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Finite-time Ruin Probability of Renewal Model with Risky Investment and Subexponential Claims

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Abstract—**In this paper, we establish a simple asymptotic formula for the finite-time ruin probability of the renewal model with risky investment in the case that the claimsize is subexponentially distributed and the initial capital is large. The result is consistent with known results for the ultimate and finite-time ruin probability and, particularly, is inconsistent with the corresponding Poisson risk model when the time-arrivals are exponentially distributed.**

Keywords: Ruin probability; renewal risk model; subexponential class; ruin probability with finite horizon

1 Renewal model with risky investment

Consider a Sparre Andersen risk model as following:

(a) the individual claim size, X_n, $n \geq 1$, form a sequence of independent identically distributed (i.i.d.) nonnegative random variables (r.v.s) with a common distribution (d.f.) $F(x) = 1-\overline{F}(x) = P(X \leq x)$ for $x \in [0, \infty)$ and a finite mean $\mu = EX_1$.

(b) The inter-occurrence times $\theta_1 = \sigma_1$, $\theta_2 = \sigma_2 - \sigma_1$, $\theta_3 = \sigma_3 - \sigma_2$, ... are i.i.d. nonnegative random variables with mean $E\theta_1 = 1/\lambda$.

(c) $\{B_0(t), t \geq 0\}$ is a standard Brownian motion and $\sigma > 0$ is the diffusion volatility parameter of diffusion term $\sigma B_0(t)$.

In the special case, where θ_n has an exponential distribution, the Sparre Andersen model is called the compound Poisson model.

The random variables $\sigma_{\rm k} = \sum_{\rm i=1}^{\rm k} \theta_{\rm i}$, $k = 1, 2, ...$ constitute a renewal counting process

$$N(t) = \sup\{n \geq 1 : \sigma_n \leq t\} \tag{1.1}$$

with mean $\lambda(t) = EN(t)$. If an insurer invests insurance capital in risky asset, then its capital value should be specified by a geometric Brownian motion

$$dV_t = V_t(rdt + \sigma dB(t)), \tag{1.2}$$

where $\{B(t), t \geq 0\}$ is a standard Brownian motion and $r \geq 0, \sigma \geq 0$ are respectively called expected rate of return and volatility coefficient. By famous Black-Scholes formula, we know that stochastic equation (1.2) has the solution

$$V_t = V(0)e^{(r-\frac{1}{2}\sigma^2)t+\sigma B(t)}.$$

Hence, the total surplus up to time t, denoted by $U_{inv}(t)$, satisfies that

$$\begin{aligned} & U_{inv}(t) \\ = \; & e^{\Delta(t)}(u + c\int_0^t e^{-\Delta(s)}ds - \sum_{i=1}^{N(t)} X_i e^{-\Delta(\sigma_i)}) \end{aligned} \tag{1.3}$$

where, $U_{inv}(0) = u$ is the initial capital, $c > 0$ is premium intensity, $\Delta(t)$ represents $\beta t + \sigma B(t)$ and $\beta = r - \sigma^2/2$. $\{X_n, n \geq 1\}$, $\{N(t), t \geq 0\}$ and $\{B(t), t \geq 0\}$ are assumed to be mutually independent.

Usually, we define the time to ruin of this process as

$$\tau(u) = \inf\{t \geq 0 : U_{inv}(t) < 0 | U_{inv}(0) = u\}. \tag{1.4}$$

Therefore, the probability of ruin within a finite time $T > 0$ is defined by

$$\psi_{inv}(u; T) = P(\tau(u) \leq T). \tag{1.5}$$

In this paper, under the assumption that the claimsize is heavy-tailed, we establish an asymptotic formula of ruin probability $\psi_{inv}(u, T)$.

2 A brief review of related results

All limit relationships in this paper, unless otherwise stated, are for $u \to \infty$. $A \sim B$ and $A \gtrsim B$ respectively mean that $\lim_{u\to\infty} \frac{A}{B} = 1$ and $\lim_{u\to\infty} \frac{A}{B} \geq 1$.

Heavy-tailed risk has played an important role in insurance and finance because it can describe large claims; see Embrechts et al. (1997) and Goldie & Klüppelberg (1998) for a nice review. We give here several important classes of heavy-tailed distributions for further references:

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We say a d.f. $F \in \mathcal{R}_{-\alpha}$, for some $\alpha > 0$, if $\overline{F}(x) = x^{-\alpha}L(x)$, where $x > 0$ and $L(x)$ is a slowly varying function as $x \to \infty$. It is well known that (see Embrechts et al. (1997)), if $F \in \mathcal{R}_{-\alpha}$, then F satisfies

$$\lim_{x\to\infty} \frac{\overline{F}(x+t)}{\overline{F}(x)} = 1$$

for any t (or, equivalently, for $t = 1$). The relation above characterizes the class of long-tailed distributions, $\mathcal{L}$. Class $\mathcal{D}$ mans that, F satisfying

$$\limsup_{x\to\infty} \frac{\overline{F}(xy)}{\overline{F}(x)} < \infty$$

for any fixed $0 < y < 1$ (or, equivalently, for $y = 1/2$). Another important class of heavy-tailed distributions is the subexponential class $\mathcal{S}$. Let F be a d.f. concentrated on $[0, \infty)$. We say $F \in \mathcal{S}$ if

$$\lim_{x\to\infty} \frac{\overline{F^{*n}}(x)}{\overline{F}(x)} = n$$

for any n (or, equivalently, for $n = 2$), where F^{*n} denotes the n-fold convolution of F, with convention that F^{*0} is a d.f. degenerate at 0. These heavy-tailed classes have the properties below (see Embrechts et al. (1997)): These heavy-tailed classes have the properties below (see Embrechts et al. (1997)):

$$\mathcal{R}_{-\alpha} \subset \mathcal{L} \cap \mathcal{D} \subset \mathcal{S} \subset \mathcal{L}. \tag{2.1}$$

The asymptotic behavior of the ultimate ruin probability $\psi_r(u)$ is an important topics in the area of risk theory. A very famous asymptotic relation was established by Veraverbeke (1979) and Embrechts and Veraverbeke (1982). Briefly speaking, they showed that, if the so-called safety loading condition holds and, the integrated tail distribution of the r.v. X is sub-exponentially distributed , then the ultimate ruin probability, $\psi(u)$, satisfies that

$$\psi(u) \sim \frac{1}{\mu}\int_u^\infty \overline{F}(y)\mathrm{d}y \qquad \text{as } u \to \infty. \tag{2.2}$$

In the recent literatures ruin probability under the constant interest force in a continuous time risk model has been extensively investigated. One of the interesting results was obtained by Klüppelberg and Stadtmuller (1998), who used a very complicated L_p transform method, proved that, in the Cramér-Lundberg risk model, if the claimsize is of regularly varying with index $-\alpha$, then

$$\psi(u) \sim \frac{\lambda}{\alpha r}\overline{F}(u), \tag{2.3}$$

where r is constant interest force. Asmussen (1998) and Asmussen et al. (2002) obtained a more general result:

$$\psi(u) \sim \frac{\lambda}{r}\int_u^\infty \frac{\overline{F}(y)}{y}\mathrm{d}y, \tag{2.4}$$

where the claimsize is assumed to be in $\mathcal{S}^*$, an important subclass of subexponential family $\mathcal{S}$. In the case of compound Poisson model with constant interest force and without diffusion term, Tang (2005a) obtained the asymptotic formula of finite time ruin probability for subexponential claims. Tang (2005b) proved that, in the renewal risk model with constant interest force, if the d.f. of claimsize belongs to regularly varying class with index $-\alpha$, then ultimate ruin probability satisfies that

$$\psi(u) \sim \frac{\mathrm{E}e^{-\mathrm{r}\alpha\theta_1}}{1 - \mathrm{E}e^{-\mathrm{r}\alpha\theta_1}}\overline{F}(u), \tag{2.5}$$

which extends (2.4) essentially. The following theorem is the main result of this paper:

Theorem 2.1 *In the renewal risk model introduced in Section 1, if $F \in \mathcal{L} \cap \mathcal{D}$, then*

$$\psi(u;T) \quad \sim \quad \int_0^T P(X_1 e^{-\Delta(s)} \geq u)dm(s), \tag{2.6}$$

where $m(s)$ is the renewal function of the process, i.e., $m(t) = EN(t)$.

Remark. *When $F \in \mathcal{R}_{-\alpha}$ and the perturbed term disappears, the results of Tang (2005b) is consistent with this Theorem. In particular case that the process is Poisson one and when $\sigma = 0$, the result turns to the case of Klüppelberg and Stadtmuller (1998). Particularly, this result is also in consistence with that of Veraverbeke (1993), who pointed out that the diffusion term $B_0(t)$ does not influence the asymptotic behavior of the ruin probability. We should point out that the diffusion term $B(t)$ that influence the interest force plays an essential role.*

To complete Theorem 2.1, some lemmas in the following are needed.

Lemma 2.1 *If F is subexponential, the tail of its n-fold convolution is bounded by F's tail in the following way: for any $\varepsilon > 0$, there exists an $A(\varepsilon) > 0$ such that, uniformly for all $n \geq 1$ and all $x \geq 0$,*

$$\overline{F^{n*}}(x) \leq A(\varepsilon)(1+\varepsilon)^n \overline{F}(x), \tag{2.7}$$

see Embrechts *et al.* (1997) (p.41-42). The following result can be found in Klebaner (1998).

Lemma 2.2 *For maximum $M(t) = \max_{0\leq s\leq t} B(s)$ of Brownian motion at [0,t]. For any $x > 0$, it holds that*

$$P(M(t) \geq x) = 2(1 - \Phi(x/\sqrt{t})),$$

here, $\Phi(x)$ stands for standard normal distribution function.

Lemma 2.3 If $F_1 \in \mathcal{S}$ and $\overline{F_2}(x) = o\left(\overline{F_1}(x)\right)$, then $\overline{F_1 * F_2}(x) \sim \overline{F_1}(x)$.

Proof. See Proposition 1 in Embrechts et al. (1979) or Lemma 4.5 of Tang (2004).

3 Proof of Theorem 2.1

Now we begin to prove Theorem 2.1. It is easy to see that

$$\begin{aligned} & \psi(u;T) \\ = \; & P(e^{-\Delta(t)}U_{inv}(t) < 0 \;\text{ for some} \\ & T \geq t > 0 | U_{inv}(0) = u). \end{aligned} \tag{3.1}$$

For each $t \in (0, T]$, we have

$$\begin{aligned} & u - \sum_{i=1}^{N(t)} X_i e^{-\Delta(\sigma_i)} \\ \leq \; & e^{-\Delta(t)} U_{inv}(t) \\ \leq \; & u + c\int_0^T e^{-\Delta(s)} ds - \sum_{i=1}^{N(t)} X_i e^{-\Delta(\sigma_i)}. \end{aligned} \tag{3.2}$$

Therefore ruin probability $\psi(u;T)$ satisfies that

$$\begin{aligned} & \psi(u;T) \\ \geq \; & P(\sum_{i=1}^{N(t)} X_i e^{-\Delta(\sigma_i)} \geq u + cTe^{\max_{0\leq s\leq T}(-\Delta(s))} \\ & \text{for some } \; T \geq t > 0) \\ = \; & P(\sum_{i=1}^{N(T)} X_i e^{-\Delta(\sigma_i)} \geq u + \\ & cTe^{\max_{0\leq s\leq T}(-\Delta(s))}) \end{aligned} \tag{3.3}$$

and

$$\psi(u;T) \;\; \leq \;\; P(\sum_{i=1}^{N(T)} X_i e^{-\Delta(\sigma_i)} \geq u) \tag{3.4}$$

respectively. First we deal with $P(\sum_{i=1}^{N(T)} X_i e^{-\Delta(\sigma_i)} \geq u)$. Notice

$$\begin{aligned} & P(\sum_{i=1}^{N(T)} X_i e^{-\Delta(\sigma_i)} \geq u) \\ = \; & \sum_{k=1}^{\infty} P(\sum_{i=1}^{k} X_i e^{-\Delta(\sigma_i)} \geq u, N(T) = k). \end{aligned} \tag{3.5}$$

From lemma 2.1, and denote by H the distribution of $\max_{t\leq s\leq T}(-\Delta(s))$. For any fixed $\varepsilon > 0$

$$\begin{aligned} & P(\sum_{i=1}^{k} X_i e^{-\Delta(\sigma_i)} \geq u, N(T) = k) \\ \leq \; & \int_0^T P(\sum_{i=1}^{k} X_i e^{-\Delta(t)} \geq ue^{-\max_{t\leq s\leq T}(-\Delta(s))}, \\ & \sum_{i=2}^{k} \theta_i \leq T - t, \sum_{i=2}^{k+1} \theta_i \geq T - t) dF_{\theta_1}(t) \\ \leq \; & \int_0^T \int_0^{\infty} P(\sum_{i=1}^{k} X_i e^{-\Delta(t)} \geq ue^{-v}) \cdot \\ & P(N(T-t) = k-1) dH(v) dF_{\theta_1}(t) \\ \leq \; & A(\varepsilon)\int_0^T \int_0^{\infty} P(X_1 e^{-\Delta(t)} \geq ue^{-v})(1+\varepsilon)^k \cdot \\ & P(N(T-t) = k-1) dH(v) dF_{\theta_1}(t) \\ \leq \; & CA(\varepsilon)\int_0^T P(X_1 e^{-\Delta(t)} \geq u)(1+\varepsilon)^k \cdot \\ & P(N(T-t) = k-1) dF_{\theta_1}(t), \end{aligned} \tag{3.6}$$

where in the last step, we have used the property of $\mathcal{D}$ class.

For some N_0 to be chosen, we have

$$\begin{aligned} & \sum_{k=N_0}^{\infty} P(\sum_{i=1}^{k} X_i e^{-\Delta(\sigma_i)} \geq u, N(T) = k) \\ \leq \; & CA(\varepsilon)(1+\varepsilon)\int_0^T P(X_1 e^{-\Delta(t)} \geq u) \cdot \\ & \sum_{k=N_0}^{\infty} (1+\varepsilon)^k P(N(T-t) = k) dF_{\theta_1}(t) \\ \leq \; & CA(\varepsilon)(1+\varepsilon)E[(1+\varepsilon)^{N(T)} I(N(T) \geq N_0)] \cdot \\ & \int_0^T P(X_1 e^{-\Delta(t)} \geq u) dF_{\theta_1}(t). \end{aligned} \tag{3.7}$$

Especially, we choose $\varepsilon > 0$ and $N_0 > 0$ such that $CA(\varepsilon)(1+\varepsilon)E[(1+\varepsilon)^{N(T)} I(N(T) \geq N_0)]$ is smaller than any arbitrarily given number, say, $\eta_0 > 0$. In fact, we have

$$\begin{aligned} & E[(1+\varepsilon)^{N(T)} I(N(T) \geq N_0)] \\ \leq \; & \sum_{k=N_0}^{\infty} (1+\varepsilon)^k P(N(T) \geq k) \\ \leq \; & \sum_{k=N_0}^{\infty} (1+\varepsilon)^k P(\sigma_k \leq T) \\ \leq \; & \sum_{k=N_0}^{\infty} ((1+\varepsilon)Ee^{-\theta_1})^k e^T, \end{aligned} \tag{3.8}$$

obviously, we first choose ε small enough such that $((1+\varepsilon)Ee^{-\theta_1})^k < 1$, and then choose N_0 large enough so that $CA(\varepsilon)(1+\varepsilon)E[(1+\varepsilon)^{N(T)} I(N(T) \geq N_0)] < \eta_0$, an arbitrarily given number.

On the other hand

$$\begin{aligned} & P(\sum_{i=1}^{k} X_i e^{-\Delta(\sigma_i)} \geq u, N(T) = k) \\ = \; & \int_{(0\leq v_1\leq v_2\leq \ldots \leq v_k \leq T, v_{k+1} > T)} P(\sum_{i=1}^{k} X_i e^{-\Delta(v_i)} \geq u) \cdot \end{aligned}$$

$$dF(v_1,...,v_{k+1})$$

$$\begin{aligned}
&= \int_{(N(T)=k)} E[E[P(\sum_{i=1}^{k} X_i e^{-\Delta(v_i)} \geq u| \\
&\quad B(v_1),...,B(v_k))]]dF(v_1,...,v_{k+1}) \\
&\sim \int_{(N(T)=k)} \sum_{i=1}^{k} P(X_i e^{-\Delta(v_i)} \geq u) dF(v_1,...,v_{k+1}) \\
&= \sum_{i=1}^{k} P(X_i e^{-\Delta(\sigma_i)} \geq u, N(T)=k), \qquad (3.9)
\end{aligned}$$

therefore, for the same $\eta_0 > 0$, when N_0 large enough and for u large enough, it holds that

$$\begin{aligned}
&\sum_{k=1}^{N_0} P(\sum_{i=1}^{k} X_i e^{-\Delta(\sigma_i)} \geq u, N(T)=k) \\
&\leq (1+\eta_0) \sum_{k=1}^{\infty} \sum_{i=1}^{k} P(X_i e^{-\Delta(\sigma_i)} \geq u, N(T)=k) \\
&= (1+\eta_0) \sum_{i=1}^{\infty} P(X_i e^{-\Delta(\sigma_i)} \geq u, N(T) \geq i) \\
&= (1+\eta_0) \sum_{i=1}^{\infty} P(X_i e^{-\Delta(\sigma_i)} \geq u, \sigma_i \leq T) \\
&= (1+\eta_0) \int_0^T P(X_1 e^{-\Delta(s)} \geq u) d \sum_{i=0}^{\infty} F_{\sigma_i}(s) \\
&= (1+\eta_0) \int_0^T P(X_1 e^{-\Delta(s)} \geq u) dm(s). \qquad (3.10)
\end{aligned}$$

Thus, combining (3.7) and (3.10) together, we get

$$\begin{aligned}
&P(\sum_{i=1}^{N(T)} X_i e^{-\Delta(\sigma_i)} \geq u) \\
&\leq (1+2\eta_0) \int_0^T P(X_1 e^{-\Delta(s)} \geq u) dm(s). \qquad (3.11)
\end{aligned}$$

On the other hand

$$\begin{aligned}
&P(\sum_{i=1}^{N(T)} X_i e^{-\Delta(\sigma_i)} \geq u) \\
&= \sum_{k=1}^{\infty} P(\sum_{i=1}^{k} X_i e^{-\Delta(\sigma_i)} \geq u, N(T)=k) \\
&\geq \sum_{k=1}^{N_0} (1-\eta_0) \sum_{i=1}^{k} P(X_i e^{-\Delta(\sigma_i)} \geq u, N(T)=k) \\
&\geq (1-\eta_0)^2 \sum_{k=1}^{\infty} \sum_{i=1}^{k} P(X_i e^{-\Delta(\sigma_i)} \geq u, N(T)=k) \\
&\geq (1-2\eta_0) \int_0^T P(X_1 e^{-\Delta(\sigma_i)} \geq u) dm(s). \qquad (3.12)
\end{aligned}$$

By the arbitrariness of η_0, we have

$$\begin{aligned}
&P(\sum_{i=1}^{N(T)} X_i e^{-\Delta(\sigma_i)} \geq u) \\
&\sim \int_0^T P(X_1 e^{-\Delta(s)} \geq u) dm(s). \qquad (3.13)
\end{aligned}$$

Next, we aim to prove that

$$\begin{aligned}
&P(\sum_{i=1}^{N(T)} X_i e^{-\Delta(\sigma_i)} \geq u + cTe^{\max_{0\leq s\leq T}(-\Delta(s))}) \\
&\sim P(\sum_{i=1}^{N(T)} X_i e^{-\Delta(\sigma_i)} \geq u). \qquad (3.14)
\end{aligned}$$

Obviously,

$$\begin{aligned}
&P(\sum_{i=1}^{N(T)} X_i e^{-\Delta(\sigma_i)} \geq u + cTe^{\max_{0\leq s\leq T}(-\Delta(s))}) \\
&\geq P(\sum_{i=1}^{N(T)} X_i e^{-\Delta(\sigma_i)} \geq u + c_0(\beta,T) e^{\max_{0\leq s\leq T}(-B(s))}) \\
&\geq P(\sum_{i=1}^{N(T)} X_i e^{-\Delta(\sigma_i)} \geq (1+\varepsilon)u) - P(\Delta \geq \varepsilon u) \\
&\geq P(\sum_{i=1}^{N(T)} X_i e^{-\Delta(\sigma_i)} \geq (1+\varepsilon)u) - \frac{E\Delta^\tau}{(\varepsilon u)^\tau} \qquad (3.15)
\end{aligned}$$

where, $c_0(\beta,T) e^{\max_{0\leq s\leq T}(-\Delta(s))}$ is denoted by Δ. Here we have used Markov inequality. From Fima (1998), we know that $E\Delta^\tau$ exists for any $\tau > 0$. We choose τ large enough such that

$$\frac{E\Delta^\tau}{(\varepsilon u)^\tau} = o(P(\sum_{i=1}^{N(T)} X_i e^{-\Delta(\sigma_i)}). \qquad (3.16)$$

Still consider the arbitrariness of ε, this ends the proof of Theorem 2.1.

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