Proceedings of the World Congress on Engineering 2009 Vol II
WCE 2009, July 1 - 3, 2009, London, U.K.

Attitude Control of Spider-type Multiple-rotor Rigid Bodies Systems

Anton V. Doroshin

Abstract—**This paper sets out to develop a new multiple-rotor system which can be used for attitude control of spacecraft. Unlike typical attitude control devices the new multirotor system has no disadvantages of reaction wheels, on the contrary, it possesses the advantages of a control moment gyroscope. The paper describes a new method of attitude reorientation of spacecraft with the help of the multirotor system. New multirotor system contains a lot of rotor-equipped rays, so it was called a "Rotor-type Spider" or a "Rotor-type Hedgehog". These systems allow using spinups and captures of conjugate rotors to perform compound attitude motion of spacecraft.**

Index Terms—**Spacecraft, Reaction Wheels, Control Moment Gyroscope, Conjugate Spinup, Rotor Capture**

I. Introduction

Most research into attitude motions of rigid bodies systems always has been and still remains one of the most important problems of theoretical and applied mechanics. Dynamics of the attitude motion of such systems is a classical mechanical research topic. Basic aspects of this motion were studied by Euler, Lagrange, Kovalevskaya, Zhukovsky, Volterra, Wangerin, Wittenburg. However, the study of the dynamics of rigid bodies is very important in modern science and engineering.

Among the basic directions of modern research into the framework of the indicated problem it is possible to highlight the following points: mathematical modeling and analysis of multibody systems motion [1], multibody spacecraft (SC) attitude dynamics and control [2]-[18], multibody systems approach to vehicle dynamics and computer-based technique [19], simulation of multibody systems motion [21], multibody dynamics in computational mechanics [20].

If we speak about practical use of system of rigid bodies dynamics research results we have to note first of all SC with momentum wheels, reaction wheels and control moment gyroscopes (dual-spin satellites, gyrostats, space stations, space telescopes, etc.) [1-18]. Let us briefly describe current rotor-type systems for attitude control of SC.

Reaction wheel is a spinning wheel that can be moved to change the orientation of SC to which the wheel is attached. Reaction wheels are used in many satellites, including the Hubble Space Telescope, to allow precise pointing. By attaching an electric motor to a heavy wheel, and spinning the wheel one way quickly, a satellite rotates the other way slowly by conservation of angular momentum. This method can provide very precise orientation, and does not require any fuel. The problem is that if there is some small continuous torque on a satellite, e.g., radiation pressure, the wheels end up spinning all the time, faster and faster, to counteract it. The solution of the problem is to have some way of dumping momentum. The usual technique is utilizing a set of electromagnets that can be used to exert a weak torque against the Earth's magnetic field.

Momentum wheels are a different type of actuator, mainly used for gyroscopic stabilization of SC: momentum wheels have high rotation speeds and mass, while reaction wheels work around a nominal zero rotation speed.

A *control moment gyroscope* (CMG) is an attitude control device which consists of a spinning rotor and one or more motorized gimbals that tilt the rotor's angular momentum. As the rotor tilts, the changing angular momentum causes a gyroscopic torque that rotates the spacecraft. Certainly, a CMG is a very powerful device which differs from reaction wheels. The latter applies torque simply by changing rotor spin speed, but the former tilts the rotor's spin axis without necessarily changing its spin speed. CMGs are also far more efficient. For a few hundred watts and about 100 kg of mass, large CMGs have produced thousands of newton meters of torque. CMG has got "gyroscopic power increase". A reaction wheel of similar capability would require megawatts of power.

Thus, we have a wide choice of devices for SC attitude control. Nevertheless, it is still possible to evolve new equipment for changing SC orientation. This paper sets out to develop a new multiple-rotor system which also can be used for attitude control of a SC. The new multirotor system differs from typical attitude control devices. It has no disadvantages of a reaction wheel, and on the contrary, it possesses the advantages of a control moment gyroscope. Due to the large number of rays with rotors, we called new systems a "Rotor-type hedgehog" and a "Rotor-type spider".

The paper has the following structure: Section 1 is an introduction of the problem background, Section 2 comprises mechanical and mathematical models of the new multiple-rotor system, Section 3 is devoted to development of a new reorientation method of the multiple-rotor system, Section 4 contains analytical solution of multiple-rotor system attitude motion, Section 5 include numerical simulation of the method implementation, Section 6 is conclusion.

Manuscript received February 26, 2009.

This work was supported in part by the Russian Federation Presidential Support Program for Russian scientists and leading scientific schools (MK-516.2008.8) and the Russian Foundation for Basic Research.

Anton V. Doroshin is an associate professor of Samara State Aerospace University, Samara, Russia (SSAU, 34, Moskovskoe Shosse str., Samara, 443086, Russia; phone: +78462674598; e-mail: doran@inbox.ru).

II. Mechanical and Mathematical Models of Multiple-Rotor Systems

We shall investigate an attitude motion about fixed point O of multirotor systems which are depicted in Fig.1.

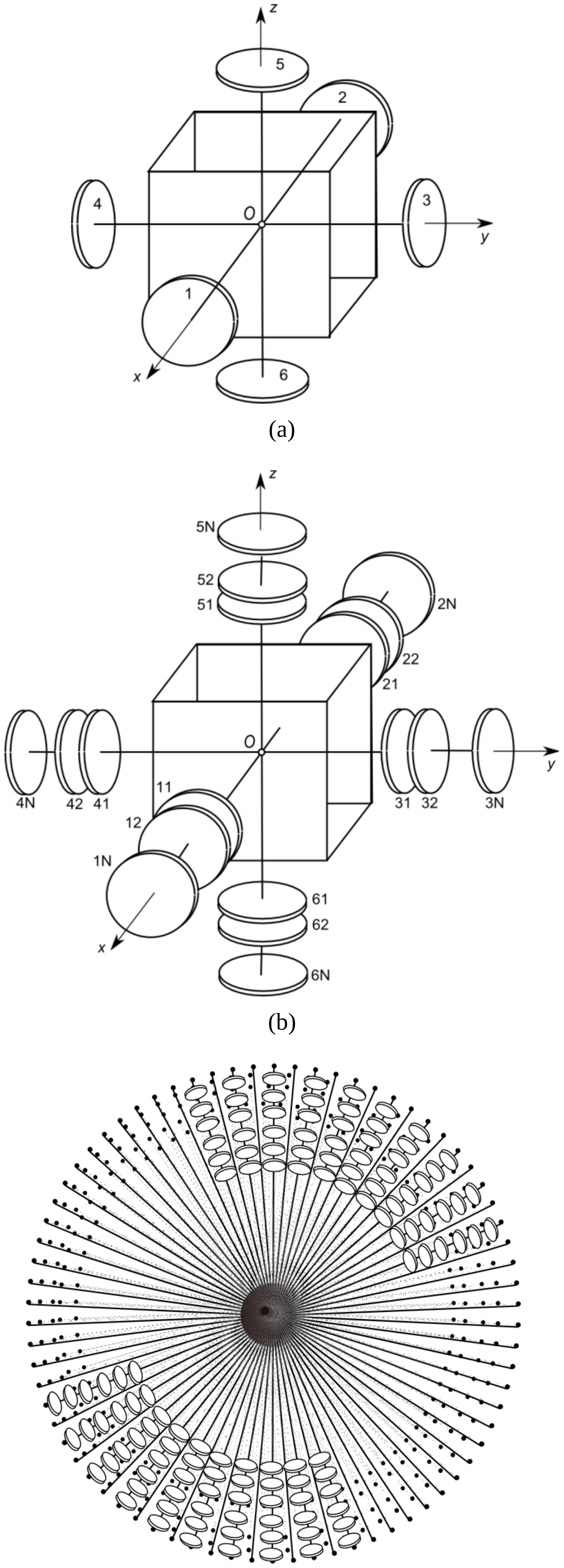

Fig.1. *Multirotor systems*

Fig.1 contains mechanical models of multirotor systems: cases (a) and (b) correspond to “Rotor-type spider” systems; case (c) corresponds to “Rotor-type hedgehog”.

Firstly we consider rotor-type spider with six rotors which spin about general orthogonal axis of main (central) body (Fig.1-a). Let’s assume symmetry of rotors disposition with respect to point O and equivalence of their mass-inertia parameters. Angular momentum of the system in projections onto the axes of frame $Oxyz$ connected with main body is defined by

$$\mathbf{K} = \mathbf{K}_m + \mathbf{K}_r \tag{1}$$

$$\mathbf{K}_m = \begin{bmatrix} \tilde{A}p \\ \tilde{B}q \\ \tilde{C}r \end{bmatrix} + \left(4J + 2I\right) \begin{bmatrix} p \\ q \\ r \end{bmatrix}, \quad \mathbf{K}_r = I \begin{bmatrix} \sigma_1 + \sigma_2 \\ \sigma_3 + \sigma_4 \\ \sigma_5 + \sigma_6 \end{bmatrix} \tag{2}$$

where $\mathbf{K}_m$ is angular moment of a main rigid body with resting (“frozen”) rotors; $\mathbf{K}_r$ is relative angular moment of rotors; $\boldsymbol{\omega}=[p, q, r]^{\mathrm{T}}$ is vector of absolute angular velocity of main body; σ_i is relative angular velocity of i-th rotor with respect to main body; $\tilde{A}, \tilde{B}, \tilde{C}$ are general moments of inertia of main body; I is longitudinal moments of inertia of single rotor; J is equatorial moments of inertia of single rotor calculated about point O.

Motion equations of the multirotor system can be obtained with help of law of changing of angular momentum in frame $Oxyz$

$$\frac{d\mathbf{K}}{dt} + \boldsymbol{\omega} \times \mathbf{K} = \mathbf{M}^e \tag{3}$$

where $\mathbf{M}^e$ is principal moment of the external forces. Eq. (3) can be rewritten as

$$\begin{cases} A\dot{p} + I\dot{\sigma}^{12} + \left(C - B\right)qr + I\left(q\sigma^{56} - r\sigma^{34}\right) = M_x^e \\ B\dot{q} + I\dot{\sigma}^{34} + \left(A - C\right)pr + I\left(r\sigma^{12} - p\sigma^{56}\right) = M_y^e \\ C\dot{r} + I\dot{\sigma}^{56} + \left(B - A\right)pq + I\left(p\sigma^{34} - q\sigma^{12}\right) = M_z^e \end{cases} \tag{4}$$

In the last equations following terms take place

$$\begin{gathered} \sigma^{ij} = \sigma_i + \sigma_j, \qquad A = \tilde{A} + 4J + 2I \\ B = \tilde{B} + 4J + 2I, \; C = \tilde{C} + 4J + 2I \end{gathered} \tag{5}$$

We need to add equations for rotors relative motion. These equations can also be written on the base of the law of the change in the angular momentum

$$\begin{cases} I\left(\dot{p} + \dot{\sigma}_1\right) = M_1^i + M_{1x}^e; \; I\left(\dot{p} + \dot{\sigma}_2\right) = M_2^i + M_{2x}^e \\ I\left(\dot{q} + \dot{\sigma}_3\right) = M_3^i + M_{3y}^e; \; I\left(\dot{q} + \dot{\sigma}_4\right) = M_4^i + M_{4y}^e \\ I\left(\dot{r} + \dot{\sigma}_5\right) = M_5^i + M_{5z}^e; \; I\left(\dot{r} + \dot{\sigma}_6\right) = M_6^i + M_{6z}^e \end{cases} \tag{6}$$

where M_j^i is a principal moment of the internal forces acting between main body and j-th rotor; $M_{jx}^e, M_{jy}^e, M_{jz}^e$ are principal moments of external forces acting only at j-th rotor.

Equation systems (4) and (6) together completely describe the attitude dynamics of the rotor-type spider (Fig.1-a).

ISBN:978-988-18210-1-0

Motion equations (4) and (6) corresponding to the spider with six rotors can be generalized for description of attitude dynamics of rotor-type spider with 6*N* rotors (Fig.1-b). As presented in Fig.1-b multirotor system has got *N* rotors on every ray - *N* rotor layers (levels). Similarly to previous case we can obtain the same equation system (4) for attitude motion of the multirotor system with *N* rotor layers (levels), but equations (4) will receive the following new terms

$$\begin{gathered}\sigma^{ij}=\sum_{l=1}^{N}\left(\sigma_{il}+\sigma_{jl}\right),\qquad A=\tilde{A}+4\sum_{l=1}^{N}J_l+2NI\\ B=\tilde{B}+4\sum_{l=1}^{N}J_l+2NI,\ C=\tilde{C}+4\sum_{l=1}^{N}J_l+2NI\end{gathered}\tag{7}$$

where σ_{kl} is the relative angular velocity of the *kl*-th rotor (Fig.1-b) with respect to main body; J_l is equatorial moments of inertia of the *kl*-th rotor (correspond to the *l*-th layer of rotors) calculated about point *O*.

Equations of rotors' relative motion are given by

$$\begin{cases}I\left(\dot{p}+\dot{\sigma}_{1l}\right)=M_{1l}^{i}+M_{1lx}^{e};\ I\left(\dot{p}+\dot{\sigma}_{2l}\right)=M_{2l}^{i}+M_{2lx}^{e}\\ I\left(\dot{q}+\dot{\sigma}_{3l}\right)=M_{3l}^{i}+M_{3ly}^{e};\ I\left(\dot{q}+\dot{\sigma}_{4l}\right)=M_{4l}^{i}+M_{4ly}^{e}\\ I\left(\dot{r}+\dot{\sigma}_{5l}\right)=M_{5l}^{i}+M_{5lz}^{e};\ I\left(\dot{r}+\dot{\sigma}_{6l}\right)=M_{6l}^{i}+M_{6lz}^{e}\end{cases}\tag{8}$$

where $l=1..N$ and therefore we have got *N* systems like (8) for each *kl*-th rotor.

Equation system (4) with terms (7) and *N* systems like (8) completely describe of attitude dynamics of the rotor-type spider with 6*N* rotors (Fig.1-b).

Thus, we have dynamic equations of attitude motion. Let's define kinematic parameters and corresponding kinematic equations. We will use well-know [23] Euler parameters $\{\lambda_0,\lambda_1,\lambda_2,\lambda_3\}$ describing a finite rotation of main body by an angle χ about an arbitrary unit vector $\mathbf{e}=\left[\cos\alpha,\cos\beta,\cos\gamma\right]^T$ in inertial fixed frame *OXYZ* which coincides with the initial position of *Oxyz* (Fig.2).

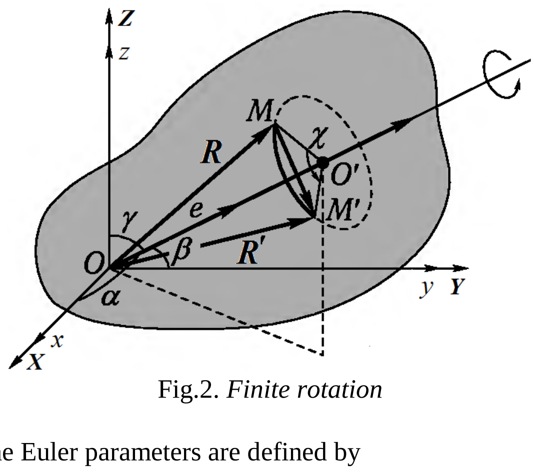

Fig.2. *Finite rotation*

The Euler parameters are defined by

$$\begin{cases}\lambda_0=\cos\dfrac{\chi}{2};\qquad \lambda_1=\cos\alpha\sin\dfrac{\chi}{2}\\ \lambda_2=\cos\beta\sin\dfrac{\chi}{2};\ \lambda_3=\cos\gamma\sin\dfrac{\chi}{2}\end{cases}\tag{9}$$

Following system of kinematical equation takes place for Euler parameters

$$2\dot{\boldsymbol{\lambda}}=\boldsymbol{\Theta}\cdot\boldsymbol{\lambda}\tag{10}$$

where

$$\boldsymbol{\lambda}=\begin{bmatrix}\lambda_0\\ \lambda_1\\ \lambda_2\\ \lambda_3\end{bmatrix},\qquad \boldsymbol{\Theta}=\begin{bmatrix}0&-p&-q&-r\\ p&0&r&-q\\ q&-r&0&p\\ r&q&-p&0\end{bmatrix}\tag{11}$$

Set of dynamic and kinematic equations completely describe of attitude motion of multirotor systems.

III. A New Method of Attitude Reorientation of Multiple-Rotor System

First of all, we will give a series of definitions.

Def.1. Conjugate rotors are pair rotors located in the same layer on the opposite rays. For example, rotor 3N and rotor 4N (Fig.1-b) are conjugate rotors (also rotor 12 and rotor 22, etc.).

Def.2. Conjugate spinup mean a process of spinning up conjugate rotors in opposite directions up to a desired value of relative angular velocity with help of internal forces moments from main body. Velocities of conjugate rotors will equal in absolute value and opposite in sign.

Def.3. Rotor capture is an immediate deceleration of rotor relative angular velocity with help of internal forces moment from the main body. So, rotor capture means an "instantaneous freezing" of rotor with respect to the main body. The capture can be performed with help of gear meshing, large friction creation or other methods.

Now we provide an explanation of essence of an attitude reorientation method.

Let's consider conjugate spinup of conjugate rotors 1 and 2 (Fig.1-a) in the absence of external forces moments $\left(M_x^e=M_y^e=M_z^e=0\right)$ assuming initial rest of main body and all rotors and mass-inertia symmetry of the system

$$p(0)=q(0)=r(0)=0;\ \ \forall i:\ \sigma_i(0)=0\tag{12}$$

$$A=B=C=D\tag{13}$$

In simplest case we can use following piecewise constant spinup internal forces moments

$$\begin{gathered}M_1^i=\begin{cases}M_{12},\ \ if\ \ t\in\left[0,t_{12}^s\right]\\ 0,\qquad otherwise\end{cases}\\ M_2^i=\begin{cases}-M_{12},\ \ if\ t\in\left[0,t_{12}^s\right]\\ 0,\qquad otherwise\end{cases}\end{gathered}\tag{14}$$

where t_{12}^s is the time point of spinup stopping of rotors 1 and 2; $M_{12}=\text{const}>0$.

After the conjugate spinup rotors 1 and 2 will reach an absolute value $S_{12}=M_{12}\cdot t_{12}^s/I$ of relative angular velocity $\left(\sigma_1=S_{12},\ \sigma_2=-S_{12}\right)$ but the main body will remain in rest; angular momentum of system will be equal null as before. After the conjugate spinup we will capture rotor 1 at time

ISBN:978-988-18210-1-0

point t_1^c $\left(t_1^c > t_{12}^s\right)$ and then the relative angular velocity of the rotor 1 will be null $\left(\sigma_1' = 0\right)$, but main body will take absolute angular velocity p and rotor 2 will change relative angular velocity up to σ_2'. Conservation of angular momentum of full system makes it possible to write

$$Ap + I\sigma_2' = 0 \quad (15)$$

Similarly, conservation of angular momentum of rotor 2 makes it possible to write

$$I\left(p + \sigma_2'\right) = -IS_{12} \quad (16)$$

Numerical values for angular velocities after caption of rotor 1 are obtained from expressions (15) and (16)

$$p = \frac{IS_{12}}{A-I}; \quad \sigma_2' = -\frac{AS_{12}}{A-I} \quad (17)$$

At the time t_2^c $\left(t_2^c > t_1^c\right)$ we will capture of rotor 2 and then all of system's bodies (main body and both conjugate rotors) will return to the absolute rest.

Thus we can conclude that conjugate spinup and two serial captures of conjugate rotors bring to piecewise constant angular velocity of main body

$$p = \begin{cases} 0, \quad t \in \left[0, t_1^c\right) \cup \left(t_2^c, \infty\right) \\ P = \dfrac{IS_{12}}{A-I}, \; t \in \left[t_1^c, t_2^c\right] \end{cases} \quad (18)$$

It can be used for main body angular reorientation about corresponding axis. In our case the main body performed the rotation about Ox axis by finite angle

$$\varphi_x = \frac{IS_{12}\left(t_2^c - t_1^c\right)}{A-I} \quad (19)$$

In the next section we will put forward this method of attitude reorientation and solve the equation of motion.

IV. Analytical Solutions

The research results presented in previous section showed that angular velocity of main body is piecewise constant value at realization of conjugate spinup and serial capture of one pair of conjugate rotors (rotors 1 and 2).

Similarly we can explain the same process at realization of conjugate spinup and serial capture of another one pair of conjugate rotors. For example, for the rotors 3 and 4 we will write

$$q = \begin{cases} 0, \; t \in \left[0, t_3^c\right) \cup \left(t_4^c, \infty\right) \\ Q = \dfrac{IS_{34}}{B-I}, \; t \in \left[t_3^c, t_4^c\right] \end{cases} \quad (20)$$

$$\sigma_4' = -\frac{BS_{34}}{B-I}; \; S_{34} = \frac{M_{34} \cdot t_{34}^s}{I}$$

where t_{ij}^s is during (ending time point) of spinup of conjugate rotors i and j; t_i^c is time point of rotor i capture; M_{ij} is absolute value of principle momentum of internal forces for spinup of conjugate rotors i and j; S_{ij} is absolute value of relative angular velocity of conjugate rotors i and j after spinup; σ_i' is relative angular velocity of rotor i after capture of its conjugate rotor $i*$

$$\sigma_i' = \sigma_i \Big|_{t \in \left(t_{i*}^c, t_i^c\right)}$$

Assuming $t_1^c = t_3^c$, $t_2^c = t_4^c$ with help of (20), (17), and (13) we can prove following auxiliary expression at $t \in \left[t_1^c, t_2^c\right]$

$$p\sigma^{34} - q\sigma^{12} = p\sigma_4' - q\sigma_2' = 0 \quad (21)$$

Expression (21) show that gyroscopic term in the third equation (4) equals null at concurrent execution of spinups and captures of conjugate rotors {1, 2} and {3, 4}. Therefore angular velocity r will remain constant. Also we can prove equality to zero of all gyroscopic terms and equality to constant of all angular velocity components at every set of concurrent spinups and captures.

Thus, angular velocity components are constant at concurrent execution of spinups and between conjugate captures of rotors

$$p = P; \; q = Q; \; r = R \quad (22)$$

Let's assume synchronical capture of all conjugate rotors {1, 2}, {3, 4}, {5, 6} and coincidence of frame $Oxyz$ initial position and fixed frame $OXYZ$ (Fig.2).

$$\begin{gathered} t_1^c = t_3^c = t_5^c = t_{start}^c; \quad t_2^c = t_4^c = t_6^c = t_{finish}^c \\ \lambda_0\left(t_{start}^c\right) = 1; \; \lambda_1\left(t_{start}^c\right) = \lambda_2\left(t_{start}^c\right) = \lambda_3\left(t_{start}^c\right) = 0 \end{gathered} \quad (23)$$

Now we can solve kinematic equation (10)

$$\begin{gathered} \lambda_0(t) = \cos\frac{\Omega \cdot \left(t - t_{start}^c\right)}{2}; \quad \lambda_1(t) = \frac{P}{\Omega}\sin\frac{\Omega \cdot \left(t - t_{start}^c\right)}{2} \\ \lambda_2(t) = \frac{Q}{\Omega}\sin\frac{\Omega \cdot \left(t - t_{start}^c\right)}{2}; \quad \lambda_3(t) = \frac{R}{\Omega}\sin\frac{\Omega \cdot \left(t - t_{start}^c\right)}{2} \quad (24) \\ \Omega = \sqrt{P^2 + Q^2 + R^2}; \quad t \in \left[t_{start}^c, t_{finish}^c\right]; \quad T = t_{finish}^c - t_{start}^c \end{gathered}$$

Solutions (24) show that at the time point t_{finish}^c the main body performed finite rotation about vector **e** by angle χ (Fig.2)

$$\mathbf{e} = \left[\cos\alpha = \frac{P}{\Omega}, \cos\beta = \frac{Q}{\Omega}, \cos\gamma = \frac{R}{\Omega}\right]^T \quad (25)$$

$$\chi = \Omega T$$

So, solutions (24) illustrate an opportunity of practical use of the multirotor system for attitude reorientation of spacecraft. How we can see, attitude reorientation of SC by desired finite rotation was performed with help of conjugate spinup and serial captures of rotors into the SC.

Above we have considered the case of motion with zero initial value (12) angular momentum of system with equal inertia moments (13). It is easy to show that without external forces at zero initial value angular momentum of system the main body with different inertia moments also has piecewise constant components of angular velocity in frame $Oxyz$. First of all, in this case vector of angular momentum identically equals zero in every coordinate frame including moving frame

Oxyz connected with the main body

$$\begin{aligned}
&\mathbf{M}^e = 0 \quad \Rightarrow \quad \mathbf{K}_{OXYZ} = \mathbf{const} \\
&\mathbf{K}_{OXYZ} = \left[K_X, K_Y, K_Z\right]^T \\
&\mathbf{K}_{Oxyz} = \left[K_x, K_y, K_z\right]^T \\
&\mathbf{K}_{OXYZ}(0) = \mathbf{0} \Rightarrow \\
&K_X = K_Y = K_Z = K_x = K_y = K_z = 0 \\
&\dot{\mathbf{K}}_{Oxyz} = \mathbf{-\omega} \times \mathbf{K}_{Oxyz} = \mathbf{-\omega} \times \mathbf{0} = \mathbf{0} \Rightarrow \\
&K_x \equiv K_y \equiv K_z \equiv 0 \quad \left(\forall A, B, C\right)
\end{aligned} \tag{26}$$

Then we can write

$$K_x \equiv 0 \Rightarrow \begin{cases} p = 0,\ \sigma_1 = -\sigma_2 = \left\{0 \middle| \pm S_{12}\right\} & if \ \ t \in \left[0, t_1^c\right) \cup \left(t_2^c, \infty\right) \\ p = P = \text{const},\ \sigma_1 = 0,\ \ \sigma_2 = \sigma_2' & if \ \ t \in \left[t_1^c, t_2^c\right] \end{cases}$$

$$K_y \equiv 0 \Rightarrow \begin{cases} q = 0,\ \sigma_3 = -\sigma_4 = \left\{0 \middle| \pm S_{34}\right\} & if \ \ t \in \left[0, t_3^c\right) \cup \left(t_4^c, \infty\right) \\ p = Q = \text{const},\ \sigma_3 = 0,\ \ \sigma_4 = \sigma_4' & if \ \ t \in \left[t_3^c, t_4^c\right] \end{cases}$$

$$K_z \equiv 0 \Rightarrow \begin{cases} r = 0,\ \sigma_5 = -\sigma_6 = \left\{0 \middle| \pm S_{56}\right\} & if \ \ t \in \left[0, t_5^c\right) \cup \left(t_6^c, \infty\right) \\ r = R = \text{const},\ \sigma_5 = 0,\ \ \sigma_6 = \sigma_6' & if \ \ t \in \left[t_5^c, t_6^c\right] \end{cases}$$

Last expression proves that main body has piecewise constant components of angular velocity in case zero value of system angular momentum.

The case of main body attitude reorientation at zero value of system angular momentum is inertialess and reaction-free method. This method has no disadvantages of reorientation method with help of reaction wheels system like a negative affect of nonlinearity of internal spinup engines, use of large value torque of internal spinup engines and, therefore, large energy consumption.

If initial angular momentum of system equals nonzero vector, then at rotors captures main body angular velocity will depend on time. In this case the effect of "gyroscopic power increase" will take place, which means gyroscopic torque initiation. We can illustrate gyroscopic torque initiation and variability of main body angular velocity

$$\begin{aligned}
&\mathbf{K}_{OXYZ} = \mathbf{const} \neq \mathbf{0} \\
&\mathbf{K}_{Oxyz}(0) \neq 0,\ \ \boldsymbol{\omega}\left(t_i^c \le t \le t_{i*}^c\right) \neq 0 \Rightarrow \\
&\dot{\mathbf{K}}_{Oxyz} = \mathbf{-\omega} \times \mathbf{K}_{Oxyz} \neq \mathbf{0} \Rightarrow \\
&K_{x,y,z} = \text{var}(t)
\end{aligned} \tag{27}$$

Gyroscopic power increase allow to use of small value of initial main body angular velocity (at conjugate rotors capture) to appear large value of torque which accelerate reorientation process. This effect is used for attitude control of spacecraft with control moment gyroscope. In addition, nonzero initial angular momentum can be transferred from main body on several (or one) rotors with help of internal forces moments (by engines): main body comes to a stop, but rotors comes to a spins.

We need to note that initiation of nonzero angular momentum can be performed by rocket jet engines. On the contrary, if this initial momentum is nonzero, then it can be reduced to null also by jet engines and internal dampers. So, it is possible to use both modes (with/without initial value of system angular momentum) of realization of attitude method.

Systems like multirotor-spider (Fig.1-b) and multirotor-hedghog (Fig.1-c) allow using sequences of serial rotor spinups and captures to perform compound attitude motion. In these cases we can write a program for realization of complex serial spinups and captures of set conjugate rotors. For rotor-type hedgehog this research and engineering problem is highly interesting.

V. Numerical Simulation of Reorientation Process

Let's calculate two sets of results of numerical simulation of multirotor system attitude reorientation. We used the following internal forces moment

$$\begin{aligned}
&M_j^i = M_{jj*} \cdot \left[H(t) - H\left(t - t^s\right)\right] - \nu\sigma_j \cdot H\left(t - t_j^c\right) \\
&M_{j*}^i = -M_{jj*} \cdot \left[H(t) - H\left(t - t^s\right)\right] - \nu\sigma_{j*} \cdot H\left(t - t_{j*}^c\right) \\
&M_{jj*} = \text{const} > 0; \quad \nu >> 1; \quad j = 1,3,5; \quad j* = 2,4,6
\end{aligned} \tag{28}$$

where $H(t)$ is Heaviside function. In expressions (28) the first term corresponds to piecewise constant spinup moment and the second – to capture moment of viscous friction type. System parameters for calculations are presented in table I. Also the following parameters are common for all calculations: $\nu = 300$ $N{\cdot}m{\cdot}s$, $t^s = 3$ s. Simulation results in Fig.3 correspond to the case of reorientation process with zero value of angular momentum (12) and different inertia moments of main body. These results demonstrate constant components of angular velocity of the body between conjugate captures. Fig.4 corresponds to reorientation process at nonzero system angular momentum

$$\boldsymbol{\omega}(0) = \sigma_1(0) = ... = \sigma_4(0) = 0; \quad \sigma_5(0) = 100 \quad 1/s$$

for main body with the same inertia moments. In this mode of motion we also can see that main body comes to permanent rotation, which illustrate redistribution of angular momentum. Certainly, it is possible to perform the next series of spinups and captures to new reorientation of main body and transfer of angular momentum back to rotors.

Table I contains all necessary numerical parameters for calculation of results, which presented in figures (Fig.3, 4).

Table I. *Numerical parameters for calculations*

	A, $kg \cdot m^2$	B, $kg \cdot m^2$	C, $kg \cdot m^2$	I, $kg \cdot m^2$	t_1^c, s	t_2^c, s	t_3^c, s	t_4^c, s	t_5^c, s	t_6^c, s	M_{12}, $N \cdot m$	M_{34}, $N \cdot m$	M_{56}, $N \cdot m$
Fig.3	60	80	100	10	4.0	5.5	4.5	5.0	4.75	6.0	10	20	30
Fig.4	100	100	100	10	4.0	4.75	4.0	4.75	4.0	4.75	10	20	0

ISBN:978-988-18210-1-0

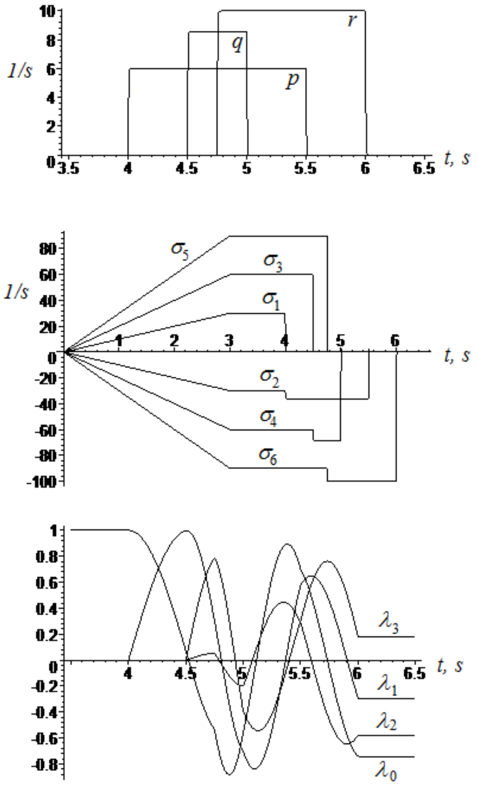

Fig.3

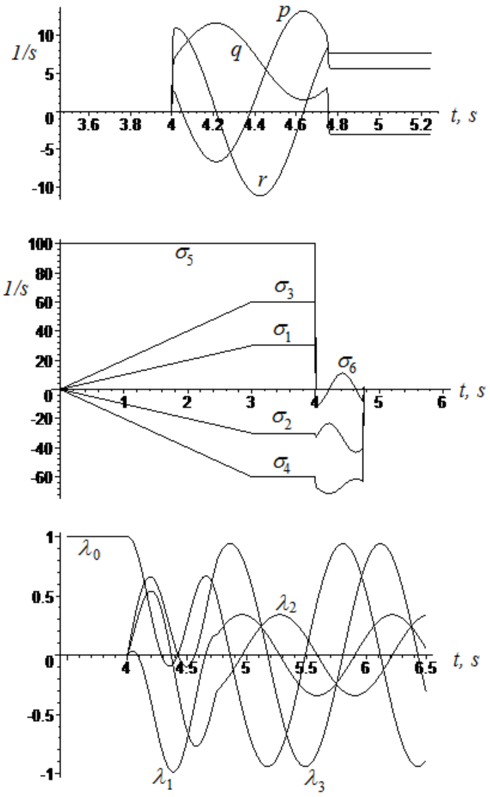

Fig.4

VI. Conclusion

In the paper new multirotor systems and their attitude reorientation method have been developed. Systems like multirotor-spider and multirotor-hedghog allow using programs for serial rotor spinups and captures to perform compound attitude motion. The analysis of such motion is a subject of further research.

References

[1] J. Wittenburg, Dynamics of Systems of Rigid Bodies, Teubner, Stuttgart Germany, 1977.
[2] P. W. Likins, Spacecraft Attitude Dynamics and Control - A Personal Perspective on Early Developments, J. Guidance Control Dyn. Vol. 9, No. 2 (1986) 129-134.
[3] D. L. Mingori, Effects of Energy Dissipation on the Attitude Stability of Dual-Spin Satellites, AIAA Journal, Vol. 7, No. 1 (1969) 20-27.
[4] P. M. Bainum, P. G. Fuechsel, D. L. Mackison, Motion and Stability of a Dual-Spin Satellite With Nutation Damping, Journal of Spacecraft and Rockets, Vol. 7, No. 6 (1970) 690-696.
[5] K. J. Kinsey, D. L. Mingori, R. H. Rand, Non-linear control of dual-spin spacecraft during despin through precession phase lock, J. Guidance Control Dyn. 19 (1996) 60-67.
[6] C. D. Hall, Escape from gyrostat trap states, J. Guidance Control Dyn. 21 (1998) 421-426.
[7] C. D. Hall, Momentum Transfer Dynamics of a Gyrostat with a Discrete Damper, J. Guidance Control Dyn., Vol. 20, No. 6 (1997) 1072-1075.
[8] A. E. Chinnery, C. D. Hall, Motion of a Rigid Body with an Attached Spring-Mass Damper, J. Guidance Control Dyn. Vol. 18, No. 6 (1995) 1404-1409.
[9] W. H. Steyn, A dual-wheel multi-mode spacecraft actuator for near-minimum-time large angle slew maneuvers, Aerospace Science and Technology, Vol. 12, Issue 7 (2008) 545-554.
[10] D. Izzo, L. Pettazzi, Command shaping for a flexible satellite platform controlled by advanced fly-wheels systems, Acta Astronautica, Vol. 60, Issues 10-11 (2007) 820-827
[11] V.S. Aslanov, A.V. Doroshin, Two cases of motion of an unbalanced gyrostat, Mechanics of solids, Allerton Press, Inc., Vol. 41, No. 4, (2006). 29-39. Available: http://cat.inist.fr/ and http://direct.bl.uk
[12] V.S. Aslanov, A.V. Doroshin, Stabilization of the descent apparatus by partial twisting when carrying out uncontrolled descent. Cosmic Research, Vol. 40, No. 2 (2002). 193-200. Available: http://springerlink.com
[13] V.S. Aslanov, A.V. Doroshin, G.E. Kruglov, The Motion of Coaxial Bodies of Varying Composition on the Active Leg of Descent, Cosmic Research, Vol.43, No. 3 (2005) 213-221. Available: http://springerlink.com
[14] V. S. Aslanov, A. V. Doroshin, The Motion of a System of Coaxial Bodies of Variable Mass, J. Appl. Math. Mech. Vol.68 (2004) 899-908. Available: http://science-direct.com/
[15] V.S. Aslanov, A.V. Doroshin, Influence of Disturbances on the Angular Motion of a Spacecraft in the Powered Section of Its Descent, Cosmic Research, Vol. 46, No. 2 (2008) 166–171. Available: http://springerlink.com
[16] A.V. Doroshin, Evolution of the Precessional Motion of Unbalanced Gyrostats of Variable Structure. Journal of Applied Mathematics and Mechanics 72 (2008) 259–269. Available: http://science-direct.com/
[17] A.V. Doroshin, Synthesis of Attitude Motion of Variable Mass Coaxial Bodies. WSEAS TRANSACTIONS on SYSTEMS and CONTROL, Issue 1, Volume 3 (2008) 50-61. Available: http://www.wseas.us/e-library/transactions/control/2008/25-273.pdf
[18] A. Pothiawala, M. A. Dahleh, H∞ Optimal Control For The Attitude Control And Momentum Management Of The Space Station, MIT, Cambridge, MA 02139. Available: http://dspace.mit.edu/bitstream/1721.1/3208/1/P-1985-22200134.pdf
[19] M. Blundell, D. Harty, Multibody Systems Approach to Vehicle Dynamics, Elsevier, 2004, 288 p.
[20] W. Schiehlen, N. Guse, R. Seifried, Multibody dynamics in computational mechanics and engineering applications, Computer Methods in Applied Mechanics and Engineering, Vol. 195, Issues 41-43 (2006) 5509-5522.
[21] T. M. Wasfy, A. K. Noor, Multibody dynamic simulation of the next generation space telescope using finite elements and fuzzy sets, Computer Methods in Applied Mechanics and Engineering, Vol.190, Issues 5-7 (2000) 803-824.
[22] P. Santini, P. Gasbarri, Dynamics of multibody systems in space environment; Lagrangian vs. Eulerian approach, Acta Astronautica, Vol. 54, Issue 1 (2004) 1-24.
[23] H. Goldstein, Classical Mechanics, 2-nd ed. Reading, MA: Addison-Wesley, 1980.