Diameter Constrained Network

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***Abstract*—These The Diameter-Constrained Minimum Spanning Tree Problem is about finding a minimum cost spanning tree, subjected to pre-defined constraints on the number of edges that can integrate the path between any pair of nodes. This problem typically models network design applications where all vertices must communicate with each other at a minimum cost, while meeting a given quality requirement. This is the typical situation addressed in an environment of Multi-Protocol Label Switching networks over Wavelength Division Multiplexing network design, in which an IP travelling pack goes through a delay queuing, proportional to the number of Label Switching Routers that it crosses. In this paper we describe different types of heuristics that were implemented in order to find optimal or near-optimal solutions to solve this problem.**

***Index Terms*— Diameter constrained spanning tree, heuristics, graphs, network optimization.**

I. Introduction

The Diameter-constrained Minimum Spanning Tree (DMST) is formally defined in an undirected graph G=(V, E), where V is the node set and E represents the edge set. A cost function c is associated to this graph and is obtained by the sum of the individual costs associated with each edge $e \in E$. The last parameter is D which defines the diameter bound. The problem consists of finding a tree at a minimum c-cost connecting the nodes with a diameter at most D.

Computing a Minimum Spanning Tree (MST) is one of the best-known network optimization problems. However, in most practical applications, the MST must also satisfy some additional constraints, like a bound on the degree or a bound on the diameter of the tree [1], which often makes the problem NP-hard. The latter, defined as the Diameter Minimal Spanning Tree (DMST) problem, is considered NP-hard (except when the diameter bound is at most 3) since it contains, as a particular case, a NP-hard version of the Simple Uncapacitated Facility Location Problem [2], [3]. Common applications of DMST can be found in the optimal design of centralized telecommunication networks with quality of service constraints, as well as in the context of Multi-Protocol Label Switching (MPLS) over Wavelength Division Multiplexing (WDM) network design, where an IP travelling packet from ingress to the egress nodes undergoes a queuing delay in the transmitting interface of each LSR it crosses. The diameter constraint defines the maximum length for every connecting path preventing transmission impairments.

Due to the complexity of the most combinatorial optimization problems, which includes DMST, exact algorithms need exponential run time to reach the optimal solution, and become useless when the size of the problems grows. For that reason, the objective of researchers is to develop heuristic methods to solve efficiently these problems and find satisfactory solutions.

In this paper we describe the constructive Heuristics that had been developed to solve the problem under study, which follows the procedure suggested by Prim to generate a Minimum Spanning Tree, with an additional test for the edge inclusion. We found that better solutions were obtained if this procedure is run n times, since it depends on the starting vertex. The heuristics developed to solve the problem followed a strategy based on the relations that can be established between this problem and the hop constrained minimal spanning tree.

In order to compare different approaches was also developed a heuristic that works on an expanded graph where each edge is a feasible path in the original network that satisfies the length restriction. Different solutions were also obtained with the use of random elements on these heuristic procedures, usually named greedy heuristics (instead of adding the best choice at any stage, the element added is chosen randomly from the best p available edges). Finally a genetic algorithm was also developed for this problem, in order to test the efficiency of this type of approach.

II. Hop Constrained Minimal Spanning Tree Problem

The Diameter Minimal Spanning Tree (DMST) being problem a generalization of the well known Hop-constrained Minimal Spanning Tree (HMST) problem, it is important to study and develop solutions to this particular problem. Its diverse structure only considers distance between one given node and all the other remains nodes, where the hop constrained limits the number of arcs in the connecting path from the root node and any other node.

The motivation for describing the HMST problem together with the DMST is given in the two following results:

Result 1: T is a tree of diameter D (D even) and is a feasible solution for the DMST (G, D) if and only if a node r exists for which all linking paths from r to any other node have at most D/2 arcs. If this happens, it means that T is also feasible for the HMST (G, r, D/2)

Proof: The only if case is obvious. To prove that if case also holds, we must assume that T is feasible for DMST and that k and s are two nodes such that the distance between them in T is the maximum between all pair of nodes. Representing by N the number of arcs between k and s, there exist a node r in this connecting path such that the distance to k or to s is equal to N/2. Consider now any other node q, $q \neq k$, s and r. Since T is a spanning tree, then either r is in the path among q and s or r is in the path among q and k (both statements can be true). Assuming that r is in the path from q to k, the distance in T

Manuscript received February 19, 2009.

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between q and r cannot be greater than N/2, otherwise the distance among q and k would be greater than N, contradicting the supposition that the linking path among k and s was the greater.

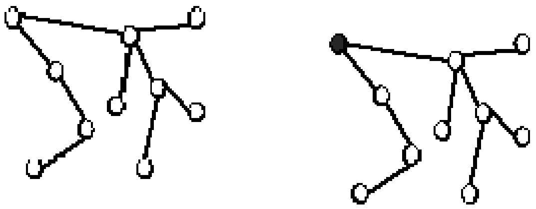

Fig. 1
Example of the relationship between a feasible tree for the DMST with diameter 6 and a feasible tree for the HMST with 3 hops

Result 2: F [DMST(G,D)]= F [HMST(G,r,D/2)], where F[problem] represents the set of the feasible solutions.

The above result states that solving the DMST problem with diameter D is the same as solving HMST with path lengths equal to D/2 choosing for root all nodes of V and picking up the best solution. Note that the optimal algorithm for solving DMST with diameter 2 is based on this observation.

When the value of diameter D of the DMST problem is odd, then an artificial variation of the HMST problem is defined [represented as HMST (G,(r, s), h)], which consists of finding a spanning tree where all connecting paths from the root edge (r, s) to any other node of the graph have less than h arcs.

In this case, the concept of distance between a node r and a edge (i, j) is given by

$$d(r, (i, j)) = \min \{d(r, i), d(r, j)\}$$

which is interpreted as the minimum distance of the set of distances between node r and node i or node r and node j.

III. Heuristics

Several approaches have been proposed for obtaining good solutions for the problem in study, which can be divided in three main classes [4]: approximation algorithms which guarantees that a solution is found within a known gap from optimality; probabilistic algorithms, which assures that for instances over a given size, the probability of getting a bad solution is small; and heuristics algorithms, which do not offer any guarantees concerning the quality of the solution, but represents a good trade-off among quality of the solution and the time spent to solve the problem. Additionally, heuristics algorithms can be divided into three sub-classes: those that center attention on structural properties of the problem and use them to define the constructive rules; the ones that focus on the guidance of a constructive or local search algorithm to avoid local optima; and finally those ones that incorporate partial results from exact methods into the heuristic framework.

A. Constructive Heuristics

Constructive heuristics generate solutions from scratch by adding solution components to an initially empty solution, in some specific order, until the solution is feasible.

In this section we incorporate the heuristics that follow the nearest node strategy using the relationship between the HMST problem and the DMST. We developed 3 different types of heuristics with these characteristics. To obtain a feasible solution, the procedure used was the same as that used to generate an MST. However, if the addition of a chord to the partial tree causes a diameter violation it is ignored and another chord is tested. Better results can be obtained by running the procedure for every node r of the V.

The next heuristic is identified as a vertex clearing algorithm. The algorithm begins with the MST, and if the MST violates any of the diameter constraints, one of the violated node is chosen. A Prim algorithm style is used to reconstruct the tree that satisfies the diameter constraints. The process is repeated until all diameter constraints are satisfied.

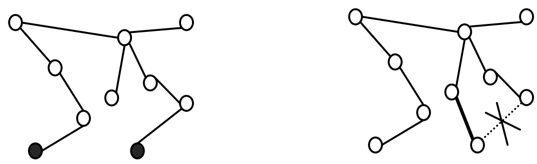

Fig. 2
Example of vertex clearing algorithm

The last heuristic follows the model proposed by Gouveia et al. [5]. Accordingly, the problem can be modeled in an expanded graph that guarantees the path length constraints, by computing the shortest paths between a root node r and any remain node of the original graph and by selecting those with length not greater than the given quantity, D/2. Afterwards the nodes were classified in levels according to the position they assume in the path. Subsequently the expanded graph G' is constructed, where each edge corresponds to a feasible path in the original network, and the solution is obtained by linking nodes from different levels, which means that a node from level k can only be connected to a node of level k+1 and to a node of level k-1, with $k=1,..,\frac{D}{2}-1$. Better results can be obtained by running the procedure for every vertex of V.

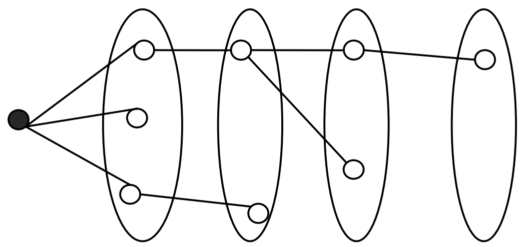

Fig. 3
Example of root paths, and division of the nodes in levels

B. Local Search Heuristics

Local search also referred to as neighborhood search or hill climbing, is the basis of many heuristics methods for combinatorial optimization problems. It is based on the iterative exploration of neighborhoods of solutions, trying to improve the current solution by local changes. The type of local changes that may be applied to a solution is defined by the neighborhood structure.

Ahuja [6] proposed a rule that usually works well in practice which states that, the larger the neighborhood of each solution, the better is the quality of the locally optimal solutions. The typical neighborhood for improving a

ISBN:978-988-18210-1-0

spanning tree solution with additional constraints is defined by the set of feasible solutions that are obtained from the current one, by adding a chosen edge and then removing the edge which is included on the single cycle formed by the previous inclusion.

In order to improve the current solution, a node level exchange is used to generate a different neighborhood structure and start a new search.

The local search heuristics starts with a feasible solution and explores the search space replacing the current solution for the best solution from the whole neighborhood, following the rules established above.

C. Local Search Heuristics- Path Based

In these types of algorithms a feasible initial solution is constructed following one of the constructive heuristics defined in section A.

T being the spanning tree of G, satisfying the diameter constraints $T \in F[DMST(G, D)]$, and r the root of the spanning tree satisfying the hop-constraints of the problem $HMST(G, r, {}^{D}/_{2})$ (see result 1), $T_i, i = 1, \ldots, s$ defines the sub trees induced in T by $V - \{r\}$. Solutions in the neighborhood of the current solution T are obtained inserting in the current tree an edge $e \notin T$ with extremities in two different components T_i and T_j. Only edges linking nodes from the same or lesser level are predisposed to be considered for insertion. This heuristic consists in exploring the different arborescence obtained by removing the paths which extremes are the nodes of the candidate-edge. The best improving neighbor generated by this scheme is chosen.

D. Greedy Randomized Adaptive Search Procedure

The greedy randomized adaptive search (GRASP) procedure described by Feo et. al. [7], is a multi-star metaheuristic divided in two main phases: a construction phase and a local search. In the construction phase, an initial feasible solution is iteratively built following an adaptive reasoning methodology; the decisions taken in the previous iteration influence the decision in the current solution. Then, the neighborhood of that solution is explored using local search procedure, and if the solution obtained is better than the best solution found in the previous run of the algorithm, that solution is kept.

To obtain an initial feasible solution at each iteration of the construction phase, all decisions that can be taken in a set of possible candidates are ranked, to evaluate the contribution to the objective function obtained by choosing that candidate. The candidates are ranked in a list and in each iteration of the construction phase, the candidate is chosen randomly among the elements of that list. After that, the ranked list is modified taking into account the previous choice. For this constructive phase were adopted the constructive heuristics defined in section A.

E. Genetic Algorithms

Genetic algorithms were first introduced by Holland [8] and they are considered a population algorithm where several solutions are looked upon in parallel, and during the procedure there is an exchange of information among them. This kind of algorithms is inspired in natural models of the species evolution. Its course of action is based in three operators that are used to exploit the space: selection, crossover and mutation.

The idea of the constructive heuristic defined in section A that uses the level procedure is used to define the offspring that represents the solution for genetic algorithm. The solution is represented by an offspring of dimension |V|, where a number between 1 and D/2, is associated with each node of V. The number assigned to each node indicates the number of arcs in the path from root node r to the node, which indicates the level of this vertex. The solution is obtained by linking the node in level k to its closest node in level k-1, for k=1,…, D/2. Using the results established between HMST e DMST the procedure builds a feasible solution for the HMST problems with root r that is also a feasible solution for the DMST with diameter D.

IV. Conclusions

The quality of solutions obtained by the different methods was compared using completed graphs with 40 and 80 nodes. We verify that an improvement is obtained with the local search procedures, while the results obtained by the Grasp heuristics are better than the ones reached with the constructive heuristics. Howeve,r the best results were obtained by the genetic algorithm approach, where the tuning parameter is a critical requirement because it influences the overall performance of the heuristic. For that reason, the work currently in progress is to implement constraint oriented neighborhood that are able to reduce the dependency of that parameter.

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