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On the General Solution of the Two-Dimensional Electrical Impedance Equation for a Separable-Variables Conductivity Function

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***Abstract*— We analyze the structure of the general solution of the two-dimensional electrical impedance equation in analytic form using Taylor series in formal powers, for the case when the conductivity σ is a separable-variables function only once derivable, using a quaternionic reformulation that leads us to a special kind of Vekua equation. Finally, we broach its applications in the field of electrical impedance tomography.**

***Index Terms*— Electrical impedance, pseudoanalytic functions, quaternions, tomography.**

I. Introduction

The study of solutions of electrical impedance equation

$$\operatorname{div}(\sigma \operatorname{grad} u) = 0, \tag{1}$$

where σ is the conductivity function and u denotes the electric potential, is crucial for approaching solutions of the inverse problem posed by Calderon in 1980 [3], whose two-dimensional case is known as electrical impedance tomography. In 2006 Astala and Päivärinta [1] posed the solution of this problem trough the path of relating the two-dimensional electrical impedance equation with the theory of pseudoanalytic functions [2], [16]. In 2007 Kravchenko and Oviedo [10], who had previously noticed the relations of the two-dimensional stationary Schrödinger equation with a special class of Vekua equation [8] (whose solutions are known as *pseudoanalytic functions*), studied the structure of the general solution of (1) for the two-dimensional case in terms of Taylor series in formal powers, and gave an explicit general solution for a special class of σ.

We will analyze an alternative way for relating (1) with a Vekua equation for the two-dimensional case, based onto a quaternionic reformulation [11], [14], broaching the structure of its general solution in terms of formal powers, and we will study an analytical approach for the case when σ is a separable-variables function only once derivable, remarking the contribution of this general solution in the field of electrical impedance tomography.

II. Preliminaries

A. Elements of quaternionic analysis

We will denote the algebra of real quaternions (see e.g. [6], [9]) by $\mathbb{H}(\mathbb{R})$. The elements $q \in \mathbb{H}(\mathbb{R})$ have the form $q = q_0 + q_1e_1 + q_2e_2 + q_3e_3$ where $q_k \in \mathbb{R}$, and e_k are the standard quaternionic units satisfying the relations

$$\begin{aligned} e_1e_2 &= e_3 = -e_2e_1, \\ e_2e_3 &= e_1 = -e_3e_2, \\ e_3e_1 &= e_2 = -e_1e_3, \\ e_k^2 &= -1, \ k = 1, 2, 3. \end{aligned}$$

We will use the notation

$$q = q_0 + \overrightarrow{q},$$

where $\overrightarrow{q} = \sum_{n=1}^{3} q_k e_k$ is usually known as the *vectorial part* of quaternion q. Notice the set of purely vectorial quaternions $q = \overrightarrow{q}$ can be identified with the set of three-dimensional vectors belonging to $\mathbb{R}^3$. This is, to every $\overrightarrow{E} = (E_1, E_2, E_3) \in \mathbb{R}^3$ corresponds one purely vectorial quaternion $\overrightarrow{E} = E_1e_1 + E_2e_2 + E_3e_3$. It is easy to see that this relation is one-to-one.

Due to this isomorphism, we can represent the multiplication of two quaternions q and p as follows

$$q \cdot p = q_0p_0 + q_0\overrightarrow{p} + p_0\overrightarrow{q} - \langle \overrightarrow{q}, \overrightarrow{p} \rangle + [\overrightarrow{q} \times \overrightarrow{p}], \tag{2}$$

where $\langle \overrightarrow{q}, \overrightarrow{p} \rangle$ denotes the scalar product and $[\overrightarrow{q} \times \overrightarrow{p}]$ is the vectorial product. We shall notice $q \cdot p \neq p \cdot q$ in general, so we will use the notation

$$M^p q = q \cdot p$$

to indicate the multiplication by the right-hand side.

The Moisil-Theodoresco differential operator D is defined as

$$D = e_1\partial_1 + e_2\partial_2 + e_3\partial_3,$$

where $\partial_k = \frac{\partial}{\partial x_k}$, and it acts on the set of at least once-derivable quaternionic-valued functions. Using the classic vectorial notation we can write

$$Dq = \operatorname{grad} q_0 - \operatorname{div} \overrightarrow{q} + \operatorname{rot} \overrightarrow{q}. \tag{3}$$

B. Elements of pseudoanalytic functions

Following [2], let F and G be a pair of complex-valued functions such that

$$\operatorname{Im}\left(\overline{F}G\right) > 0, \tag{4}$$

where $\overline{F}$ is the complex conjugation of F:

$$\overline{F} = \operatorname{Re} F - i \operatorname{Im} F,$$

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and i denotes the standard complex unit $i^2 = -1$ as usual. Then any complex function W can be expressed as a linear combination of the form

$$W = \phi F + \psi G,$$

where ϕ and ψ are purely real-valued functions. A pair of complex-valued functions satisfying (4) is called a *generating pair*. The derivative in the sense of Bers, or (F,G)-*derivative*, of a function W is defined as

$$\frac{d_{(F,G)}W}{dz} = (\partial_z \phi) F + (\partial_z \psi) G \tag{5}$$

where $\partial_z = \partial_1 - i\partial_2$, and it exists iff

$$(\partial_{\overline{z}} \phi) F + (\partial_{\overline{z}} \psi) G = 0 \tag{6}$$

where $\partial_{\overline{z}} = \partial_1 + i\partial_2$ (usually the operators ∂_z and $\partial_{\overline{z}}$ are introduced with the factor $\frac{1}{2}$, nevertheless it will result more convenient for us to work without it). Let us introduce the following functions

$$\begin{aligned} A_{(F,G)} &= -\frac{\overline{F}\partial_z G - \overline{G}\partial_z F}{F\overline{G} - \overline{F}G}, \\ B_{(F,G)} &= \frac{F\partial_z G - G\partial_z F}{F\overline{G} - \overline{F}G}, \\ a_{(F,G)} &= -\frac{\overline{F}\partial_{\overline{z}} G - \overline{G}\partial_{\overline{z}} F}{F\overline{G} - \overline{F}G}, \\ b_{(F,G)} &= \frac{F\partial_{\overline{z}} G - G\partial_{\overline{z}} F}{F\overline{G} - \overline{F}G}. \end{aligned} \tag{7}$$

This functions are known as *characteristic coefficients* of the generating pair (F,G). According to this notations, equation (5) can be expressed as

$$\frac{d_{(F,G)}W}{dz} = \partial_z W - A_{(F,G)} W - B_{(F,G)} \overline{W}, \tag{8}$$

and (6) will turn into

$$\partial_{\overline{z}} W - a_{(F,G)} W - b_{(F,G)} \overline{W} = 0, \tag{9}$$

which is known as *Vekua equation* [16]. The complex-valued functions that fulfill (9) are named (F,G)-*pseudoanalytic functions*.

The following statements were originally posed in [2].

Remark 1: The complex-valued functions of the generating pair (F,G) are (F,G)-pseudoanalytic, and in agreement with (8) their (F,G)-derivatives satisfy

$$\frac{d_{(F,G)}F}{dz} = \frac{d_{(F,G)}G}{dz} = 0.$$

Definition 2: Let (F,G) and (F_1,G_1) be two generating pairs such that their characteristic coefficients satisfy

$$a_{(F,G)} = a_{(F_1,G_1)} \text{ and } B_{(F,G)} = -b_{(F_1,G_1)}. \tag{10}$$

Hence the generating pair (F_1,G_1) is called *successor* pair of (F,G), as well (F,G) is called *predecessor* pair of (F_1,G_1).

Theorem 3: Let W be a (F,G)-pseudoanalytic function, and let (F_1,G_1) be a successor of (F,G). Then the (F,G)-derivative of W

$$\frac{d_{(F,G)}W}{dz}$$

will be (F_1,G_1)-pseudoanalytic.

Definition 4: Let (F,G) be a generating pair. Its *adjoint* pair (F^*,G^*) is defined by the formulas

$$F^* = -\frac{2\overline{F}}{F\overline{G} - \overline{F}G}, \quad G^* = \frac{2\overline{G}}{F\overline{G} - \overline{F}G}.$$

The (F,G)-*integral* of a complex-valued function W is posed as

$$\int_{z_0}^{z_1} W d_{(F,G)}z =$$
$$= F(z_1) \operatorname{Re} \int_{z_0}^{z_1} G^* W dz + G(z_1) \operatorname{Re} \int_{z_0}^{z_1} F^* W dz.$$

If $W = \phi F + \psi G$ is (F,G)-pseudoanalytic, then

$$\int_{z_0}^{z} \frac{d_{(F,G)}W}{dz} d_{(F,G)}z =$$
$$= W(z) - \phi(z_0) F(z) - \psi(z_0) G(z),$$

and since

$$\frac{d_{(F,G)}F}{dz} = \frac{d_{(F,G)}G}{dz} = 0$$

this integral represents the *antiderivative* of

$$\frac{d_{(F,G)}W}{dz}.$$

A continuous function w is said to be (F,G)-*integrable* iff

$$\operatorname{Re} \oint G^* w dz + i \operatorname{Re} \oint F^* w dz = 0.$$

Theorem 5: The (F,G)-derivative of a (F,G)-pseudoanalytic function W is (F,G)-integrable.

Theorem 6: Let (F,G) be a predecessor pair of (F_1,G_1). A complex-valued function $\mathcal{E}$ will be (F_1,G_1)-pseudoanalytic iff it is (F,G)-integrable.

Definition 7: Let $\{(F_m,G_m)\}$, $m = \pm 1, \pm 2, \pm 3, \ldots$ be a sequence of generating pairs. If every (F_{m+1},G_{m+1}) is a successor of (F_m,G_m) we say that $\{(F_m,G_m)\}$ is a *generating sequence*. If $(F_0,G_0) = (F,G)$ we say that (F,G) is *embedded* in $\{(F_m,G_m)\}$.

Let W be a (F,G)-pseudoanalytic function, and let $\{(F_m,G_m)\}$ be a generating sequence in which (F,G) is embedded. Then we can express the higher derivatives in the sense of Bers of W as

$$W^{[0]} = W; \quad = \frac{d_{(F_m,G_m)}W^{[m]}}{dz}; \; m = 0, 1, \ldots$$

Definition 8: The formal power $Z_m^{(0)}(a, z_0; z)$ with center at z_0, coefficient a and exponent 0 is defined as the linear combination of the generators F_m and G_m with real constant coefficients λ and μ such that

$$\lambda F_m(z_0) + \mu G_m(z_0) = a.$$

The formal powers with exponents $n = 1, 2, \ldots$ are defined by the formulas

$$Z_m^{(n)}(a, z_0; z) = n \int_{z_0}^{z} Z_{m+1}^{(n-1)}(a, z_0; \varsigma) \, d_{(F_m,G_m)}\varsigma.$$

It is possible to verify that formal powers posses the following properties:

1) $Z_m^{(n)}(a, z_0; z)$ is (F_m,G_m)-pseudoanalytic.

2) If a_1 and a_2 are real constants, then

$$Z_m^{(n)}(a_1 + ia_2, z_0; z) = \\ = a_1 Z_m^{(n)}(1, z_0; z) + a_2 Z_m^{(n)}(i, z_0; z).$$

3) The formal powers satisfy the differential relations

$$\frac{d_{(F_m,G_m)} Z_m^{(n)}(a, z_0; z)}{dz} = Z_{m+1}^{(n-1)}(a, z_0; z).$$

4) The formal powers satisfy the asymptotic formulas

$$\lim_{z \to z_0} Z_m^{(n)}(a, z_0; z) = a(z - z_0)^n.$$

Remark 9: As it has been proved in [2], any complex-valued function W, solution of (9), accepts the expansion

$$W = \sum_{n=0}^{\infty} Z^{(n)}(a_n, z_0; z), \tag{11}$$

where the missing subindex m indicates that all formal powers belong to the same generating pair. This is: *expression (11) is an analytic representation of the general solution of (9).*

The Taylor coefficients a_n are obtained according to the formulas

$$a_n = \frac{W^{[n]}(z_0)}{n!}.$$

III. Quaternionic reformulation of the electrical impedance equation, and its relation with pseudoanalytic function theory

Consider the electrical impedance equation

$$\operatorname{div}(\sigma \operatorname{grad} u) = 0.$$

Indeed, the electric field vector $\overrightarrow{E}$ for the static case is defined as

$$\overrightarrow{E} = -\operatorname{grad} u, \tag{12}$$

so we can write

$$\operatorname{div}\left(\sigma \overrightarrow{E}\right) = 0.$$

But

$$\operatorname{div}\left(\sigma \overrightarrow{E}\right) = \left\langle \operatorname{grad}\sigma, \overrightarrow{E} \right\rangle + \sigma \operatorname{div} \overrightarrow{E},$$

then

$$\operatorname{div} \overrightarrow{E} = -\left\langle \frac{\operatorname{grad}\sigma}{\sigma}, \overrightarrow{E} \right\rangle. \tag{13}$$

Beside, from (12) we immediately obtain

$$\operatorname{rot} \overrightarrow{E} = 0. \tag{14}$$

Following [11], [14], let us consider now $\overrightarrow{E}$ as a purely vectorial quaternionic-valued function

$$\overrightarrow{E} = E_1 e_1 + E_2 e_2 + E_3 e_3.$$

Substituting the equalities (13) and (14) in (3) we have

$$D\overrightarrow{E} = -\left\langle \frac{\operatorname{grad}\sigma}{\sigma}, \overrightarrow{E} \right\rangle,$$

or using again (3)

$$D\overrightarrow{E} = -\left\langle \frac{D\sigma}{\sigma}, \overrightarrow{E} \right\rangle.$$

According to expression (2) we can write

$$\left\langle \frac{D\sigma}{\sigma}, \overrightarrow{E} \right\rangle = \frac{1}{2}\left(\frac{D\sigma}{\sigma} \cdot \overrightarrow{E} + \overrightarrow{E} \cdot \frac{D\sigma}{\sigma} \right),$$

and it follows

$$D\overrightarrow{E} = -\frac{1}{2}\left(\frac{D\sigma}{\sigma} \cdot \overrightarrow{E} + \overrightarrow{E} \cdot \frac{D\sigma}{\sigma} \right). \tag{15}$$

Notice

$$\frac{1}{2}\frac{D\sigma}{\sigma} = \frac{D\sqrt{\sigma}}{\sqrt{\sigma}},$$

thus introducing the notations

$$\overrightarrow{\mathcal{E}} = \sqrt{\sigma}\overrightarrow{E}, \tag{16}$$

and

$$\overrightarrow{\sigma} = \frac{D\sqrt{\sigma}}{\sqrt{\sigma}}, \tag{17}$$

equality (15) turns into the quaternionic equation

$$\left(D + M^{\overrightarrow{\sigma}}\right)\overrightarrow{\mathcal{E}} = 0, \tag{18}$$

which is a quaternionic reformulation of (1).

A. The two-dimensional case

Let us consider the special situation when

$$\overrightarrow{\mathcal{E}} = \mathcal{E}_1 e_1 + \mathcal{E}_2 e_2 \tag{19}$$

and when σ depends upon two spatial variables $\sigma = \sigma(x_1, x_2)$. We obtain that (17) adopts the form

$$\overrightarrow{\sigma} = \frac{\partial_1\sqrt{\sigma}}{\sqrt{\sigma}} e_1 + \frac{\partial_2\sqrt{\sigma}}{\sqrt{\sigma}} e_2.$$

Let us denote

$$\sigma_1 = \frac{\partial_1\sqrt{\sigma}}{\sqrt{\sigma}}, \quad \sigma_2 = \frac{\partial_2\sqrt{\sigma}}{\sqrt{\sigma}}. \tag{20}$$

Substituting the expressions (19) and (20) in (18) we have

$$D(\mathcal{E}_1 e_1 + \mathcal{E}_2 e_2) + (\mathcal{E}_1 e_1 + \mathcal{E}_2 e_2)(\sigma_1 e_1 + \sigma_2 e_2) = 0,$$

which is equivalent to the system

$$\begin{aligned} \partial_1\mathcal{E}_1 + \partial_2\mathcal{E}_2 &= -\mathcal{E}_1\sigma_1 - \mathcal{E}_2\sigma_2, \\ \partial_1\mathcal{E}_2 - \partial_2\mathcal{E}_1 &= \mathcal{E}_3\sigma_1 - \mathcal{E}_1\sigma_2, \\ \partial_3\mathcal{E}_1 &= \partial_3\mathcal{E}_2 = 0. \end{aligned}$$

Multiplying the second equation by $-i$ and adding to the first, it yields

$$\partial_{\overline{z}}(\mathcal{E}_1 - i\mathcal{E}_2) + (\sigma_1 - i\sigma_2)(\mathcal{E}_1 - i\mathcal{E}_2) = 0,$$

but according to (20)

$$\sigma_1 - i\sigma_2 = \frac{\partial_z\sqrt{\sigma}}{\sqrt{\sigma}}.$$

Taking this into account and introducing the notation

$$\mathcal{E} = \mathcal{E}_1 - i\mathcal{E}_2,$$

we have

$$\partial_{\overline{z}}\mathcal{E} + \frac{\partial_z\sqrt{\sigma}}{\sqrt{\sigma}}\overline{\mathcal{E}} = 0. \tag{21}$$

This equation is closely related with a Vekua equation of the form

$$\partial_{\overline{z}}W - \frac{\partial_{\overline{z}}\sqrt{\sigma}}{\sqrt{\sigma}}\overline{W} = 0, \tag{22}$$

as we shall expose in the following paragraphs [10].

Let

$$F = \sqrt{\sigma} \text{ and } G = \frac{i}{\sqrt{\sigma}}. \tag{23}$$

It is easy to verify that these functions satisfy (4), so they constitute a generating pair whose characteristic coefficients, according with (7), are

$$A_{(F,G)} = a_{(F,G)} = 0,$$
$$B_{(F,G)} = \frac{\partial_z\sqrt{\sigma}}{\sqrt{\sigma}}, \quad b_{(F,G)} = \frac{\partial_z\sqrt{\sigma}}{\sqrt{\sigma}}.$$

In concordance with *Definition 2*, a successor generating pair (F_1, G_1) of $\left(\sqrt{\sigma}, \frac{i}{\sqrt{\sigma}}\right)$ must have characteristic coefficients

$$a_{(F_1,G_1)} = 0, \quad b_{(F_1,G_1)} = -\frac{\partial_z\sqrt{\sigma}}{\sqrt{\sigma}};$$

and a (F_1, G_1)-pseudoanalytic function $\mathcal{E}$ must fulfill equation (21). Thus by *Theorem 3*, the $\left(\sqrt{\sigma}, \frac{i}{\sqrt{\sigma}}\right)$-derivative of any solution of (22) will be a solution of (21).

Moreover, since the general solution of (22) can be represented asymptotically by means of (11), once we achieve to build a generating sequence where the generating pair $\left(\sqrt{\sigma}, \frac{i}{\sqrt{\sigma}}\right)$ is embedded, we will be able to express the general solution of (21) as the $\left(\sqrt{\sigma}, \frac{i}{\sqrt{\sigma}}\right)$-derivative of the general solution of (22), in the way we have explained it in the last paragraph of Preliminaries.

B. Explicit generating sequence for the case when σ is a separable-variables function of the form $\sigma = U^2(x_1)V^2(x_2)$

Since the early appearing of Bers pseudoanalytic function theory [2], a very interesting problem was located around the techniques for constructing explicit generating sequences, once a generating pair for a Vekua equation is given. The explicit generating sequence is required because it is the only way to express analytically the solution of such Vekua equation in terms of Taylor series in formal powers.

When considering the electrical impedance equation, a very important physical case is referred to a conductivity σ that can be expressed as a separable-variables function of the form

$$\sigma(x_1, x_2) = U^2(x_1)V^2(x_2),$$

because this is a very useful approach for the problem of electrical impedance tomography.

For this case, an explicit generating sequence was recently introduced by V. Kravchenko in [7] as follows.

Theorem 10: Let (F, G) be a generating pair of the form

$$F = \sqrt{\sigma} = U(x_1)V(x_2),$$
$$G = \frac{i}{\sqrt{\sigma}} = \frac{i}{U(x_1)V(x_2)}.$$

It is possible to check that this generating pair is embedded in the generating sequence $\{(F_m, G_m)\}$, $m = \pm1, \pm2, \pm3, ...$ defined as

$$F_m = (x_1 + ix_2)^m U(x_1)V(x_2),$$
$$G_m = i\frac{(x_1 + ix_2)^m}{U(x_1)V(x_2)};$$

for even m, and

$$F_m = \frac{(x_1 + ix_2)^m}{U(x_1)}V(x_2),$$
$$G_m = i\frac{(x_1 + ix_2)^m}{V(x_2)}U(x_1)$$

for odd m.

Remark 11: Given the explicit generating sequence for two arbitrary non-vanishing functions $U(x_1)$ and $V(x_2)$, such that the conductivity function $\sigma = U^2(x_1)V^2(x_2)$, we are in the possibility of calculating analytically the Taylor series in formal powers that will constitute the general solution of the Vekua equation (22). The $\left(\sqrt{\sigma}, \frac{i}{\sqrt{\sigma}}\right)$-derivative of such solution, according to *Theorem 3*, will be the general solution of equation (21). The real and the imaginary components of the solution of (21) will constitute the general solution for the two-dimensional case of the quaternionic equation (18). Hence, using (16), it immediately follows we are able to write the general solution of the two-dimensional electrical impedance equation (1).

IV. Conclusions

Since the study of equation (1) is the base for well understanding the electrical impedance tomography problem, the possibility to express the general solution of (1) by means of Taylor series in formal powers, opens a new path for improving the convergence speed of numerical methods designed for image reconstruction.

We should notice that the mathematical methods exposed before, minimally restrict the conductivity function σ. Indeed, it is only necessary for σ to be a separable-variables function in the Cartesian plane, and to be at least once derivable. This is a very general case which includes most part of mathematical approaches for physical situations in electrical impedance tomography (see e.g. [4], [12]).

Beside, the numerical methods involved in these procedures concern almost exclusively to the evaluation of the integral operators related to the construction of formal powers, which in fact can be accomplished by quite standard techniques. This leads our further discussions to approach the constants for Taylor series at the moment of solving the problem of electrical impedance tomography.

Notice also that the equivalence of the electrical impedance equation, in the two-dimensional case, with a Vekua equation is precisely the key that warrants the uniqueness of the solution of Calderon problem [1], hence from a proper point of view, it is justified to compare directly the value of the computed potentials with the physical lectures. In the opinion of the authors, this will work as a powerful complement to the well developed electronic systems [5][13] designed for detecting with high accuracy the potentials around the domains of interest of tomography.

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