in Table 1 where no uncertainty was introduced, the dimensional uncertainties resulted in the increase of cross-sectional areas, as may be expected. There is, however, a marked difference between minimizing the weighted mean and minimizing the variance of the fitness function in the results.

In Table 2, the increase in cross-sectional area varied from section to section: 16.6% increase for Channel, 8.5 % for T-Section and 9.3% for I-Section. In Table 3, on the other hand, the increase is much larger: 73.6% for Channel, 63.0% for T-Section and 39.2% for I-Section. Note that in the dimensions in Table 3, there is a significant decrease in x_1 and x_2, while x_3 and x_4 increase in all sections. Evidently, there is a wide berth between the results of minimizing the mean and minimizing the variance of the fitness function.

Minimizing the variance tends to move the solution far away from the optimal (no uncertainty) results. Although of academic interest, it has no practical value. Minimizing the weighted mean of the fitness function is, therefore, the preferred option as it is realistic and stays within the periphery of the optimal solution.

By way of recapitulating the results obtained, Fig. 3 illustrates graphically the three (3) T-Beam solutions, namely, (a) no uncertainty; (b) with uncertainty, minimizing the weighted mean; and (c) with uncertainty, minimizing the variance.

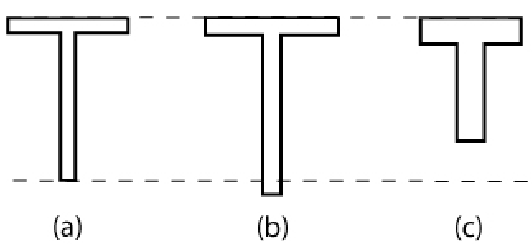

Figure 3. The three T-Beam solutions.

III. Conclusions and Future Work

The approach presented in this paper showed that through the use of Genetic Algorithms in comparative strength assessment of common structural shapes, a methodology for definitive selection of structural shape and its appropriate proportions (x_1 through x_4) can be developed to arrive at an optimal solution in beam design. The results yielded the least cross-sectional area to carry the given loading and support conditions. This will translate to savings in the cost of structural materials and to lighter structural members. The intended beneficiary of this work is the structural engineer who may wish to optimize his beam design instead of just satisfying stress and deflection specifications as heretofore. The foregoing optimization process is rather simple and straightforward and can be carried out easily by a structural engineer employing genetic algorithms.

Moreover, the structural engineer is made aware of the effects of uncertainties on his design variables. Although sensitivity to the effects of dimensional uncertainties on account of manufacturing limitations vary among the structural shapes studied, perception of the order of magnitude of the effects can be gathered from the Study. For the case at hand, the effect ranges from 8% to 16% increase in cross-sectional area if minimizing the weighted mean of the fitness function is used, and 39% to 74% if minimizing the variance is employed. In practice, the structural engineer would consider the former option a reasonable allowance as a result of introducing the dimensional uncertainties. The authors, moreover, consider minimizing the variance as untenable because it drifts far away from the optimal solution and negates whatever benefits that may accrue from optimizing the beam design.

The application of this approach can be further widened by assessing the strength of the other structural shapes not included in this paper. Additionally, the approach presented can be expanded to increase its scope and utility by investigating the effect of other uncertainties, such as variations in the composition of the structural material (the case of recycled materials), and that of corrosion on beams spanning bodies of water, especially salt water, as well as other uncertainties due to uncontrollable factors.

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