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LQR-Based Trajectory Control Of Full Envelope, Autonomous Helicopter

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***Abstract*-Controlling rotary wing platforms, especially on helicopters, is a difficult task because of the nonlinearity of the structure and strong coupled motion dynamics. In this paper, a linear, quadratic regulator method is used to control the trajectory and mission paths of the autonomous helicopter. Nonlinear motion dynamics is linearized at certain operating points and linear model is obtained by Taylor's series expansion. This model is integrated into MATLAB® program. By using LQR (Linear Quadratic Regulator) methodology, the attitude of [1]the autonomous Puma helicopter is controlled and two simulations are realized. The results show that this approach can effectively be applied to control rotary wing platforms on helicopters.**

I. INTRODUCTION

In recent years the concept of controlling autonomous helicopters has gained a big attraction because of their vertical take-off/landing advantages and hovering. Although the coupled and nonlinear dynamics of the helicopter make the attitude control a difficult task, numerous control techniques are applied to perform missions of hovering, aggressive manoeuvring, course keeping etc. But conventional techniques like PD or PID have shown to be insufficient to control such a platform. Even for an experienced engineer it is hard to regulate considerable amount of parameters of the 6-degrees of freedom helicopter. In the literature, LQR has shown to be a very efficient and relatively easy way to utilize with respect to other control methods that have been applied to helicopters.

Despite the fact that many researchers applied optimal control techniques to small scale helicopters [1], there is only a few studies focusing on full envelope helicopter control. Though, in war/tactical simulators it is necessary for the full envelope platforms have middle/high fidelity relative to real helicopters. The helicopter's control members of the simulator must hover, take-off and follow a path, etc. So this study aims to clarify the main points of modelling, trajectory/attitude control of the helicopter by LQR contributing to easy control of rotary wing platforms.

II. MANUAL CONTROL OF HELICOPTER

Due to the strong coupling between the longitudinal and lateral motion of the helicopter, the work of the pilot is harder than an aircraft pilot. Pilot should simultaneously control three controllers, collective, cyclic, and tail pedals. Collective controller helps pilots to adjust the altitude. Cyclic controller enables pilots to change the angle of attack so longitudinal and lateral motion can be performed. By tail pedals the angle of attack of the tail rotor is changed so yaw motion is performed. Any small mistake can cause collapse of the platform. In this paper, owing to the optimal control methods, the controller gains will be hold at the optimum values. These inputs will be defined in this study as $\theta_{0mr}, a_1 / b_1, \theta_{0tr}$ respectively.

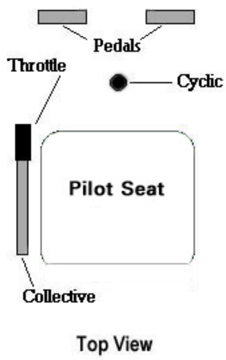

Figure 2.1 Helicopter's input controllers

III. MATHEMATICAL MODELLING OF HELICOPTER

A. Coordinate Frames and Transformations

Body fixed and earth fixed frames are needed to demonstrate the motion of the helicopter.. Force, moment and other effects follow these two main frames. The origin of the body fixed frame is the center of gravity of the platform, and it moves with the motion of the fuselage. In body fixed frame, x shows longitudinal, y shows lateral, and z shows up/down movement. In the latter coordinate system, x points the north, y points east, and z points the center of the earth. Earth fixed frame notation is necessary for the calculation of the displacements. Figure 3.1 shows body fixed and earth fixed frames.

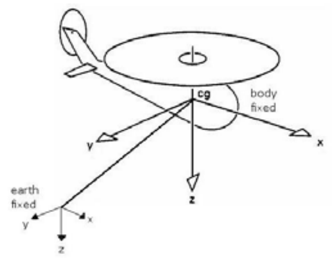

Figure 3.1 Helicopter's two main frames

To transform between body and earth frames, orthonormal rotation matrix R is used. Motion equations are multiplied with R, which is the result of the rotation by Euler angles. [2] yaw, pitch, roll rotation is the standard in aircraft modelling. As cΘ shows cos(Θ) and sΘ shows, two rotation matrices is as follows.

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Earth frame to body frame :

$$R_{be} = \begin{bmatrix} c\Theta c\Psi & c\Theta s\Psi & -s\Theta \\ s\Phi s\Theta c\Psi - c\Phi s\Psi & s\Phi s\Theta s\Psi + c\Phi c\Psi & s\Phi c\Theta \\ c\Phi s\Theta c\Psi + s\Phi s\Psi & c\Phi s\Theta s\Psi - s\Phi c\Psi & c\Phi c\Theta \end{bmatrix} (3.1)$$

Body frame to earth frame :

$$R_{eb} = \begin{bmatrix} c\Theta c\Psi & s\Phi s\Theta c\Psi - c\Phi s\Psi & c\Phi s\Theta c\Psi + s\Phi s\Psi \\ c\Theta s\Psi & s\Phi s\Theta s\Psi + c\Phi c\Psi & c\Phi s\Theta s\Psi - s\Phi c\Psi \\ -s\Theta & s\Phi c\Theta & c\Phi c\Theta \end{bmatrix} (3.2)$$

B. Dynamic Equations of Motion

By assuming that the platform as a rigid body, any two points on the helicopter does not change during the mission. The fuselage can make two types of movements: translational and rotational. They define change in position and rotate around an axis respectively.

Translational motion, which is the motion of the center of gravity, can be defined by Newton's second law and Coriolis Effect. Linear accelerations along x, y, and z axes can be defined as:

$$\begin{bmatrix} \dot{u} = vr - qw + \frac{F_x}{m} & (3.3) \\ \dot{v} = pw - ur + \frac{F_y}{m} & (3.4) \\ \dot{w} = uq - pv + \frac{F_z}{m} & (3.5) \end{bmatrix}$$

Angular accelerations around x, y, and z axes can be defined as:

$$\begin{bmatrix} \dot{p} = qr\frac{I_{yy} - I_{zz}}{I_{xx}} + \frac{M_x}{I_{xx}} & (3.6) \\ \dot{q} = pr\frac{I_{zz} - I_{xx}}{I_{yy}} + \frac{M_y}{I_{yy}} & (3.7) \\ \dot{r} = pq\frac{I_{xx} - I_{yy}}{I_{zz}} + \frac{M_z}{I_{zz}} & (3.8) \end{bmatrix}$$

C. Kinematic Equations

Kinematic equations must be used to represent the motion of the helicopter with respect to earth fixed frame,. For translational kinematics, relation between body and earth fixed frame is as follows, where x_E, y_E, z_E identifies position of the helicopter with respect to earth-fixed frame.

$$\frac{dy}{dx}\begin{bmatrix} x_E \\ y_E \\ z_E \end{bmatrix} = R_{eb}\begin{bmatrix} u_B \\ v_B \\ w_B \end{bmatrix} (3.9)$$

Rotational kinematic equations of helicopter are as fallows, where ϕ, θ, ψ defines Euler angles of roll, pitch, and yaw respectively.

$$\begin{bmatrix} \dot{\phi} = p + \tan(\theta)\left[q\sin(\phi) + r\cos(\phi)\right] \\ \dot{\theta} = q\cos(\phi) - r\sin(\phi) \\ \dot{\psi} = \left[q\sin(\phi) + r\cos(\phi)\right]\sec(\theta) \end{bmatrix} \begin{matrix} (3.10) \\ (3.11) \\ (3.12) \end{matrix}$$

D. Force and Moments Acting on Helicopter

In order to represent the motion of the helicopter, force and moment effects must be taken into account.

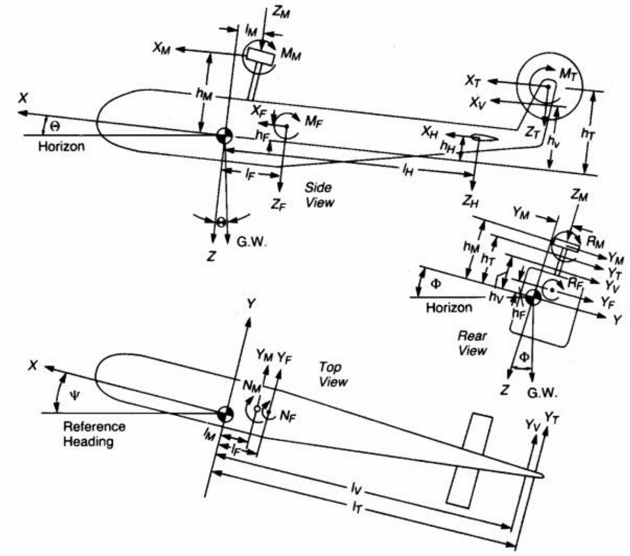

Figure 3.2 Force and moments acting on the platform [3]

Helicopter can be modeled by combining five subsystems: main-rotor, fuselage, empennage (consist of horizontal stabilizer and vertical fin), tail rotor and engine [4]. To define the force and moment effects originated from main rotor, tail rotor, gravity and drag on main rotor; mr, tr, g, and d subscripts are used respectively.

$$\begin{bmatrix} F_x = X_{mr} + X_{tr} + X_g \\ F_y = Y_{mr} + Y_{tr} + Y_g \\ F_z = Z_{mr} + Z_{tr} + Z_g \\ L = L_{mr} + L_{tr} + L_d \\ M = M_{mr} + M_{tr} + M_d \\ N = N_{mr} + N_{tr} + N_d \end{bmatrix} \begin{matrix} (3.13) \\ (3.14) \\ (3.15) \\ (3.16) \\ (3.17) \\ (3.18) \end{matrix}$$

As T, which is equal to $C_T \rho(\Omega R)^2 \pi R^2$, shows thrust, a1 and b1 shows longitudinal flapping angle and lateral flapping angle respectively, we obtain combined force equation matrix:

$$\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} -T_{mr} * \sin(a_1) - \sin(\theta) * mg \\ T_{mr} * \sin(b_1) + T_{tr} + \sin(\phi) * \cos(\theta) * mg \\ -T_{mr} * \cos(b_1) * \cos(a_1) + \cos(\phi) * \cos(\theta) * mg \end{bmatrix} \begin{matrix} (3.19) \\ (3.20) \\ (3.21) \end{matrix}$$

As h_{mr}, h_{tr} represents distance between cog and main/tail rotor along z axis, l_{mr}, l_{tr} represents distance between cog and main/tail rotor along x axis, Q_{mr} defines counter torque that comes from the drag of main rotor, we can obtain combined torque equation matrix:

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$$\begin{bmatrix}\tau_x\\ \tau_y\\ \tau_z\end{bmatrix}=\begin{bmatrix}Y_{mr}*h_{mr}-Z_{mr}*y_{mr}+Y_{tr}*h_{tr}+Q_{mr}*\sin(a_1)\\ -X_{mr}*h_{mr}-Z_{mr}*l_{mr}-Q_{mr}*\sin(b_1)\\ X_{mr}*y_{mr}+Y_{mr}*l_{mr}-Y_{tr}*l_{tr}+Q_{mr}*\cos(a_1)*\cos(b_1)\end{bmatrix}\begin{matrix}(3.22)\\(3.23)\\(3.24)\end{matrix}$$

Mathematical model can be build by using nonlinear variables given above . To apply a linear controller the model must be linearized about certain operating points which will be covered in the following section.

E. Trimming and Linearization

Nonlinear motion equations must be linearized about certain operating points. Although many researchers have generally used one operating point in their models, to increase the fidelity of the model 8 trim points (0, 20, 40, 60, 80, 100, 120 and 140 knots) have been used. First assuming that linear and angular accelerations are zero; setting the trimming forward/ side/ vertical velocity and heading rate to our desired values. Then the trimming algorithm is run until the estimated values of ϕ, θ, a_1 and b_1 converge.

By using Taylor's series expansion, external forces acting on platform become linear functions of perturbed states. Total force along x axis, by the advantage of small perturbation theory $(x = x_e + \Delta x)$, can be written as follows:

$$\begin{bmatrix}F_x = X = X_e+\frac{\partial x}{\partial u}\Delta u+\frac{\partial x}{\partial w}\Delta w+\frac{\partial x}{\partial q}\Delta q+\frac{\partial x}{\partial \theta}\Delta\theta+\frac{\partial x}{\partial v}\Delta v\\ +\frac{\partial x}{\partial p}\Delta p+\frac{\partial x}{\partial \phi}\Delta\phi+\frac{\partial x}{\partial r}\Delta r+\frac{\partial x}{\partial \theta_{0mr}}\Delta\theta_{0mr}+\frac{\partial x}{\partial a_1}\Delta a_1\\ +\frac{\partial x}{\partial b_1}\Delta b_1+\frac{\partial x}{\partial \theta_{0tr}}\Delta\theta_{0tr}\end{bmatrix}(3.25)$$

Then

$$\begin{bmatrix}X = X_e+X_u\Delta u+X_w\Delta w+X_q\Delta q+X_\theta\Delta\theta+X_v\Delta v\\ +X_p\Delta p+X_\phi\Delta\phi+X_r\Delta r+X_{\theta_{0mr}}\Delta\theta_{0mr}+X_{a_1}\Delta a_1\\ +X_{b_1}\Delta b_1+X_{\theta_{0tr}}\Delta\theta_{0tr}\end{bmatrix}(3.26)$$

If we consider that the motion can be described nonlinearly as $\dot{x} = F(x,u,t)$, the linearized model can be defined as $\dot{x} = Ax + Bu$,

where $x = \begin{bmatrix}u & w & q & \theta & v & p & \phi & r\end{bmatrix}$

and $u = \begin{bmatrix}\theta_{0mr} & a_1 & b_1 & \theta_{0tr}\end{bmatrix}$

The coefficients like $X_u, X_w, \ldots$ are called stability derivatives in flight dynamics.

The result of these formulations can be found in appendix as A and B matrices.

F. Obtaining The Stability Derivatives

Stability derivatives can be calculated by using numerical (which is mentioned in the previous section) and/or analytical methods. By using platform's main characteristics [5], [6] all derivatives can be figured. For example X_u can be found analytically by the equations below:

$$\begin{bmatrix}\frac{\partial X_{mr}}{\partial u}=\frac{\partial(Ta_1)}{\partial u}=\frac{\partial T}{\partial u}a_1+T\frac{\partial a_1}{\partial u}\\ \frac{\partial T}{\partial u}=\rho(\Omega R)^2\pi R^2\frac{\partial C_T}{\partial u}\\ \frac{\partial C_T}{\partial u}=\frac{a\sigma}{2}\left[\theta_o\left(\frac{\partial\mu}{\partial u}\right)-\frac{\partial\lambda_o}{\partial u}\right]\\ \frac{\partial\mu}{\partial u}=\frac{u}{(\sqrt{u^2+v^2})\Omega R}\end{bmatrix}(3.27)$$

Calculation of other derivatives was not mentioned in this paper. Further equations can be found from [4] PUMA type helicopter's stability derivatives was used for the control study. Calculated derivatives for 140 knots and the specific platform data can be found in appendix as tables.

IV. CONTROLLER DESIGN

As seen in Figure 4.1 proposed LQR based controllers consist of three subsystem:. state feedback controller, state integrator and PI controller. And gain scheduling is used to reflect the change in platform dynamics with respect to the forward velocity.

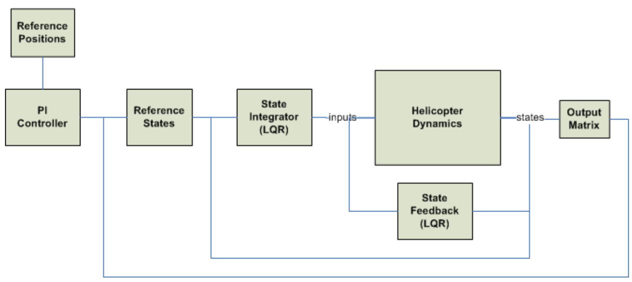

Figure 4.1 Block diagram of controllers

$\dot{e} = \dot{x}_{desired} - \dot{x}_{actual}$. *After integrating* $\dot{e}$, *e is obtained. Then new control input becomes* $u = K_2 * e - K * \dot{x}$

A. State Feedback Controller

Full-state feedback control algorithm tries to minimize the performance index (J) , where x: states, u: inputs, Q and R are weighting matrices

$$J = \frac{1}{2}\int_0^\infty (xQx^T + uRu^T)dt$$

After deciding appropriate weighting matrices according to set/rise time, overshoot and controlling effort and feedback gain (K) are calculated. By solving Riccati equation, which is calculated and used offline, LQR gains are obtained. Then the input becomes $u = -Kx$

Step response of open loop (left) and LQR closed loop (right) systems from collective input to longitudinal velocity can be seen in figure below.

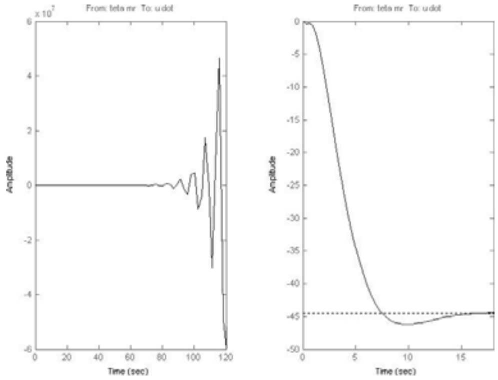

Figure 4.2 Step response

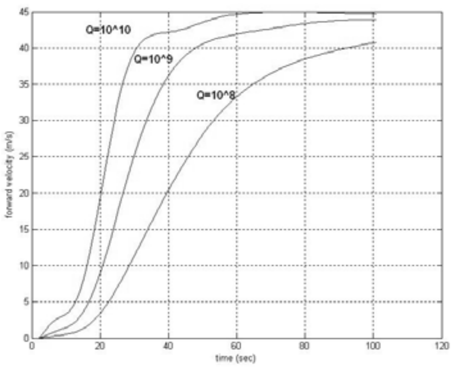

Figure 4.3 Velocity for different Q values

B. State Integrator

Full state feedback control gives adequate results. But controller will adjust the system to make states zero. For tracking control of the helicopter, the error between reference states and actual states must be taken into account. So the error term is defined as

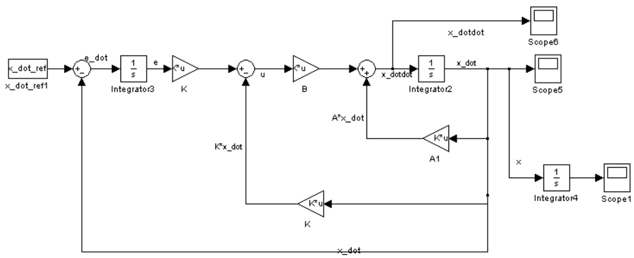

Figure 4.4 Matlab Simulink block diagrams of controller

Comparison of the desired and actual longitudinal, vertical and lateral velocity can be seen from figure below respectively. Thick lines are desired values, thin lines show actual velocities.

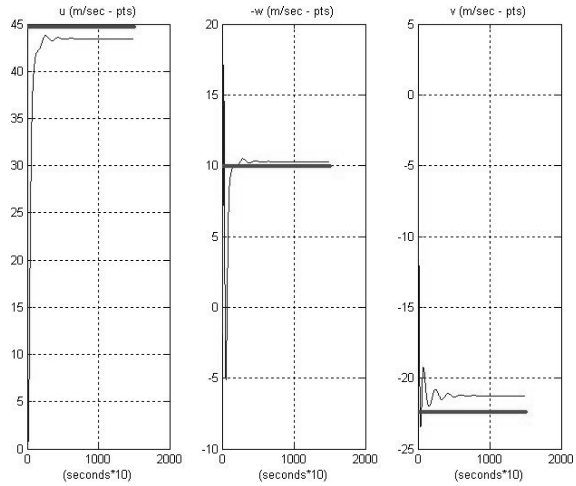

Figure 4.5 Comparisons of u, w, and v

C. PI Controller

After controlling the states and setting that values according to the reference states, for position control, which has slower dynamics than attitude control a proportional-integral feedback controller is used. Position error is calculated as $e_{pos} = [x, y, z]_{desired} - [x, y, z]_{actual}$. By using classical $K_P * e_{pos} + K_I \int e_{pos} dt$ formula the trajectory control is realized.

V. SIMULATIONS

Following two scenarios, namely movement to point and movement through waypoints, are formed for testing the controllers.

A. Movement To Point

Initial point=[x z y]=[0 0 0]
Target point=[2000 -300 -1500] meters
Composite velocity (of u and v) is 20 m/sec

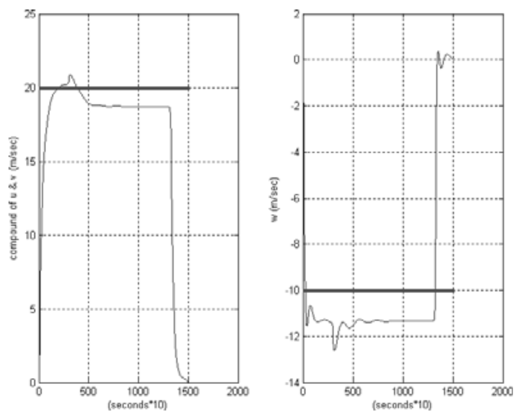

Figure 5.1 Resultant velocities

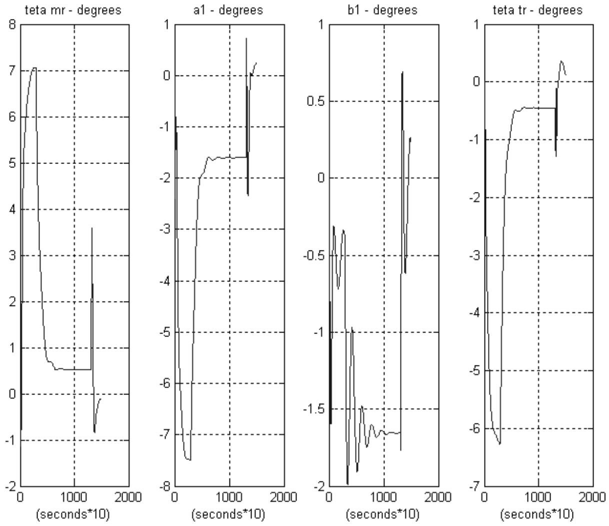

Figure 5.2 Control efforts (u)

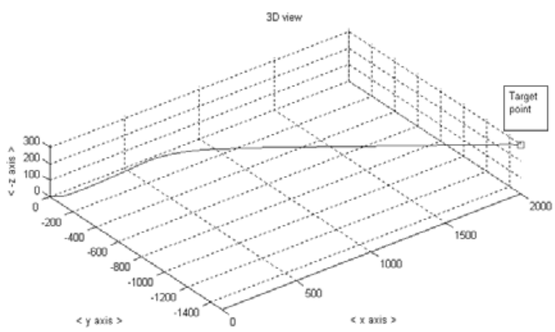

Figure 5.3 3D view of the motion

B. Movement through waypoints

Initial points: [x z y] = [0 0 0] meters

Four waypoints were selected as follows.
Waypoint 1= [2000 -300 -1500]
Waypoint 2= [4000 -300 -1500]
Waypoint 3= [2000 -300 0]
Waypoint 4= [0 -300 0]

Composite velocity (of u and v) was commanded as 20 m/sec. In figure below, squares show the waypoints and lines show the actual way of the helicopter.

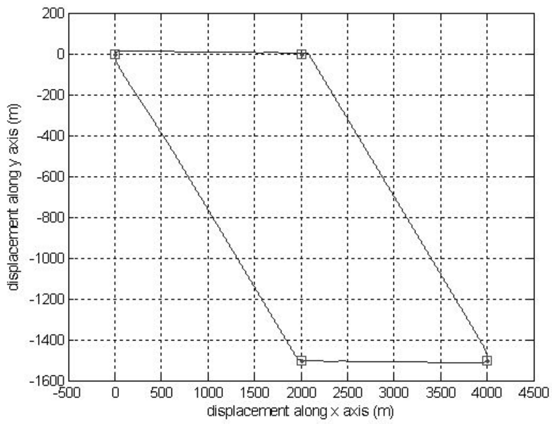

Figure 5.4 2D view of motion through waypoints

VI. CONCLUSION

To control an unmanned air vehicle, kinematics, dynamics and mathematical modelling of the platform was examined in detail. Optimal and classical control techniques were applied to achieve the missions.

Basic results of the study and future work can be summarized as follows:

- A platform which has strong coupling affects can effectively be controlled by LQR method. Fast dynamics and control efforts can easily be optimized to reflect the real motion of the helicopter in simulators.
- For full envelope platforms, PI control is sufficient to control slow dynamics like position control.
- Obstacle avoidance algorithms can be integrated to this study to use in tactic environment.

APPENDIX

$$A=\begin{bmatrix} X_u & X_w-Q & X_q-w & -g\cos(\theta) & X_v+R & X_p & 0 & X_r+V \\ Z_u+Q & Z_w & Z_q+U & -g\cos(\phi)\sin(\theta) & Z_v-P & Z_p-V & -g\sin(\phi)\cos(\theta) & Z_r \\ M_u & M_w & M_q & 0 & M_v & M_p-2P\frac{I_{xz}}{I_{yy}}-R\frac{(I_{xx}-I_{zz})}{I_{yy}} & 0 & M_r-2R\frac{I_{xz}}{I_{yy}}-P\frac{(I_{xx}-I_{zz})}{I_{yy}} \\ 0 & 0 & \cos(\phi) & 0 & 0 & 0 & -\Omega_a\cos(\theta) & -\sin(\phi) \\ Y_u-R & Y_w+P & Y_q & -g\sin(\phi)\sin(\theta) & Y_v & Y_p+W & g\cos(\phi)\cos(\theta) & Y_r-U \\ L_u' & L_w' & L_q'+k_1P-k_2R & 0 & L_v' & L_p'+k_1Q & 0 & L_r'-k_2Q \\ 0 & 0 & \sin(\phi)\tan(\theta) & \Omega_a\sec(\theta) & 0 & 1 & 0 & \cos(\phi)\tan(\theta) \\ N_u' & N_w' & N_q'-k_1R-k_3P & 0 & N_v' & N_p'+k_3Q & 0 & N_r'-k_1Q \end{bmatrix}$$

-0.0331	0.0035	4.0531	-9.7998	0.0045	0.3157	0.0000	0.0000
-0.0191	-0.9537	71.6757	0.4557	0.0394	1.1947	-0.3726	0.0000
0.0072	-0.0357	-0.9712	0.0	-0.0033	-0.2196	0.0000	0.0000
0.0000	0.0000	0.9992	0.0	0.0000	0.0000	0.0000	-0.0380
-0.0040	-0.0245	0.3172	0.173	-0.2064	-4.0045	9.7927	-71.6501
-0.0050	-0.0672	0.7554	0.0	-0.0657	-1.4244	0.0000	0.1696
0.0000	0.0000	-0.0017	0.0	0.0000	1.0000	0.0000	-0.0465
0.0071	0.0196	-0.3186	0.0	-0.0010	-0.1610	0.0000	-0.6825

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$$B = \begin{bmatrix} X_{\theta_{0mr}} & X_{a_1} & X_{b_1} & X_{\theta_{0tr}} \\ Z_{\theta_{0mr}} & Z_{a_1} & Z_{b_1} & Z_{\theta_{0tr}} \\ M_{\theta_{0mr}} & M_{a_1} & M_{b_1} & M_{\theta_{0tr}} \\ 0 & 0 & 0 & 0 \\ Y_{\theta_{0mr}} & Y_{a_1} & Y_{b_1} & Y_{\theta_{0tr}} \\ L'_{\theta_{0mr}} & L'_{a_1} & L'_{b_1} & L'_{\theta_{0tr}} \\ 0 & 0 & 0 & 0 \\ N'_{\theta_{0mr}} & N'_{a_1} & N'_{b_1} & N'_{\theta_{0tr}} \end{bmatrix}$$

-3.3699	-9.5029	0.4596	0.0000
-134.3683	-59.9275	0.0000	0.0000
3.1013	6.7591	-0.3088	0.0000
0.0000	0.0000	0.0000	0.0000
-3.9131	-1.2629	9.7500	5.6673
-10.6581	-3.6219	23.2288	2.8177
0.0000	0.0000	0.0000	0.0000
-7.5059	1.5310	2.4590	-11.2488

Characteristic coefficients of Puma helicopter:

a_0	5.73/rad	I_{zz}	25 889 kg m ^{2}	x_{cg}	0.005
a_{0T}	5.73/rad	K_β	48 149 N m/rad	δ_0	0.008
α_{tp0}	−0.0262 rad	l_{fn}	9 m	δ_2	9.5
β_{fn0}	0.0175 rad	l_{tp}	9 m	δ_3	−45°
c	0.5401 m	l_T	9 m	δ_{T0}	0.008
g_T	4.82	M_a	5805 kg	δ_{T2}	9.5
h_R	2.157 m	N_b	4		
h_T	1.587 m	R	7.5 m	γ	9.374
I_β	1280 kg m ^{2}	R_T	1.56 m	γ_s	0.0873 rad
I_{xx}	9638 kg m ^{2}	S_{fn}	1.395 m ^{2}	λ_β^2	1.052
I_{xz}	2226 kg m ^{2}	S_{tp}	1.34 m ^{2}	θ_{tw}	−0.14 rad
I_{yy}	33 240 kg m ^{2}	s_T	0.19	Ω_{idle}	27 rad/s

ACKNOWLEDGMENT

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