Computer Aided Kinematic and Dynamic Analysis of a Horizontal Slider Crank Mechanism Used For Single-Cylinder Four Stroke Internal Combustion Engine

H. D. Desai

Abstract - **The reciprocating engine mechanism is often analysed, since it serves all the demands required for the convenient utilization of natural sources of energy, such as steam, gaseous and liquid fuels, for generation of power. Further, it is widely employed as suitable mechanism for pumps and compressors. In this paper the complete kinematic and combined static and inertia force analysis of a horizontal, single – cylinder, four-stroke internal combustion engine is discussed. The analytical approach is used as it is more accurate and is less time consuming if it is programmed for the computer solution. The data for the analysis of the engine has been taken from the available literature. The present investigation furnishes the complete kinematic history of the driven links and the bearing loads for the complete working cycle of the engine mechanism. The complete force analysis of the engine is simplified by a summation of the static forces and inertia forces ignoring the friction forces which makes the analysis linear. The computer program is prepared in fortran language for both kinematic and dynamic analysis of the engine at the crank interval of 15^0.**

I. INTRODUCTION

The internal combustion engine employs a very popular mechanism known as slider crank mechanism. In this paper, complete kinematic and dynamic analysis of reciprocating group of machines is carried out, as it has reached to high state of development and is of more general interest than other group of machines.

Kinematic analysis of the slider-crank mechanism helps to answer many questions pertaining to the motions of various links of the mechanism viz. displacement, velocity and accelerations of driven members like connecting rod and piston. In the present work, the complete kinematic analysis of the engine has been carried out by analytical method using complex-algebra method as this method is more accurate than graphical method and can give results for all the phases of the mechanism.

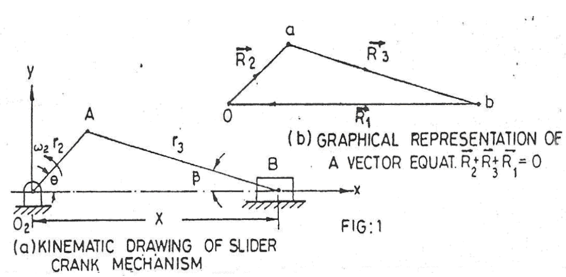
H. D. Desai, Asst. Professor, Department of Mechanical Engineering, S.V. National Institute of Technology, Surat- 395007, Gujarat, India. (e-mail: hdd@med.svnit.ac.in).

Also, once the algebraic form of the solution has been derived, it can be programmed for the computer solution.

Dynamic analysis of the engine includes static and inertia force analysis for all the possible phases of the engine which leads to an important aspect of the estimation of the loads carried by different members of the engine. The complete force analysis of the reciprocating engine mechanism using principle of superposition is carried out neglecting the effect of friction by a summation of the individual effects of gas forces and inertia forces. With the help of analytical expressions derived, a complete kinematic and dynamic analysis of the horizontal single-cylinder, four-stroke internal combustion engine with small crank interval of 15^0 is made. With engine parameters available in the literature an investigation of variation of crankpin load, pistonpin load and main bearing load is carried out for all the above phases with help of programming.

II. THEORETICAL ANALYSIS

Kinematic Analysis by Complex – Algebra Method



The kinematic drawing of slider-crank mechanism is shown in Fig .1 (a). This planar mechanism is represented by a vector equation as shown in Fig. 1 (b) for any phase diagram as

$$\overrightarrow{R_1}+\overrightarrow{R_2}+\overrightarrow{R_3}=0 \quad (1)$$

ISBN:978-988-18210-1-0

Expressing the above vectors in complex rectangular notation,

$$\left(-r_1 + j0\right) + \left(r_2 \mathrm{Cos}\theta + jr_2 \mathrm{Sin}\theta\right) + \left(r_3 \mathrm{Cos}\beta - jr_3 \mathrm{Sin}\beta\right) = 0 \qquad \text{……..(2)}$$

Where,
r_1 = Linear Displacement of the Slider, cm.
r_2 = Radius of the crank, cm
r_3 = Length of the connecting rod, cm
θ = Angular displacement of the crank, deg.
β = Angular displacement of connecting rod, deg.

Therefore

$$-r_1 + r_2 \cos\theta + r_1 \cos\beta = 0 \qquad (3)$$

$$r_2 \sin\theta - r_3 \sin\beta = 0 \qquad (4)$$

As the mobility of the mechanism is one and the rotational speed of the crankshaft is constant, input parameters are r_2, r_3, ω_2 and θ. Differentiation of Eqn. 3 and 4 gives the expressions of displacement, velocity and acceleration of the driven members, connecting rod (link 3) and the slider (link 4) [9].

$$\beta = \sin^{-1}\left(r_2 \sin\theta / r_3\right) \qquad (5)$$

$$\omega_3 = \frac{\left(r_2\omega_2 \cos\theta\right)}{\left(r_3 \cos\beta\right)} \qquad (6)$$

$$V_p = r_1^{\cdot} = \left(r_2\omega_2 \sin\theta + r_3\omega_3 \sin\beta\right) \qquad (7)$$

$$\alpha_3 = \frac{\left(r_2 \cos\theta - r_2\omega_2^{\,2} \sin\theta + r_3\omega_3^{\,2} \sin\beta\right)}{\left(r_3 \mathrm{Cos}\beta\right)} \qquad \text{---------- (8)}$$

$$A_p = \ddot{r}_1 = \left(r_2 \sin\theta + r_2\omega_2^{\,2} \mathrm{Cos}\theta + r_3\omega_3^{\,2} \mathrm{Cos}\beta + r_3 \mathrm{Sin}\beta\right) \qquad \text{---------- (9)}$$

Where,
ω_2 = Angular velocity of the crank, rad/sec.
ω_3 = Angular velocity of the connecting rod, rad/sec.
α_2 = Angular acceleration of the crank, rad/sec.^{2}
α_3 = Angular acceleration of connecting rod, rad/sec.^{2}
V_p = Velocity of the piston, cm/sec.
A_p = Acceleration of the piston, cm/sec.^{2}.

With the use of above equations the values of all the above kinematic parameters at the crank angle of 15^0 are calculated with the help of computer program.

III. Static Force Analysis

The gas force acts on the piston due to the combustion of fuel and this force varies during the cycle of operation. The gas force variation is obtained from the engine indicator diagram [9]. Fig. 2 shows graphical analysis of the gas force. From the force polygon,

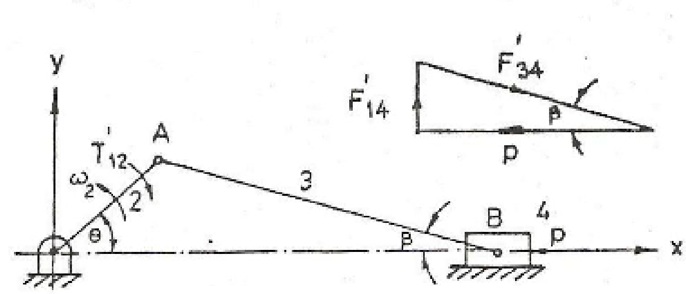


FIG: 2 ANALYSIS OF GAS FORCES

$$F'_{14} = P \tan\beta j \qquad (10)$$

$$F'_{34} = P / \mathrm{Cos}\beta \qquad (11)$$

Where,

P = Gas Force, N.

Crankshaft torque or turning moment delivered to the crank is obtained by taking the product of the gas force and piston coordinate x. Therefore crankshaft torque in vector form is,

$$T'_{21} = \left(F'_{14}.x\right)k \qquad (12)$$

IV. Inertia Force Analysis

The resultant bearing loads are made up of the following components:

1. The gas force components, designated by a single prime.
2. Inertia force due to the weight of the piston assembly, designated by a double prime.

ISBN:978-988-18210-1-0

3. Inertia force of that part of the connecting rod assigned to the pistonpin end, triple primed.
4. Connecting rod inertia force at the crankpin end, quadruple primed.

Equations for the gas force components have been determined earlier and reference shall be made to them in finding the total bearing loads. Fig. 3, shows graphical analysis of the forces in the engine mechanism with zero gas force and subjected to an inertia force resulting only from the weight of the piston assembly.

From Fig. 3, the analytical expressions for the forces are given as

$$F''_{41} = -m_4\, \ddot{x} \tan\beta\, j \tag{13}$$

$$F''_{34} = m_4 \ddot{x}\, i - m_4\, \ddot{x}\, \tan\beta\, j \tag{14}$$

$$F''_{32} = -F''_{34} \tag{15}$$

Where,

m_4 = Mass of the piston assembly, Kg.

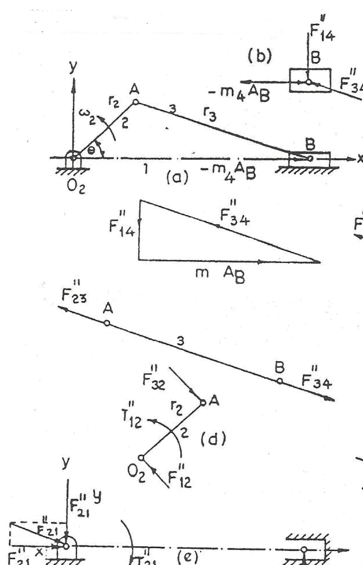


FIG:3 ANALYSIS OF THE FORCES IN THE ENGINE MECHANISM WHEN ONLY THE INERTIA FORCE DUE TO THE WEIGHT OF THE PISTON ASSEMBLY IS CONSIDERED.

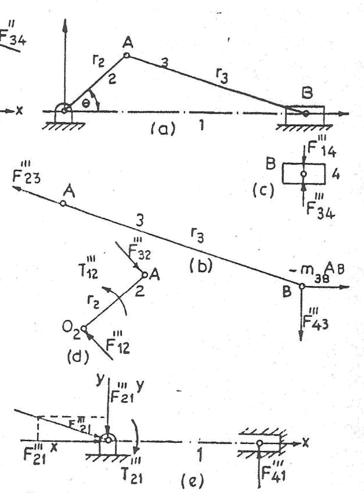


FIG:4 GRAPHICAL ANALYSIS OF FORCES RESULTING SOLELY FROM THE MASS OF THE CONNECTING ROD, ASSUMED TO BE CONCENTRATED AT THE WRIST-PIN END.

In Fig. 4, the analysis neglecting all forces except those which result because of the inertia of that part of the mass of the connecting rod which is assumed to exist at the pistonpin center is given.

The analytical expressions for the forces are given as

$$F'''_{41} = -m_{3B}\, \ddot{x} \tan\beta\, j \tag{16}$$

$$F'''_{34} = F'''_{41} \tag{17}$$

$$F'''_{32} = -m_{3B} \ddot{x} i + m_{3B}\, \ddot{x} \tan\beta j \tag{18}$$

Where m_{3B} = Mass of connecting rod lumped at wristpin B, Kg.

Fig. 5, shows the forces which result because of that part of the connecting rod mass which is concentrated at the crankpin end.

The analysis gives ,

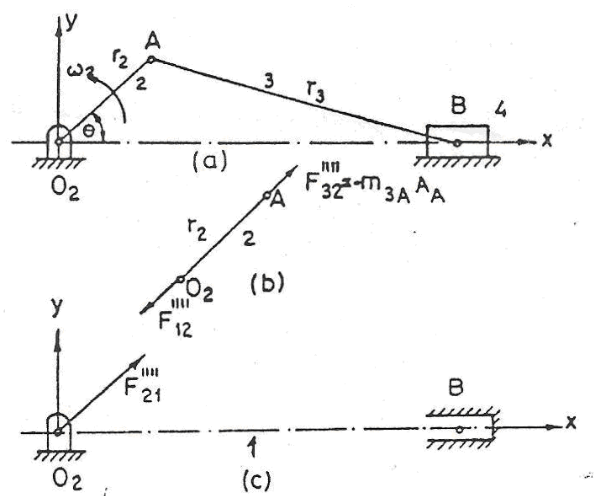


FIG. 5: GRAPHICAL ANALYSIS OF FORCES RESULTING SOLELY FROM THE MASS OF THE CONNECTING ROD, ASSUMED TO BE CONCENTRATED AT THE CRANK PIN END.

The analysis gives,

$$F''''_{32} = m_{3A}\, r_2\, \omega_2 2 (\cos\theta\, i + \sin\theta\, j) \tag{19}$$

Where,

m_{3A} = Mass of connecting rod lumped at crankpin A, Kg.

From the above analysis of inertia forces the total force of the piston against the cylinder wall, pistonpin and crankpin respectively is given by,

$$F_{41} = F'_{41} + F''_{41} + F'''_{41} = -\left[\left(m_{3B} + m_4\right)\ddot{x} + P\right]\tan\beta\, j \quad \text{......(20)}$$

$$F_{34} = \left(m_4\,\ddot{x} + P\right)i - \left[\left(m_{3B} + m_4\right)\ddot{x} + P\right]\tan\beta j \quad \text{......(21)}$$

$$F_{32} = \left[m_{3A} r_2 \omega_2^{\,2}\, \mathrm{Cos}\,\theta - \left(m_{3B} + m_4\right)\ddot{x} - P\right]i + \left\{m_{3A} r_2 \omega_2^{\,2}\, \mathrm{Sin}\,\theta + \left[\left(m_{3B} + m_4\right)\ddot{x} + P\right]\tan\beta\right\} j \quad \text{.....(22)}$$

Where,
F_{34} = Wristpin load, N.
F_{32} = Crankpin load, N.
F_{41} = Cylinder wall force, N.

The crankshaft torque or turning moment delivered by the engine is obtained from the equation

$$T_{21} = -\left(F_{41}.x\right)k = \left[\left(m_{3B} + m_4\right)\ddot{x} + P\right]x\tan\beta\, k \quad \text{....(23)}$$

The computer program has been developed in fortran language to compute the total bearing loads for the horizontal single-cylinder, four-stroke petrol engine for the complete working cycle at the crank interval of 15^0. The analysis of the engine is carried out for the gas forces available in the literature which have been obtained from the ideal air standard Otto cycle and the data is given in the Table: 1.

TABLE 1: GAS FORCES

SUCTION		COMPRESSION		EXPANSION		EXHAUST	
θ	P	θ	P	θ	P	θ	P
(Deg)	(N)	(Deg)	(N)	(Deg)	(N)	(Deg)	(N)
0	1166	360	14210	360	36309	720	1166
15	1166	345	12348	375	30772	705	1166
30	1166	330	8524	390	21560	690	1166
45	1166	15	5733	405	14308	675	1166
60	1166	300	3930	420	9800	660	1166
75	1166	285	2852	435	7105	645	1166
90	1166	270	2254	450	5439	630	1166
105	1166	255	1725	465	4302	615	1166
120	1166	240	1509	480	3773	600	1166
135	1166	225	1328	495	3308	585	1166
150	1166	210	1235	510	3087	570	1166
165	1166	195	1191	525	2960	555	1166
180	1166	180	1166	540	2901	540	1166

Crank speed = 1800 rpm.
Stroke = 14 cm.
r_2/r_3 = 0.228.

Mass of piston, pistonpin and main bearing, m_4 = 1.125 Kg.
Mass of connecting rod, m_3 = 1.75 Kg.
Centre of gravity of connecting rod is 7.44 cm from the crankpin centre.

From the above data,

m_{3B} = [7.44 /(16.86+7.44)] x 1.75 = 0.535 Kg.

m_{3A} = 1.75 – 0.535 = 1.215 Kg.

RESULTS AND DISCUSSION

The results computed from the program developed for kinematic analysis of the engine for the complete working cycle are given at the crank interval of 60 degree.

TABLE 2: KINEMATIC VALUES

θ (degree)	r_1 (cm)	β (degree)	ω_2 (rad/s)	V_p (cm/s)	α_3 (rad/s^2)	Ap. (cm/s^2)
0	31.30	0.00	54.30	0.00	0.00	-320375.20
60	27.03	14.45	26.29	-1307.31	-9190.64	-88538.17
120	20.03	14.45	-26.29	-978.13	-9190.64	160187.60
180	17.30	0.00	-54.30	0.00	0.00	177076.40
240	20.03	-14.45	-26.29	978.13	9190.64	160187.60
300	27.03	-14.45	26.29	1307.31	9190.64	-88538.32

The results computed from the computer program developed for complete dynamic analysis of the engine are shown in the Table: 3.

TABLE 3: BEARING LOADS

θ (Degree)	F_{32} (N)	F_{34} (N)	F_{41} (N)	T_{21} (N-cm)
0.00	7174.25	2438.22	0.00	0.00
60.00	3121.14	188.81	78.49	-24.57
120.00	6437.44	3126.72	-985.17	308.36
180.00	7127.49	3158.11	0.00	0.00
240.00	6772.19	3480.17	1073.53	336.02
300.00	3386.20	3000.94	633.56	-198.30
360.00	27968.76	10605.78	0.00	0.00
420.00	8317.13	9061.03	-2145.78	671.63
480.00	9019.11	5815.45	-1656.78	518.57
540.00	8862.49	4893.11	0.00	0.00
600.00	6437.44	3126.72	985.17	-308.36
660.00	3121.14	186.61	-78.49	24.57
720.00	7124.25	2438.22	0.00	0.00

The results of the kinematic analysis of the engine depicts the variation of the kinematic parameters like displacement, velocity and acceleration of the driven members i.e. connecting rod, link 3 and slider, link 4. The results of the combined static and inertia force analysis show variation of the bearing loads F_{32}, F_{34}, F_{41} and turning moment T_{21}. The knowledge of these forces is essential for the design of the bearings. The present investigation can be applied to multicylinder in-line engines, radial engines and V-engines.

REFERENCES

[1] Noxling, R. L., 1978, "Continuous Time Simulation of Forces and Motion within an Automobile Engine", SAE Paper No. 780665.

[2] Shiao, Y. J., Pan C. H., and Moskwa, J. J., 1994, "Advanced Dynamic Spark Ignition Engine Modeling for Diagnostics and Control, Int. J. Veh. Des., 15, PP. 578-596 (Inspec) [ISI].

[3] Suh, C. H., and Smith, C. G., 1997, "Dynamic Simulation of Engine – Mount Systems", SAE Paper No. 971940.

[4] Hoffman, D. M. W. and Dowling, D. R., 1999, "Modeling Fully coupled Rigid Engine Dynamics and Vibrations", SAE Paper No. 1999-01-1749, Proceedings, 1999 SAE Norse and Vibration Conference, Vol. 2, Traverse City, MI, Society of Automotive Engineers, Warrendale, PA, PP 747-755.

[5] Hoffman, D. M. W., 1999, "In-Line Internal Combustion Engine Dynamics and Vibration", Ph. D. thesis, University of Michigan, Ann Arbor, MI.

[6] Hoffman, D. M. W., and Dowling, D. R., 2001, "Fully Coupled Rigid Engine Dynamics and Vibrations Part – II: Model Experiment Comparisons" ASME J. Eng. Gas Turbine Power, 123, pp. 685-692 [ISI].

[7] Cowie, A., 1961, "Kinematics and Design of Mechanisms", International Textbook company, Scranton, Pennsylvania.

[8] Charles E. Wilson, J., Peter Sadler, Walter, J. Michels, 1983,"Kinematics and Dynamics of Machinery", Haper and Row Publisher, New York.

[9] Shigley, Joseph Edward, 2003, "Theory of Machines and Mechanisms", Tata McGraw Hill, New York.

[10] Greenwood, D. T., 1988, "Principles of Dynamics", 2nd Ed. Prentice-Hall.

ISBN:978-988-18210-1-0