Unsteady Free Convective Flow of a Temperature Varying Electrically Conducting Fluid

Krishna Gopal Singha and P. N. Deka

***Abstract*—An unsteady viscous incompressible free convective flow of an electrically conducting fluid between two heated vertical parallel plates is considered in presence of an induced magnetic field applied transversely to the flow. Assuming that the magnetic field induces a field along the lines of motion which varies transversely to the flow and the fluid temperature changing with time analytical solutions for velocity, induced magnetic field and temperature distributions are obtained for small and large magnetic Reynolds numbers. The skin friction on the two plates are obtained and plotted graphically. The problem has also been solved for thermometric case i.e. when the lower plate is adiabatic.**

Index Terms— **Hartmann number, induced magnetic field, magnetic Reynolds number, skin friction, unsteady free convective flow.**

I. INTRODUCTION

Borkakati and Srivastava[1] investigated free and forced convection and MHD flow. In a fluid, the variation of temperature causes variation of density. This in turn raises force of buoyancy which governs the fluid motion. This type of unsteady fluid motion under the action of uniform magnetic field applied externally reduces the heat transfer and the skin friction considerably. This process of reduction of heat transfer and skin friction of the fluid motion has various engineering applications such as nuclear reactor, power transformation etc. Borkakati and Chakraborty[2] investigated the nature and behaviour of a viscous, incompressible, electrically conducting fluid over a flat plate which is moving with a uniform speed in a quiescent fluid in presence of a uniform magnetic field. In their conclusion they have found that for an incompressible fluid, both the fluid velocity and temperature gradually decreases with the increase of viscosity parameter. Elbashbeshy[3] studied heat and mass transfer in the same problem in presence of variable transverse magnetic field. The unsteady problem in a channel was studied numerically by Attia[4] with temperature dependence viscosity. He also considered steady state solution for velocity and temperature. In his study he analyzed the effect of viscosity parameter defined as ratio of viscosity of the fluid at two different temperatures. In the recent years, Attia[5] studied an unsteady magnetohydrodynamic flow and heat transfer problem of dusty fluid between two parallel plates with variable physical properties. Takhar[6] considered the effect of radiation on free convection flow along semi-infinite vertical plate in presence of transverse magnetic field. Very recently Singha[7] investigated the effect of heat transfer on unsteady hydromagnetic flow in a parallel-plate channel of an electrically conducting, viscous, incompressible fluid. He found that velocity distribution increases near the plates and then decreases very slowly at the central portion between the two plates. The principal numerical results presented in his work showed that the flow field is appreciably influenced by the applied magnetic field. Gourla and Katoch[8] discussed an unsteady free convection flow through the vertical parallel plates in the presence of uniform magnetic field.

In this paper, we are investigating the fully developed free convection laminar flow of an incompressible viscous electrically conducting fluid between two vertical parallel plates in the presence of a uniform induced magnetic field applied transversely to the flow. This induces a field along the lines of motion which varies transversely to the flow. The temperature of the fluid motion is assumed to be changing with time. The analytical solutions for velocity, induced magnetic field and the temperature distributions are obtained for small and large magnetic Reynolds numbers, R_m. The skin frictions at the two plates are obtained for different magnetic field parameters and are plotted graphically. The rate of heat transfer are also obtained and are plotted graphically. The problem has also been solved for thermometric case i.e. when the lower plate is adiabatic.

II. FORMULATION OF THE PROBLEM

We are considering an unsteady laminar convective flow of a viscous incompressible electrically conducting fluid between two vertical parallel plates. Let X -axis be taken along vertically upward direction through the central line of the channel and Y -axis is taken perpendicular to the X -axis. The plates of the channel are at $y = \pm h$ .The uniform magnetic field $\vec{B_0}$ is applied parallel to Y -axis and the induced field so produced is along X -axis that varies along Y -axis. The velocity and magnetic field distributions are $\vec{V} = \left[u(y), 0, 0\right]$ and $\vec{B} = \left[B(y), B_0, 0\right]$ respectively. Here B_0 and $B(y)$ are applied and induced magnetic field respectively.

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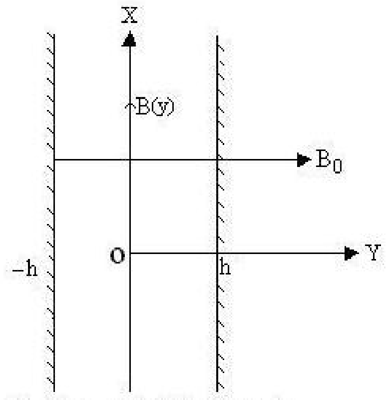

Fig. 1 Geometrical Configuration

In order to derive the governing equations of the problem the following assumptions are made

(i) the fluid is finitely conducting and the viscous dissipation and the Joule heat are neglected
(ii) Hall effect and polarization effect are negligible
(iii) initially (i.e. at time $t = 0$) the plates and the fluid are at zero temperature (i.e. $T = 0$) and there is no flow within the channel
(iv) at time $t > 0$, the temperature of the plate ($y = \pm h$) change according to $T = T_0(1 - e^{-nt})$, where T_0 is a constant temperature and $n \geq 0$ is a real number, denoting the decay factor and
(v) the plates are considered to be infinite and all the physical quantities are functions of y and t only.

III. Governing Equations

Under the above assumptions the non-dimensional governing equations are as follows:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + \left(\frac{R_a}{P_r R_e}\right)\overline{T} + \left(\frac{M^2}{R_e R_m P_r}\right)\frac{\partial b}{\partial y} \quad (1)$$

$$\frac{\partial b}{\partial t} - R_e \frac{\partial u}{\partial y} - \left(\frac{1}{R_m P_r}\right)\frac{\partial^2 b}{\partial y^2} = 0 \quad (2)$$

$$\frac{\partial \overline{T}}{\partial t} = \frac{1}{P_r}\frac{\partial^2 \overline{T}}{\partial y^2} \quad (3)$$

The following non-dimensional terms are used:

$t^* = \nu t / h^2$, $b = B / B_0$, $y^* = y / h$,
$u^* = u / u_0$,where $u_0 = (\beta g T_0 h^2) / \nu$,
$\overline{T} = (T_0 - T) / T_0$, (4)

The asterisks have been dropped with the understanding that all the quantities are now dimensionless.

where $M = \sqrt{(B_0{}^2 h^2 \sigma) / (\rho \nu)}$ is the Hartmann number,
$P_r = \nu / \alpha_1$ is the Prandtl number,
$R_e = (u_0 h) / \nu$ is the Reynolds number,
$R_a = (\beta g h^3 T_0) / (\nu \alpha_1)$ is the Rayleigh number,
$R_m = \alpha_1 \mu_e \sigma$ is the Magnetic Reynolds number,
$\alpha_1 = k / (\rho C_p)$ α_1 is the thermal diffusivity,
k is the thermal conductivity,
$\nu_e = 1 / (\sigma \mu_e)$ is the magnetic diffusivity,
$\nu = \mu / \rho$ is the kinetic viscosity,
σ is the electrical conductivity, ρ is the fluid density,
μ_e is the permeability of the medium and
μ is the co-efficient of viscosity.

The non-dimensional boundary condition becomes

$$\left.\begin{array}{l} t = 0 : u = 0 , b = 1 , \overline{T} = 1 \text{ at } y = \pm h \\ t > 0 : u = 1 , b = B' / B_0 , \overline{T} = e^{-nt} \text{ at } y = \pm h \end{array}\right\} \quad (5)$$

IV. Solutions

To solve (1) to (3) subject to the boundary condition (5), we apply the transformation of variables

$$u = f(y)e^{-nt} , \; b = g(y)e^{-nt} \text{ and } \overline{T} = \phi(y)e^{-nt} . \quad (6)$$

Substituting (6) in (1-3), we have from (1)

$$\frac{\partial^2 f}{\partial y^2} + nf + \left(\frac{R_a}{P_r R_e}\right)\phi + \left(\frac{M^2}{R_e R_m P_r}\right)\frac{\partial g}{\partial y} = 0 . \quad (7)$$

From (2)

$$\frac{\partial^2 g}{\partial y^2} + (nR_m P_r)g + (R_e R_m P_r)\frac{\partial f}{\partial y} = 0 \quad (8)$$

From (3)

$$\frac{\partial^2 \phi}{\partial y^2} + (nP_r)\phi = 0 \quad (9)$$

The corresponding boundary conditions are :

$$\left.\begin{array}{l} \text{for } t = 0 : f = 0, g = 1, \varphi = 1 \text{ at } y = \pm 1 \\ \text{for } t > 0 : f = e^{nt} , g = \left(B' / B_0\right)e^{nt} , \varphi = 1 \text{ at } y = \pm 1 \end{array}\right\} \quad (10)$$

The solutions of (7-9) subject to the boundary conditions (10) are

$$\phi(y) = \frac{\cos(ay)}{\cos a} \quad (11)$$

$$\begin{aligned} f(y) = {} & C_1\left(\cosh(\alpha y) - \sinh(\alpha y)\right) + C_2\left(\cosh(\alpha y) + \sinh(\alpha y)\right) \\ & + C_3\left(\cosh(\beta y) - \sinh(\beta y)\right) + C_4\left(\cosh(\beta y) + \sinh(\beta y)\right) \\ & - A_8 \cos(ay) \end{aligned} \quad (12)$$

$$\begin{aligned} g(y) = {} & C_5 \sin\left(y\sqrt{A_3}\right) + C_6 \cos\left(y\sqrt{A_3}\right) + \frac{1}{A_9}(A_4(a\sin(ay)) \\ & (\alpha^2 + A_3)(\beta^2 + A_3)A_8 + (a^2 - A_3)(\alpha(\cosh(\alpha y)) \\ & - \sinh(\alpha y))(\beta^2 + A_3)(C_1 - (\cosh(2\alpha y) + \sinh(2\alpha y))C_2) \\ & + \beta(\cosh(\beta y) - \sinh(\beta y))(\alpha^2 + A_3)C_3 - \beta(\cosh(\beta y) \\ & + \sinh(\beta y))(\alpha^2 + A_3)C_4))) \end{aligned} \quad (13)$$

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where

$$a=\sqrt{nP_r}\ ,\quad A_1=R_a/(P_rR_e)\ ,$$
$$A_2=M^2/(R_eR_mP_r)\ ,$$
$$A_3=nR_mP_r\ ,\quad A_4=R_eR_mP_r\ ,\quad A_5=A_1\sec a\ ,$$
$$A_6=n+A_3-A_2A_4,\quad A_7=A_3A_5-a^2A_5\ ,$$
$$A_8=A_7/(a^4+nA_3-a^2A_6)\ ,$$
$$\alpha=\left\{\sqrt{-A_6-\sqrt{-4nA_3+A_6^2}}\right\}/\sqrt{2}\ ,$$
$$\beta=\left\{\sqrt{-A_6+\sqrt{-4nA_3+A_6^2}}\right\}/\sqrt{2}\ ,$$
$$A_9=(a^2-A_3)(\alpha^2+A_3)(\beta^2+A_3),$$
$$C_1=-\frac{\cos a\left(A_7+(-a^4+\beta^4+(a^2+\beta^2)A_6)A_8\right)}{2\cosh\alpha(\alpha^2-\beta^2)(\alpha^2+\beta^2+A_6)}\ ,$$
$$C_2=-\frac{\cos a\left(A_7+(-a^4+\beta^4+(a^2+\beta^2)A_6)A_8\right)}{2\cosh\alpha(\alpha^2-\beta^2)(\alpha^2+\beta^2+A_6)}\ ,$$
$$C_3=-\frac{\cos a\left(A_7+\left(-a^4+\alpha^4+(a^2+\alpha^2)A_6\right)A_8\right)}{2\cosh\beta(\alpha^2-\beta^2)(\alpha^2+\beta^2+A_6)}\ ,$$
$$C_4=-\frac{\cos a\left(A_7+\left(-a^4+\alpha^4+(a^2+\alpha^2)A_6\right)A_8\right)}{2\cosh\beta(\alpha^2-\beta^2)(\alpha^2+\beta^2+A_6)}\ ,$$
$$C_5=-\frac{1}{A_9}(\operatorname{cosec}(\sqrt{A_3})A_4(a\sinh a(\alpha^2+A_3)$$
$$(\beta^2+A_3)A_8+(a^2-A_3)(-\alpha\sinh\alpha(\beta^2+A_3)$$
$$(C_1+C_2)-\beta\sinh\beta(\alpha^2+A_3)(C_3+C_4)))),$$
$$C_6=-\frac{1}{2A_9}\sec(\sqrt{A_3})\{2(a^2-A_3)A_4(\alpha\cosh\alpha(\beta^2+A_3)$$
$$(C_1-C_2)+\beta\cosh\beta(\alpha^2+A_3)(C_3-C_4))-2A_9\},$$

V. Thermometric Case

Let us assume that the lower plate is adiabatic i.e. the plate $y=-1$ is adiabatic (thermally insulated wall). The boundary conditions in the thermometric case are

$$\left.\begin{aligned} f=0,\ g=1,\ \varphi=1 \ \text{ at } \ y=+1\\ f=0,\ g=1,\ \varphi'=0 \ \text{ at } \ y=-1\end{aligned}\right\}\tag{14}$$

The solutions of (7-9) subject to the boundary conditions (14) are given by

$$\phi_t(y)=\frac{\cos\{a(1+y)\}}{\cos(2a)}\tag{15}$$

$$\begin{aligned}f_t(y)&=C_7\left(\cosh(\alpha y)-\sinh(\alpha y)\right)+C_8\left(\cosh(\alpha y)+\sinh(\alpha y)\right)\\&+C_9\left(\cosh(\beta y)-\sinh(\beta y)\right)+C_{10}\left(\cosh(\beta y)+\sinh(\beta y)\right)\\&-A_8\cos(a(1+y))\end{aligned}\tag{16}$$

$$g_t(y)=C_{11}\sin\left[y\sqrt{A_3}\right]+C_{12}\cos\left[y\sqrt{A_3}\right]+\frac{a\sin[a(1+y)]A_4A_8}{a^2-A_3}$$
$$-\frac{\sinh[\alpha y]A_9+\cosh[\alpha y]A_{10}}{\alpha^2+A_3}\frac{\sinh[\beta y]A_{11}+\cosh[\beta y]A_{12}}{\beta^2+A_3}\tag{17}$$

where

$$A_5=A_1\sec[2a]\ ,\quad A_9=\alpha A_4C_7+\alpha A_4C_8,$$
$$A_{10}=\alpha A_4C_8-\alpha A_4C_7\ ,$$
$$A_{13}=2(A_3-a^2)(A_3+\alpha^2)(A_3+\beta^2)\ ,$$
$$A_{14}=A_7+(\beta^4-a^4+(a^2+\beta^2)A_6)A_8\ ,$$
$$A_{15}=2(\alpha^2-\beta^2)(\alpha^2+\beta^2+A_6)\ ,$$
$$A_{16}=A_7+(\alpha^4-a^4+(a^2+\alpha^2)A_6)A_8\ ,$$
$$C_7=-\frac{A_{14}(1+\coth[a]\sin[a]^2+\cos[a]^2\tanh[\alpha])}{A_{15}(\cosh[\alpha]+\sinh[\alpha])}\ ,$$
$$C_8=-\frac{(\cos[a]^2\operatorname{sech}[\alpha]-\operatorname{cosech}[\alpha]\sin[a]^2)A_{14}}{A_{15}}$$
,
$$C_9=-\frac{(\cos[a]^2\operatorname{sech}[\beta]+\operatorname{cosech}[\beta]\sin[a]^2)A_{16}}{A_{15}},$$
$$C_{10}=-\frac{2\operatorname{cosech}[2\beta]\left(\cosh[\beta]\sin[a]^2-\cos[a]^2\sinh[\beta]\right)A_{16}}{A_{15}}\ ,$$
$$C_{11}=\frac{1}{A_{13}}(\operatorname{cosec}[\sqrt{A_3}](a\sin[2a](\alpha^2+A_3)(\beta^2+A_3)A_4A_8-2(a^2-A_3)$$
$$(\sinh[\alpha](\beta^2+A_3)A_9+\sinh[\beta](\alpha^2+A_3)A_{11})))$$
$$C_{12}=\frac{1}{A_{13}}(\sec[\sqrt{A_3}](a\sin[2a](\alpha^2+A_3)(\beta^2+A_3)A_4A_8-2(a^2-A_3)$$
$$(\cosh[\alpha](\beta^2+A_3)A_{10}+\cosh[\beta](\alpha^2+A_3)A_{12})))$$

VI. Skin Friction

The skin friction at the plates $y=\pm1$, is defined as

$$\tau=-\left[\mu\frac{du}{dy}\right]_{\pm1}\tag{18}$$

The non-dimensional skin friction after removing the asterisks takes the form:

$$\tau=-\left(\frac{\mu\beta gT_0h}{\nu}\right)\left[\frac{\partial u}{\partial y}\right]_{y=\pm1}\tag{19}$$

using (16),we get

$$\tau=-\left(\frac{\mu\beta gT_0h}{\nu}\right)\left[\frac{df}{dy}e^{-nt}\right]_{y=\pm1}\tag{20}$$

VII. Skin Friction for Thermometric Case

The skin friction at the plates $y=\pm1$, is defined as

$$\tau=-\left[\mu\frac{du}{dy}\right]_{\pm1}\tag{21}$$

The skin friction in the non-dimensional form for thermometric case is given by

$$\tau_t=-\left(\frac{\mu\beta gT_0h}{\nu}\right)\left[\frac{df_t}{dy}e^{-nt}\right]_{y=\pm1}\tag{22}$$

VIII. Results and Discussion

The velocity distribution f against the distance from the fixed plates y are plotted at different values of magnetic Hartmann number (M) for small and large magnetic Reynolds number (R_m) in the Fig. 1(a) to Fig. 1(d).

On the basis of same consideration Fig. 2(a) to Fig. 2(d) and Fig. 3(a) to Fig. 3(b) are plotted. The following fluid parameters are used:

$R_a = 1.0$, $R_e = 1.0$, $P_r = 0.71$, $n = 1.0$. All these plotting are done by using MATHEMATICA V 3.0.

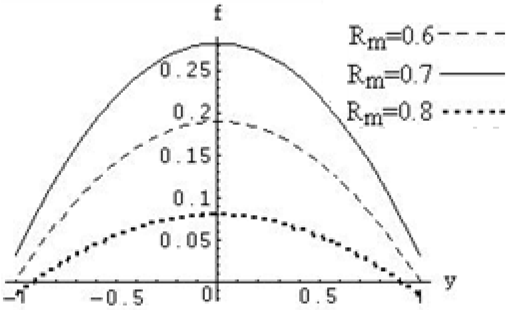

Fig.-1(a) Velocity distribution for small Rm at
R_a=1.0; M=1.5; R_e=1.0; n=1.0; P_r=0.71;

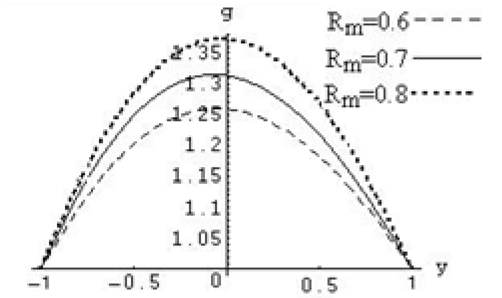

Fig.-2(a) Induced magnetic field distribution for small Rm at
R_a=1.0; M=1.5; R_e=1.0; n=1.0; P_r=0.71;

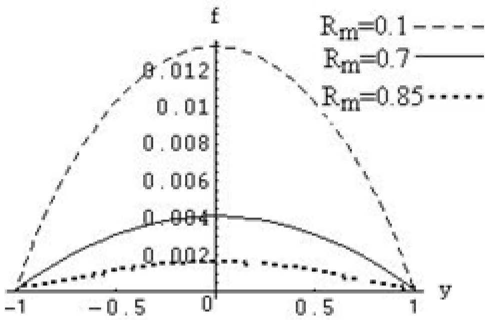

Fig.-1(b) Velocity distribution for small Rm at
R_a=1.0; M=7.5; R_e=1.0; n=1.0; P_r=0.71;

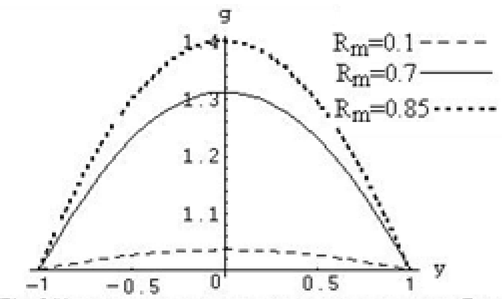

Fig.-2(b) Induced magnetic field distribution for small Rm at
R_a=1.0; M=7.5; R_e=1.0; n=1.0; P_r=0.71;

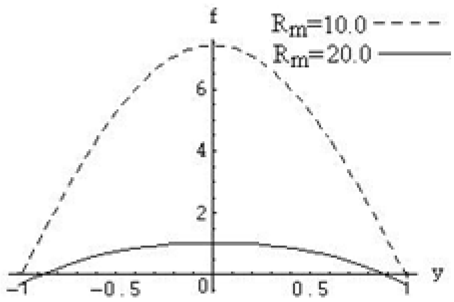

Fig.-1(c) Velocity distribution for large Rm at
R_a=1.0; M=1.5; R_e=1.0; n=1.0; P_r=0.71;

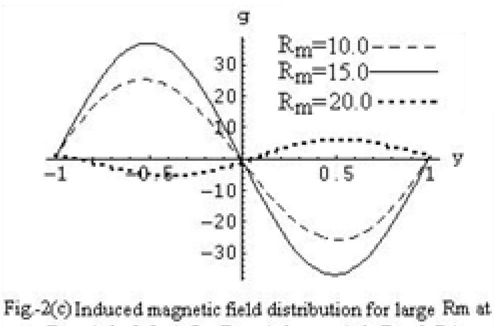

Fig.-2(c) Induced magnetic field distribution for large Rm at
R_a=1.0; M=1.5; R_e=1.0; n=1.0; P_r=0.71;

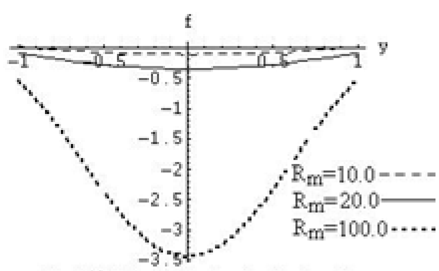

Fig.-1(d) Velocity distribution for large Rm at
R_a=1.0; M=7.5; R_e=1.0; n=1.0; P_r=0.71;

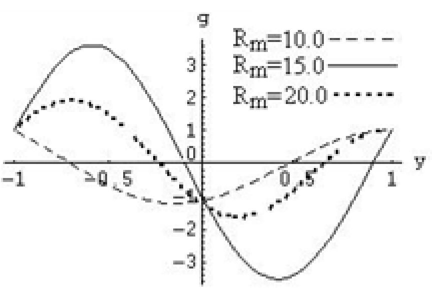

Fig.-2(d) Induced magnetic field distribution for large Rm at
R_a=1.0; M=7.5; R_e=1.0; n=1.0; P_r=0.71;

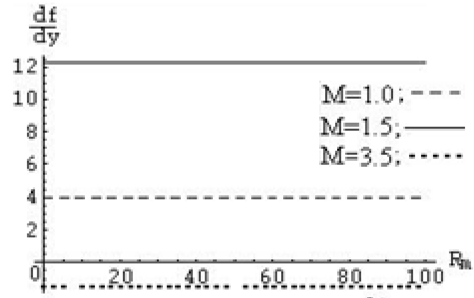

Fig.-3(a) Variation of friction factor $\frac{df}{dy}$ in non-thermometric case at y=+1

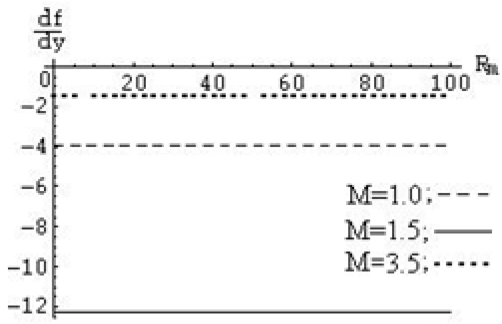

Fig.-3(b) Variation of friction factor $\frac{df}{dy}$ in non-thermometric case at y=-1

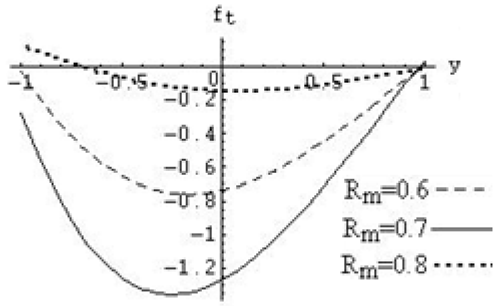

Fig.-4(a) Velocity distribution (f_t) in thermometric case for small Rm at R_a=1.0; M=1.5; R_e=1.0; n=1.0; P_r=0.71;

The velocity and induced magnetic field distributions in thermodynamic case are shown in Fig.4(a-d) to Fig.6(a-b).

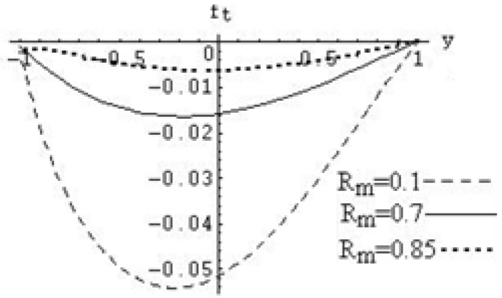

Fig.-4(b) Velocity distribution (f_t) in thermometric case for small Rm at R_a=1.0; M=7.5; R_e=1.0; n=1.0; P_r=0.71;

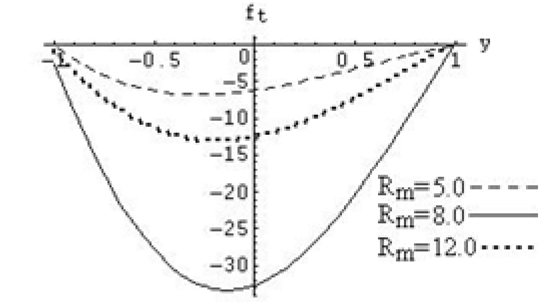

Fig.-4(c) Velocity distribution (f_t) in thermometric case for large Rm at R_a=1.0; M=1.5; R_e=1.0; n=1.0; P_r=0.71;

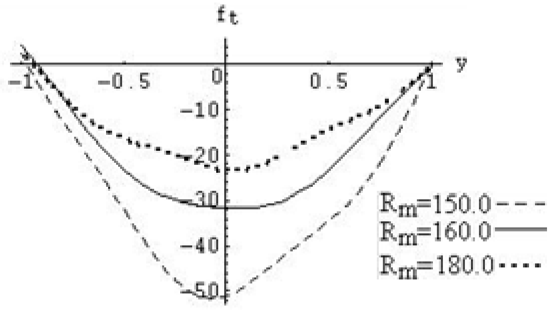

Fig.-4(d) Velocity distribution (f_t) in thermometric case for large Rm at R_a=1.0; M=7.5; R_e=1.0; n=1.0; P_r=0.71;

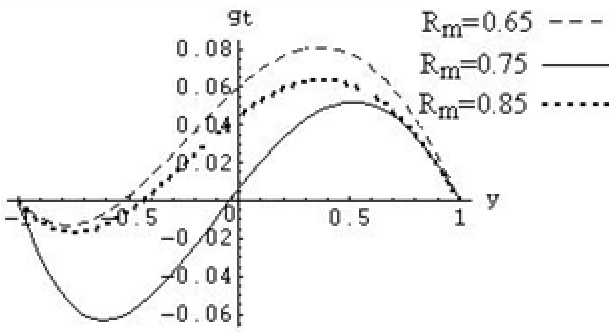

Fig.-5(a) Induced magnetic field distribution for small Rm at R_a=1.0; M=1.5; R_e=1.0; n=1.0; P_r=0.71;

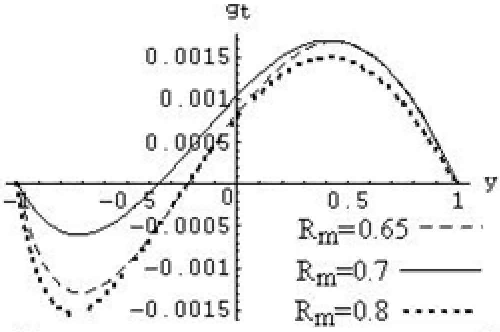

Fig.-5(b) Induced magnetic field distribution for small Rm at R_a=1.0; M=7.5; R_e=1.0; n=1.0; P_r=0.71;

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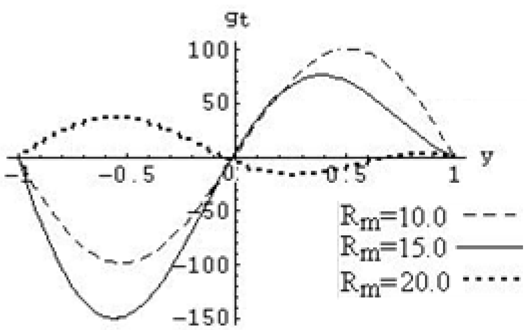

Fig.-5(c) Induced magnetic field distribution for large Rm at R_a=1.0; M=1.5; R_e=1.0; n=1.0; P_r=0.71;

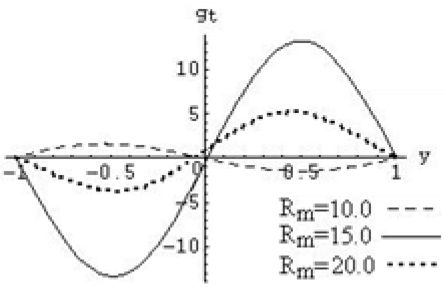

Fig.-5(d) Induced magnetic field distribution for large Rm at R_a=1.0; M=7.5; R_e=1.0; n=1.0; P_r=0.71;

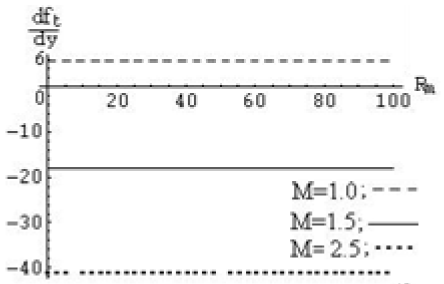

Fig.-6(a) Variation of friction factor $\frac{df_t}{dy}$ in the thermometric case at y= +1

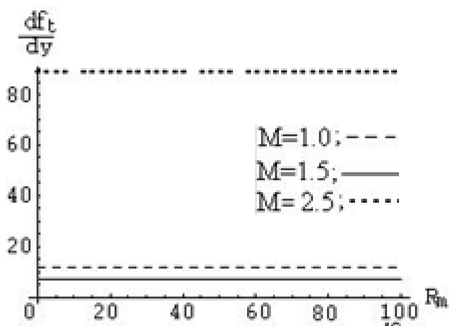

Fig.-6(b) Variation of friction factor $\frac{df_t}{dy}$ in the thermometric case at y= -1

The skin frictions at the plates $y = \pm 1$ in the non-thermometric and thermometric cases are shown in Fig.3(a-b) and Fig.6(a-b) for different values M and R_m.The results obtained from Fig. 1(a-d) to Fig. 6(a-b) are as follows:

From Fig 1(a-d) and Fig.(a-d) 4 it is observed that the velocity distributions in thermometric case are almost opposite in nature to those in non-thermometric case. Further, it is also observed that velocities at the central plane of the channel in both the cases are maximum but opposite in direction. In Fig. 1(d) it is observed that in the non-thermometric case the velocity gradually decreases with the increase of R_m. But in Fig. 4(d) it is observed that in the thermometric case the velocity gradually increases with the increase of R_m.

In Fig. 2(a-d) and Fig.5(a-d) the induced magnetic field strength are plotted against distance from the plates at point equal distance from the plates and at points on the plates. It is observed that the induced field strength in thermometric case are almost opposite in nature to those in non-thermometric case.

The effect of M and R_m on the frictional factor at the plates in thermometric case are almost opposite in nature to those in non-thermometric case. From Fig.3(a-d) and Fig.6(a-b) it is observed that the skin-friction first increases then gradually decreases with the increase of M while in thermometric case skin-friction gradually decreases with the increase of M at $y = \pm 1$.

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