Performance Analysis of an Irreversible Otto Cycle using Finite Time Thermodynamics

Hemant B. Mehta*, Omprakash S. Bharti

***Abstract*—In this paper, performance of an air standard irreversible Otto-cycle is analyzed using finite-time thermodynamics (FTT). In the irreversible cycle model, the friction loss computed according to the mean velocity of the piston, the internal irreversibility described by using the compression and expansion efficiencies, the heat-transfer loss and the heat leak rate between the two reservoirs are considered. The relations between the power output and the compression ratio, between the thermal efficiency and the compression ratio, the optimal relation between the net power output and the efficiency of the cycle, Ecological function and power output, net heat input and efficiency, as well as power loss due to entropy generation rate and efficiency are indicated by numerical example.**

***Index Terms*— FTT, Heat leak rate, Heat transfer Loss, Friction, Internal irreversibility, Ecological function.**

I. Introduction

Applications of Finite Time Thermodynamics (FTT) method have been applied to various industrial fields like refrigerators, heat pumps, engines with heat regeneration, several coupled engines, solar engines, photovoltaic cells so on and forth. *Finite Time Thermodynamics* is the non equilibrium Theory. Its aim is to provide performance bounds and extremes for irreversible thermodynamic processes. *Finite Time Thermodynamics* is developed from a macroscopic point of view with heat conductance, friction coefficients, overall reaction rates, etc. rather than based on a microscopic knowledge of the processes involved. The methods of *Finite Time Thermodynamics* are used to find the optimal time path of any cyclic processes with friction and heat leakage. Optimality is defined by maximization of the work per cycle; the system is constrained to operate at a fixed frequency, so the maximum power is obtained. *Finite time processes*, besides their practical importance as more realistic models in case of *irreversible processes* than those provided by *reversible* thermodynamics, create a deeper understanding of how irreversibility affects the performance of thermodynamically processes. [1-4]

Recently, the analysis and optimization of the thermodynamic cycles for different optimization objectives have made tremendous progress by using *finite time thermodynamics*. The generalization of the cycle model is one of the important tasks of the finite time thermodynamics. It includes two aspects. One is the generalization of the cycle process, i.e., model can involve various thermodynamic cycles. Another is the generalization of cycle process's loss, i.e., this model involves various losses such as internal irreversibilities, heat leaks and frictional losses.

Angulo-Brown [17] proposed the ecological criterion $E = f(P, T_L, \sigma)$ for finite-time Carnot heat-engines, where T_L is the temperature of cold reservoir, P is the power-output and σ is the entropy-generation rate. Yan [18] showed that it might be more reasonable to use $E = f(P, T_o, \sigma)$ if the cold-reservoir temperature T_L is not equal to the environment temperature T_0 from the point-of-view of exergy analysis. This criterion function is more reasonable than that presented by Angulo-Brown. The optimization of the ecological function represents a compromise between the power-output P and the power loss W_{otto}, which is produced by entropy-generation in the system and its surroundings. The work was carried out for power, efficiency, entropy generation rate and ecological optimization for a class of generalized irreversible universal heat engines [20]. The performance of an air standard Otto-cycle was analyzed using finite-time thermodynamics [21].

In this paper, the authors have proposed a new generalized solution procedure for the irreversible Otto cycle by considering the second aspect of generalization discussed earlier. An irreversible air standard Otto-cycle model consisting of one heating branch, one cooling branch as well as two adiabatic branches is investigated. In this model, the non-linear relation between the heat leak rate, friction losses, heat transfer losses and internal irreversibilities are computed according to the mean velocity of the piston and considering the entropy generation rate as well as ecological function as the objective. The internal irreversibility is being described by using the compression and expansion efficiency of the compressors and expanders. The performance characteristics of the cycle have been obtained by detailed numerical example. This example shows the realistic case of cycle maximum power output and efficiency in various cases .The effects of heat leak C_i, internal irreversibilities (for compression and expansion processes) η_c *and* η_e, heat-transfer loss B and friction loss μ on the performance of the cycle were analyzed by varying the compression ratio γ. The results obtained in this paper may provide guidance for the performance evaluations and improvement of real heat-engines.

II. Irreversible Otto Cycle

An air standard Otto-cycle model with heat resistance, heat leak and internal irreversibility coupled to two constant

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temperatures, one at hot reservoir T_H and other at cold reservoir T_L is shown in fig. 1.

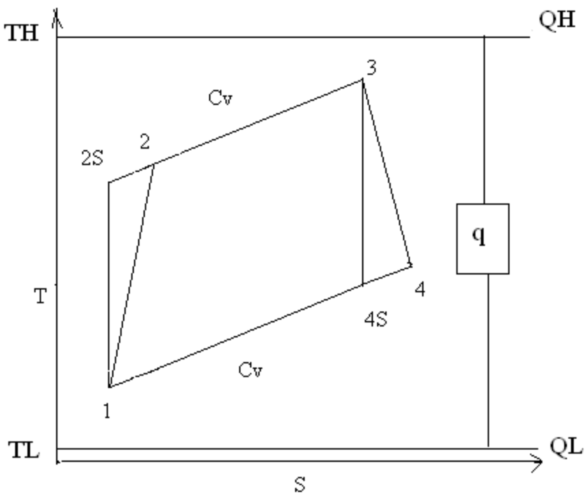

Fig. 1 Temperature-Entropy (T-S) diagram of an irreversible Otto cycle

It is assumed that there exists a constant rate of bypass heat leakage (q) from the heat source to the heat sink. This bypass heat-leakage model was advanced first by Bejan [16]. Thus $Q_H = Q_{H1} + q$ and $Q_L = Q_{L1} + q$, where Q_{H1} is due to the driving force of $(T_H - T_2)$ and Q_{L1} is due to the driving force of $(T_4 - T_L)$. Q_H is the rate of heat-transfer supplied by the heat source. Q_L is the rate of heat-transfer released to the heat sink.

Process *'1 - 2S'* is a reversible adiabatic compression, while process *'1 – 2'* is an irreversible adiabatic process that takes into account the internal irreversibility in the real compression process. The heat addition is an isochoric process *'2 – 3'*. Process *'3 - 4S'* is a reversible adiabatic expansion, while *'3 – 4'* is an irreversible adiabatic process that takes into account the internal irreversibility in the real expansion process. The heat rejection is an isochoric process *'4 – 1'*.

According to the properties of the working fluid and the theory of heat exchangers, the rate of heat-transfer (Q_H) supplied by the heat source,

$$Q_H = Q_{H1} + q \tag{1}$$

The heat added Q_{H1} to the working fluid from the hot source during the process 2-3 is,

$$Q_{H1} = \dot{m}\int_{T_2}^{T_3} C_v dT = \dot{m}\, C_v (T_3 - T_2) = \dot{m}\, C_v E_H (T_H - T_2) \tag{2}$$

The rate of heat transfer (Q_L) released to the heat sink is,

$$Q_L = Q_{L1} + q \tag{3}$$

The heat rejected Q_{L1} by the working fluid to the sink during the process 4-1 is,

$$Q_{L1} = \dot{m}\int_{T_1}^{T_4} C_v dT = \dot{m}\, C_v (T_4 - T_1) = \dot{m}\, C_v E_L (T_4 - T_L) \tag{4}$$

Here $\dot{m}$ (Kg/s) is Mass flow rate of the working fluid and C_V (kJ/kg-K) is the specific heat of the working fluid with constant volume. E_H and E_L are the effectiveness of the hot and cold side heat-exchanges respectively, and are defined by

$$E_{H1} = 1 - \exp(-N_{H1}) \text{ and } E_{L1} = 1 - \exp(-N_{L1}) \tag{5}$$

Here N_{H1} and N_{L1} are the numbers of heat-transfer units of the hot and cold side heat exchangers respectively, and are defined as

$$N_{H1} = U_{H1}/(\dot{m}C_v) \text{ and } N_{L1} = U_{L1}/(\dot{m}C_v) \tag{6}$$

Here U_{H1} and U_{L1} are the heat conductances.
The bypass Heat leak rate from the hot reservoir to the hot sink is given as

$$q = C_i (T_H - T_L) \tag{7}$$

Here C_i (kJ/kg-K) is heat-leak coefficient.
Combining (3) and (6), one can obtain

$$T_1 = E_{L1} T_L / [1 - (1 - E_{L1})] \tag{8}$$

$$T_3 = [(1 - E_{H1}) T_2] + E_{H1} T_H \tag{9}$$

The temperature ratio of both the reservoirs can be obtained by $\tau = T_H / T_L$. For the two adiabatic processes *1-2* and *3-4*, considering the interval irreversibilities of the heat engine during the compression and expansion process, the respective efficiencies are defined as,

$$\eta_C = (T_{2s} - T_1)/(T_2 - T_1) \tag{10}$$

$$\eta_e = (T_4 - T_3)/(T_{4s} - T_3) \tag{11}$$

For any process, when an infinitesimally-small change in temperature *dT*, and volume *dV* of the working fluid takes place, the equation for reversible adiabatic process with variable *k* can be written as follows [21],

$$TV^{k-1} = (T + dT)(V + dV)^{k-1} \tag{12}$$

Using (11), upon integration with limits *i* and *j*, we get,

$$C_v \times \ln(T_j / T_i) = R_g \times \ln(V_i / V_j) \tag{13}$$

Where the temperature in (12) is $T = (T_j - T_i) / \ln (T_j/T_i)$.

Therefore, equations for reversible adiabatic process *1-2S* and *3-4S* are respectively as follows,

$$(C_{v2} \times \ln(T_{2s})) - (C_{v1} \times \ln(T_1)) = +R_g \times \ln(V_1 / V_2) \tag{14}$$

$$(C_{v4} \times \ln(T_{4s})) - (C_{v3} \times \ln(T_3)) = -R_g \times \ln(V_1 / V_2) \tag{15}$$

For an ideal Otto-cycle model, there are no heat-transfer losses. However, for a real Otto-cycle, heat-transfer irreversibility between the working fluid and the cylinder wall is not negligible. The heat loss through the cylinder wall is assumed to be proportional to the average temperature of both the working fluid and the cylinder wall and the wall temperature is constant.
So the heat leak is given by the following linear equation [7-13],

$$Q_{leak} = \dot{m}B(T_2 + T_3 - 2T_o) \tag{16}$$

Where *B* is the constant related to heat-transfer, T_o is the average temperature of the working fluid and cylinder walls, and it is given by, $T_0 = (T_3 - T_2)/ \ln (T_3 / T_2)$.Taking into account the friction loss (μ) of the piston, as recommended by Chen [19] for the Dual cycle, and assuming a dissipation term represented by a friction force which in a linear function of the velocity gives,

$$f_\mu = \mu v = \mu (dx/dt) \tag{17}$$

Then the lost power is,

$$P_\mu = dW_\mu / dt = \mu(dx/dt)(dx/dt)$$
$$P_\mu = \mu(v_{mean})^2 \qquad (18)$$

If one specifies the engine is a four-stroke cycle engine, the total distance the piston travels per cycle is,

$$4L = 4(x_1 - x_2) \qquad (19)$$

For a four-stroke cycle engine, running at N cycles per second, the mean velocity of the piston is,

$$V_{mean} = 4*L*N \qquad (20)$$

Thus, the *power output* is,

$$P_{otto} = Q_H - Q_L - P_\mu \qquad (21)$$

The *efficiency* of the cycle is,

$$\eta_{otto} = P_{otto} / (Q_H + Q_{leak}) \qquad (22)$$

The Power loss (W_{otto}) which is produced due to the entropy generation (σ) in the system and its surrounding is

$$\sigma = (Q_L / T_L) - (Q_H / T_H) \qquad (23)$$

$$W_{otto} = \sigma * Tsurr \qquad (24)$$

Here T_{surr} (K) is the ambient temperature.
The Ecological Function of the model cycle is

$$Eco_{otto} = P_{otto} - W_{otto} \qquad (25)$$

When γ, T_H, U_{H1}, U_{L1}, η_c and η_e are given, T_L, T_1, T_2, T_3 and T_4 are obtained using (8)–(15) . Substituting all the obtained temperatures in above equations, one can yield the power and efficiency using (21) and (22). Entropy generation rate and Ecological function are determined using (24) and (25).

III. NUMERICAL EXAMPLE

The following numerical parameters [18] and [21] are used for the further study of irreversible Otto cycle: $T_L = 320$ K, $T_{surr} = 300$ K, $x_1 = 8 * 10^{-2}$ m, $x_2 = 1 * 10^{-2}$ m, N=30 rpm, m=4.553 * 10^{-3} kg/s, U_{H1}=U_{L1}=3kW/K. The example is solved to investigate the effect of compression ratio 'γ', the coefficient of friction 'μ' and heat transfer coefficient 'B'.

IV. RESULTS AND DISCUSSION

The results are obtained and plotted on the graphs to investigate the effects of an internal irreversibility (η_c & η_e), heat-transfer loss (**B**) and friction loss (μ) on the performance of the cycle.

From the definitions of the power output and the efficiency, the heat-transfer loss has no effect on the power output of the cycle. Fig.2 shows the effects of the internal irreversibility, heat leak and friction loss on the power output of the cycle. The curves show that the power output decreases with the increase of the internal irreversibility whether there is or not a friction loss; the power output decreases with the increase of friction loss whether or not there is internal irreversibility and there is no effect of heat transfer loss and heat leak rate.

Figs. 3 & 4 show the effects of the heat leak, internal irreversibility, heat transfer loss and friction loss on the efficiency of the cycle. In Fig. 3, curve 1 is the efficiency versus compression-ratio characteristic without irreversibility (η_c= η_e=1) and any heat leak i.e. C_i=0. For this circumstance, the efficiency increases with increases of compression ratio. But in fig. 4 it has been observed that the introduction of any heat leak in the cycle i.e. C_i = 0.05, the efficiency decreases with the increase of compression ratio instantaneously and curve is of parabolic like shape.

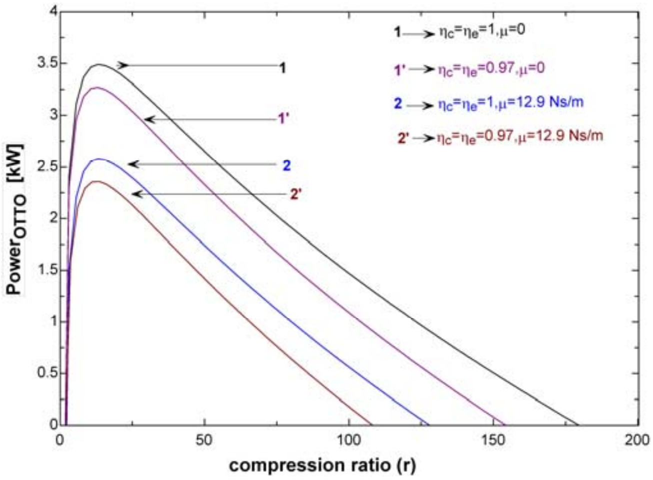

Fig. 2 Effect of an internal irreversibility and friction loss on the net power output with both C_i= 0 and C_i = 0.05.

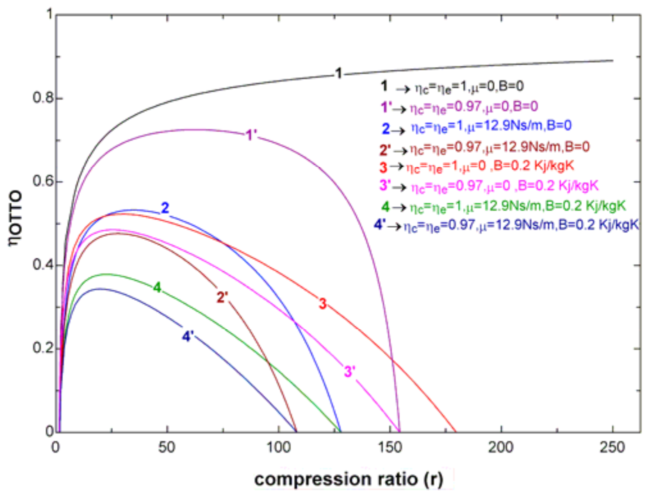

Fig. 3 Effect of an internal irreversibility, heat-transfer loss, and friction loss on the efficiency with C_i = 0.

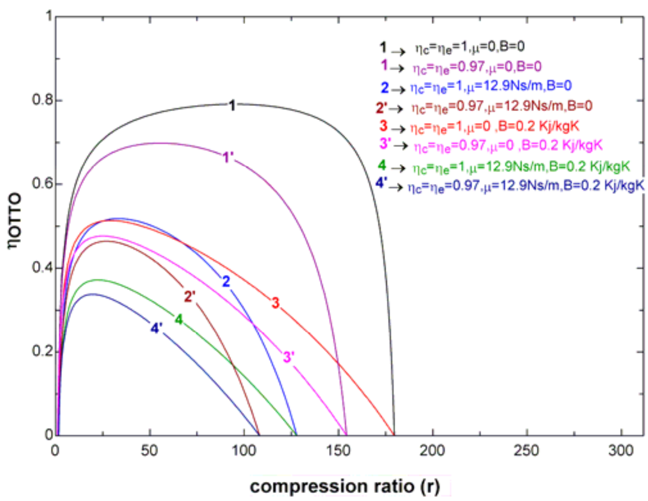

Fig. 4 Effect of an internal irreversibility, heat-transfer loss, and friction loss on the efficiency with C_i = 0.05.

Other curves in both figs. 4 & 5 are efficiency versus compression-ratio characteristic with one or more irreversibilities and these curves are parabolic-like ones. Comparing curve 1 with 1’, 2 with 2’, 3 with 3’ and 4 with 4’, it is seen that the efficiency increases with the decrease of internal irreversibility. Comparing curve 1 with 3, 2 with 4, 1’ with 3’ and 2’ with 4’, it is observed that the efficiency decreases the increasing heat-transfer loss. Comparing curve 1 with 2, 3 with 4, 1’ with 2’ and 3’ with 4’, one can see that the efficiency decreases with increases of friction loss.

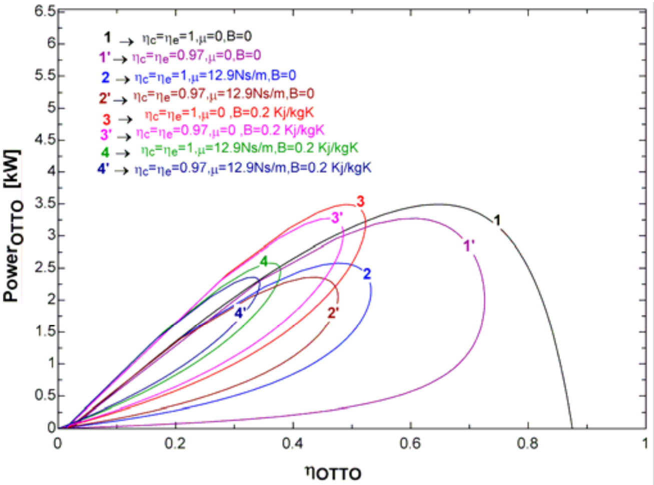

Fig. 5 Effect of an internal irreversibility, heat-transfer loss, and friction loss on the net power output v/s efficiency with $C_i = 0$.

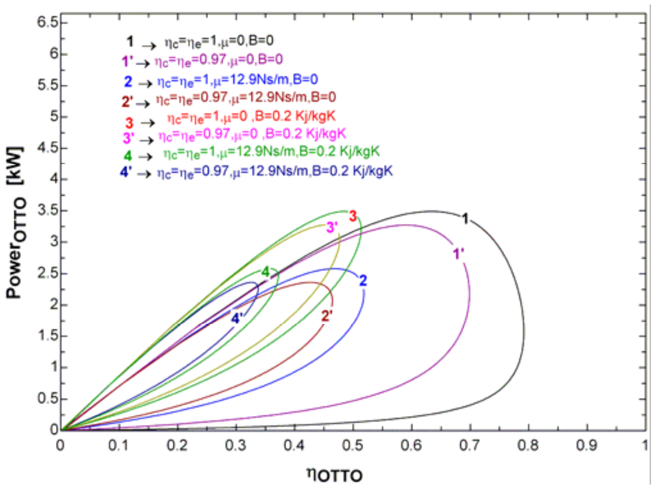

Fig.6 Effect of an internal irreversibility, heat-transfer loss, and friction loss on the net power output v/s efficiency with $C_i = 0.05$.

Figs. 5 & 6 show the effects of the heat leak, internal irreversibility, heat-transfer loss and friction loss on the power output versus the efficiency characteristic. In fig. 5, curve 1 which is a parabolic-like curve is the power output versus efficiency characteristic of the cycle without heat leak ($C_i=0$) and irreversibility ($\eta_c=\eta_e=1$), while in fig. 6, introduction of the heat leak ($C_i=0.05$) and irreversibility ($\eta_c=\eta_e=1$), curve 1 becomes loop shaped. The other curves in the both figures are loop-shaped ones with one or more irreversibilities. There is no effect of heat leak and heat transfer loss on the net power output while the efficiency decreases upon the inclusion of irreversibilities.

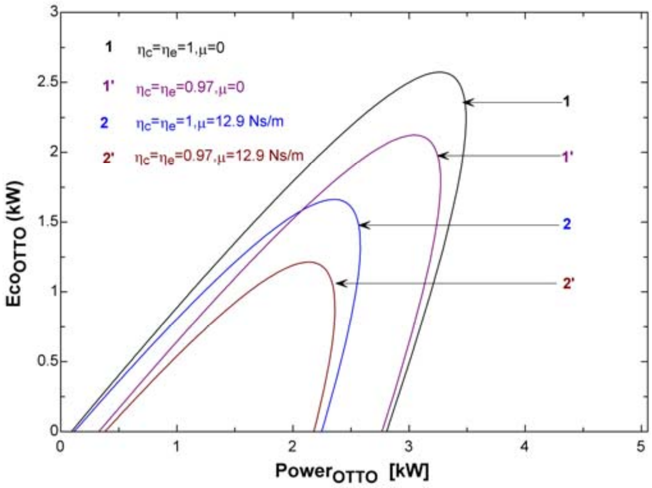

Fig. 7 Effect of an internal irreversibility and friction loss on the ecological function v/s net power output with $C_i = 0$.

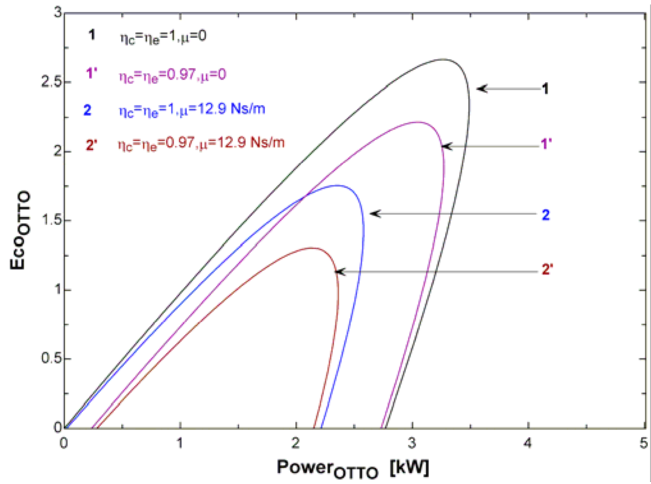

Fig. 8 Effect of an internal irreversibility and friction loss on the ecological function v/s net power output with $C_i =0.05$.

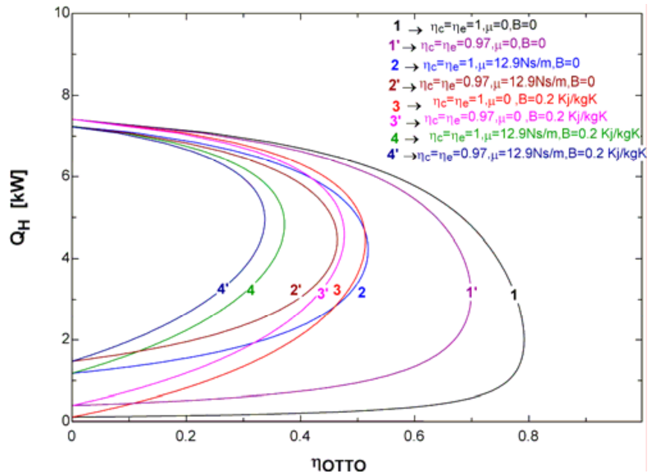

Fig. 9 Effect of an internal irreversibility, heat-transfer loss, and friction loss on the net heat input v/s efficiency with $C_i = 0.05$.

Figs. 7 & 8 show the effects of the heat leak, internal irreversibility and friction loss on the power output versus the Ecological function characteristic. In fig. 7, the effect of heat leak is not influenced on the power output and Ecological function. While in Fig.8, the effect of heat leak ($C_i=0.05$) more influence on the ecological function and no effect on power output. The curves as obtained in both cases are parabolic in nature. The maximum value for the power output and ecological function is obtained at maximum efficiency.

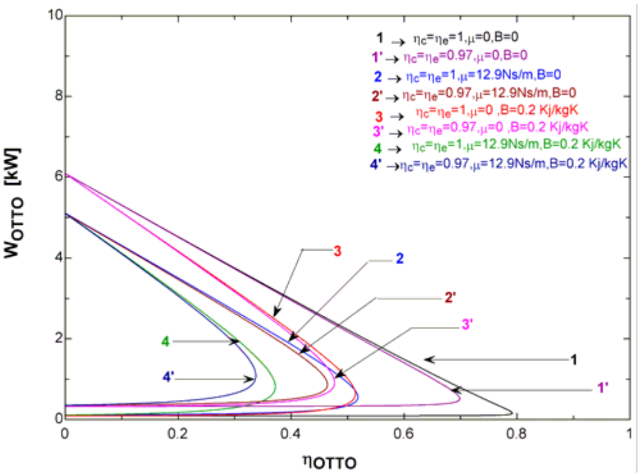

Fig. 10 Effect of an internal irreversibility, heat-transfer loss, and friction loss on the Power loss v/s efficiency with $C_i = 0.05$

Figs. 9 & 10 show the effects of heat leak internal irreversibility and friction loss on the heat input as well as power loss v/s efficiency and the curves obtained are the parabolic in nature. The efficiency of the cycle decreases with the increase of the power loss while the efficiency of the cycle increases with the increase of the input heat without any heat leak, heat transfer loss, friction loss and internal irreversibility. Introduction of any of the above parameters decreases both power loss and heat input.

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