Outer Bounds for the Symmetric Gaussian Cognitive Radio Channel with DPC Encoded Cognitive Transmitter

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Abstract—**We present new outer bounds to the individual rates and the conditions under which these bounds become tight for the symmetric Gaussian cognitive radio (CR) channel in the low interference gain regime. The CR transmitter is assumed to use dirty paper coding while deriving the outer bounds. The capacity of the CR channel in the low interference scenario is known when the CR employs "polite" approach by devoting some portion of its power to transmit primary user's (PU's) message. However, this approach does not guarantee any quality of service for the CR users. Hence, we focus on the scenario when the CR goes for the "rude" approach, does not relay PU's message and tries to maximize its own rates only. It is shown that when both CR and the PU operate in low interference gain regime, then treating interference as additive noise at the PU receiver and doing dirty paper coding at the CR is nearly optimal.**

Keywords: Cognitive radio, capacity, achievable rates, dynamic spectrum utilization

1 Introduction

The scarcity of the radio frequency (RF) spectrum along with its severe under utilization, as suggested by various government bodies like the Federal Communications Commission (FCC) in USA and Ofcom in UK, has triggered immense research activity on the concept of cognitive radio (CR) all over the world. Of the many facets that need to be dealt with, the information theoretic modeling of CR is of core importance, as it helps predict the fundamental limit of its maximum reliable data transmission.

The information theoretic model proposed in [1] represents the real world scenario that the CR will have to encounter in the presence of primary user (PU) devices. Authors in [1] characterize the CR system as an interference channel with degraded message sets (IC-DMS), since the spectrum sensing nature of the CR may enable its transmitter (TX) to know PU's message provided the PU is in close proximity of the CR. Elegantly using a combination of rate-splitting [2] and Gel'fand Pinsker (GP) [3] coding, [1] has given an achievable rate region of the so called CR-channel or IC-DMS. Further, in [1] time sharing is performed between the two extreme cases when either the CR dedicates zero power ("highly polite") or full power ("highly rude") to its message. A complete review of information theoretic studies can be found in [4] and [5].

Later, [6] determined the capacity of CR channel in a special and more reality depicting scenario when the CR is close to its own base station than the PU receiver (RX), also called *low interference regime*. [6] adopts the in-between ("polite") approach where the CR keeps some power for itself and the remaining to relay PU's message. This has an additional advantage of keeping the PU user oblivious of the CR. However, fading conditions may cause the CR to dedicate a large chunk of its power to keep the PU unaware of its presence and hence this may result in very poor data rates for the CR itself. A natural remedy to this problem for the CR is to adopt the "rude" approach i.e., use the whole of its power for its own message to maximize its data rates and thus maintain quality of service (QoS).

In this paper we characterize outer bounds for this scenario in the low interference gain situation. However, we assume that the CR TX must transmit utilizing dirty paper coding (DPC) [7]. It is to be noted that this assumption is less general for deriving outer bounds and further research needs to be done to derive the outer bounds for the most general scenario. It is worth mentioning that that capacity region for the CR channel working in exactly an opposite scenario to ours i.e., *high interference gain regime* has been computed in [8].

The rest of the paper is organized as follows: In Section 2 we present achievable rates of the CR channel. We compute the outer bounds in Section 3. Finally, we present our conclusions in Section 4.

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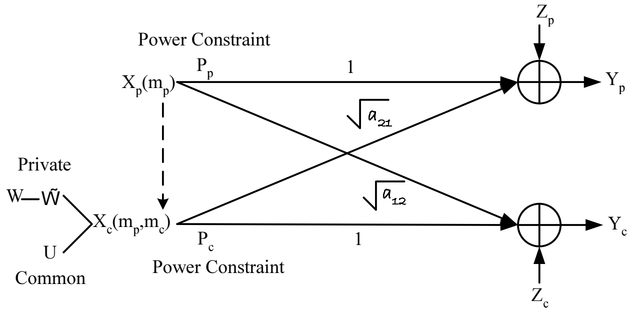

Figure 1: Cognitive radio or Interference channel with degraded message sets

2 Achievable Rates

The Gaussian IC-DMS is shown in Fig. 1. The channel in the *standard* [9] form can be mathematically represented as follows:

$$Y_p = X_p + \sqrt{a_{21}} X_c + Z_p, \quad (1)$$

$$Y_c = \sqrt{a_{12}} X_p + X_c + Z_c. \quad (2)$$

where X_p, X_c are inputs and Y_p, Y_c are outputs. For Gaussian channel the inputs follow the Gaussian distribution i.e., $X_p \sim \mathcal{N}(0, P_p)$ and $X_c \sim \mathcal{N}(0, P_c)$. P_p and P_c represent average power constraints at the PU and the CR TXs respectively. Z_p and Z_c are zero mean unit variance additive white Gaussian noise random variables at the primary and cognitive RXs respectively. $\sqrt{a_{21}}$ and $\sqrt{a_{12}}$ are the normalized link gains in the Gaussian IC-DMS. Initially, it is assumed that the CR splits its power into two portions, $(1-\alpha)P_c$ for its own usage and αP_c for relaying PU's message. Further, rate splitting at the CR TX results in the common U and private $\tilde{W}$ codebooks with $U \sim \mathcal{N}(0, \bar{\alpha}\beta P_c)$ and $\tilde{W} \sim \mathcal{N}(0, \bar{\alpha}\bar{\beta} P_c)$. It is to be noted that both $\alpha, \beta \in [0,1]$ with $\alpha = 1 - \bar{\alpha}$ and $\beta = 1 - \bar{\beta}$. W is the auxiliary random variable used for DPC purposes [7]. It should be mentioned that the common message has not been dirty paper coded. Thus, $X_c = U + \tilde{W} + \sqrt{\alpha P_c} X_p$ and (1) and (2) become:

$$Y_p = \left(\sqrt{P_p} + \sqrt{a_{21}\alpha P_c}\right) F + \sqrt{a_{21}} U + \sqrt{a_{21}} \tilde{W} + Z_p, \quad (3)$$

$$Y_c = U + \tilde{W} + \left(\sqrt{\alpha P_c} + \sqrt{a_{12} P_p}\right) F + Z_c. \quad (4)$$

where $F \sim \mathcal{N}(0,1)$ and $X_p = \sqrt{P_p} F$. With these assumptions and definitions, the mutual information terms (see [10], corollary 2) for the Gaussian IC-DMS evaluate as follows:

$$R_p \leq \frac{1}{2}\log_2\left(1 + \frac{(\sqrt{P_p} + \sqrt{a_{21}\alpha P_c})^2}{1 + a_{21}(1-\alpha)\bar{\beta} P_c}\right), \quad (5)$$

$$\begin{aligned} R_c \leq\ & \frac{1}{2}\log_2(1 + (1-\alpha)\bar{\beta} P_c) \\ & + \min\left\{\frac{1}{2}\log_2\left(1 + \frac{a_{21}\bar{\alpha}\beta P_c}{(\sqrt{P_P} + \sqrt{a_{21}\alpha P_c})^2 + a_{21}\bar{\alpha}\bar{\beta} P_c + 1}\right),\right. \\ & \left. + \frac{1}{2}\log_2\left(1 + \frac{\bar{\alpha}\beta P_c}{\bar{\alpha}\bar{\beta} P_c + (\sqrt{\alpha P_c} + \sqrt{a_{21} P_p})^2 + 1}\right)\right\}. \quad (6) \end{aligned}$$

Substituting, $\alpha = 0$, $\bar{\beta} = 1$, $P_p = P_c = P$ and $a_{12} = a_{21} = a$, thus making the channel symmetric, in the above two equations results in the following rate inequalities:

$$R_p \leq \frac{1}{2}\log_2\left(1 + \frac{P}{1 + aP}\right), \quad (7)$$

$$R_c \leq \frac{1}{2}\log_2\left(1 + P\right). \quad (8)$$

Following two important observations need to be noted here:

- The substitution $\alpha = 0$ corresponds to the fact that the CR is *not* adopting the "polite" strategy.
- Similarly, $\bar{\beta} = 1$ shows that no rate splitting is being done at the CR TX.

Achievability Argument: When the CR channel operates in low interference gain regime the interfering signal cannot be decoded by the PU RX and hence it is best to consider it as additive noise [11]. And if the CR works "selfishly" with no rate splitting we get equation (7). Similarly, if the CR TX completely knows the interfering signal it can use it for DPC purposes. And as shown in [7], it can transmit at a rate as if there was no interference at all. Thus, we obtain equation (8).

3 Outer Bounds

We will compute outer bounds for the IC-DMS in the low interference gain regime using the genie aided IC-DMS as shown in Fig. 2. However, before proceeding we again note that our channel definition is that the CR TX must transmit according to DPC. With this assumption we calculate bounds on the individual rates of the two TXs and explore the conditions under which they become tight. Clearly the capacity region of the channel in Fig. 2 is an outer bound to the capacity region of an IC-DMS that involves no rate splitting at both TXs and incorporates DPC at the CR TX. It is to be noted that, one should be

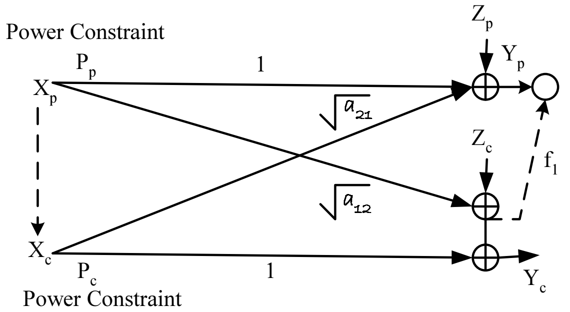

Figure 2: Genie-aided IC-DMS

careful while computing outer bounds when some side information is provided by a genie as being done here. For example, if the genie is too generous in providing side information, it may result in loose outer bounds. Thus, it is required for the genie to provide just an adequate amount of information that not only helps us derive an outer bound to the capacity, but also results in it being tight. The genie f_1 shown in the Fig. 2 is taken to be (see [12]):

$$f_1 = \sqrt{a_{12}} X_p + Z_c. \tag{9}$$

The mutual information terms representing maximum rates of the genie aided IC-DMS are:

$$R_p^* \leq I(X_p; Y_p, f_1), \tag{10}$$

$$R_c^* \leq I(X_c; Y_c | X_p). \tag{11}$$

where (11) follows from the fact that the capacity of a channel with additive Gaussian noise and power constrained input remains the same if the interfering signal is known to the encoder [7].

The mutual information term in (10) can be expanded as:

$$\begin{aligned} I(X_p; Y_p, f_1) &= I(X_p; f_1) + I(X_p; Y_p | f_1) \\ &= h(f_1) - h(f_1 | X_p) + h(Y_p | f_1) - h(Y_p | X_p, f_1) \\ &\leq h(f_1) - h(Z_c) + h(Y_p | f_1) - h(Y_p | X_p, f_1, X_c) \\ &= h(f_1) - h(Z_c) + h(Y_p | f_1) - h(Z_p). \end{aligned} \tag{12}$$

For the symmetric channel scenario $P_p = P_c = P$ and $a_{12} = a_{21} = a$. Considering the fact that X_c is DPC encoded and thus depends upon X_p, the conditional entropy term in (12) is evaluated following the usual method [13] as:

$$h(Y_p | f_1) = \frac{1}{2} \log_2 \left(2\pi e \left(1 + aP + \frac{P}{1 + aP} \right) \right). \tag{13}$$

For a unit variance additive white Gaussian noise, Z_i where $i = p, c$, the entropy is:

$$h(Z_i) = \frac{1}{2} \log_2 (2\pi e), \quad i = p, c \tag{14}$$

Similarly, for the symmetric channel under consideration the differential entropy of the genie is:

$$h(f_1) = \frac{1}{2} \log_2 (2\pi e (1 + aP)). \tag{15}$$

Combining (13), (14) and (15) the upper bound on the rate of primary is given by:

$$R_p^* \leq \frac{1}{2} \log_2 \left(1 + aP + \frac{P}{1 + aP} \right) + \frac{1}{2} \log_2 (1 + aP). \tag{16}$$

This outer bound is tight only under some particular situations. Now we explore the channel condition that make this bound a tighter one. For this, first note that the following information theoretic inequality holds:

$$\begin{aligned} I(X_p; Y_p, f_1) &= I(X_p; f_1) + I(X_p; Y_p | f_1) \\ &= I(X_p; Y_p) + I(X_p; f_1 | Y_p). \end{aligned} \tag{17}$$

For the outer bound to be tight we want that $I(X_p; f_1 | Y_p) = 0$. To derive the conditions that accomplish this we first consider a modified genie given by:

$$f_{1g} = \sqrt{a} X_p + g; \tag{18}$$

where g is arbitrarily correlated to Z_p having correlation coefficient ρ. With this modified genie we have,

$$I(X_p; f_{1g} | Y_p) = I(X_p; \sqrt{a} X_p + g | \sqrt{a} X_p + a X_c + \sqrt{a} Z_p). \tag{19}$$

The above equation equates to zero when $g = aX_c + \sqrt{a} Z_p$. Now consider the following set of equalities that will enable us to determine the conditions that result in tight outer bounds:

$$\begin{aligned} g(aX_c + \sqrt{a} Z_p) &= (aX_c + \sqrt{a} Z_p)^2 \\ \mathbb{E}\big(g(aX_c + \sqrt{a} Z_p)\big) &= \mathbb{E}\big((aX_c + \sqrt{a} Z_p)^2\big) \\ \mathbb{E}\big(\sqrt{a} g Z_p\big) &= \mathbb{E}\big(a^2 X_c^2 + a Z_p^2 + 2a^{\frac{3}{2}} X_c Z_p\big) \\ \rho &= a^{\frac{3}{2}} P_c + \sqrt{a}. \end{aligned} \tag{20}$$

Since the correlation coefficient, $\rho \leq 1$, we have:

$$a^{\frac{3}{2}} P_c + \sqrt{a} \leq 1. \tag{21}$$

The constraint given in (21) implies that $a \leq 0.25$. It means that when the interference coefficient $a \leq 0.25$, for a Gaussian IC-DMS the primary rates given in (16) are achievable by the scheme presented in the paper. It is interesting to note that in a recent work, [14], same channel condition has been obtained while deriving the sum rate capacity for Gaussian interference channels.

4 Conclusion And Future Work

In this paper we have presented outer bounds on the individual rates of Gaussian IC-DMS in low interference gain regime. We have explored the the scenarios under which the bounds become tight. However, while deriving the outer bounds we have restricted our channel definition to incorporate DPC at the CR TX. Further works needs to be done to obtain bounds in more general situation. Similarly, using the upper bounding technique presented, outer bounds on the sum rate need to be evaluated. This work will then enable the computation of outer bounds on the whole capacity region.

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