Performance Evaluation of Control Schemes Under Drift in Simple Linear Profiles

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Abstract— **Sometimes, quality of a process is characterized by a relationship between two variables referred to as profile. So far, several methods have been proposed for monitoring simple linear profiles. In all of the researches, the performance of the proposed methods is only investigated under step shifts. However, the other types of shifts such as drift can occur in the process and control charts should detect them as well. In this paper, phase II monitoring of simple linear profiles is investigated and the performance of more popular methods in the literature including T^2, EWMA-R, and EWMA-3 is evaluated under drift shift. Average run length criterion is used for this purpose. The results show that the EWMA-3 method roughly performs better than the other methods. It is roughly similar to the results obtained under the step shifts.**

Index Terms—**Average Run Length (ARL), Exponentially Weighted Moving Average (EWMA) control chart, Profile, Step shift, Drift.**

I. Introduction

Sometimes, the quality of processes or products is characterized by a relationship between a response variable and one or more explanatory variables, which is referred to as profile. Some applications in the area of Profiles monitoring have been introduced by researchers including Stover and Brill [1], Kang and Albin[2], Mahmoud and Woodall [3], Woodall et al. [4], Wang and Tsung [5], Woodall [6], Zou et al. [7], and Kazemzadeh et al. [8]. Different methods have been developed to monitor simple linear profiles in both phases I and II. In phase I, one evaluates the process stability and estimates its parameters based on a historical data set. However, the purpose of phase II analysis is to detect shifts in the process parameters as soon as possible. Kang and Albin [2] proposed two methods including T^2 and EWMA-R for monitoring simple linear profiles in both phases I and II. Kim et al. [9] coded the *x*-values to change the average to zero and make the regression parameters independent. Then they applied three distinct EWMA control charts to monitor intercept, slope and error variance. Mahmoud and Woodall [3] used a global F-test for monitoring the regression coefficients in conjunction with a univariate control chart for monitoring error variance in phase I. Noorossana and Amiri [10] proposed using a MCUSUM control chart in combination with a χ^2 control chart to improve the performance of the existing methods in phase II monitoring of simple linear profiles. Mahmoud et al. [11] and Zou et al. [12] proposed methods based on likelihood ratio statistics to monitor simple linear profiles in phases I and II, respectively. Gupta et al. [13] compared the performance of Kim et al. [9] method with a method developed by Croarkin and Varner [14]. Noorossana et al. [15] considered the case in which there is autocorrelation between simple linear profiles and proposed some methods based on time series approach to account for the AR(1) structure between profiles. The effect of non-normality on the performance of control charts for monitoring simple linear profiles is investigated by Noorossana et al. [16] and [17]. Some methods are proposed to monitor more complicated models such as multiple linear regression, polynomial and nonlinear profiles. Zou et al. [18] proposed a MEWMA control chart for monitoring general linear profiles in phase II. Kazemzadeh et al. [18] and [19] proposed some methods to monitor polynomial profiles in phases I and II, respectively. Kazemzadeh et al. [20], [21] also developed the methods by Noorossana et al. [15] to monitor autocorrelated polynomial profiles. They assumed that there is no autocorrelation within linear profiles. Jensen et al. [22] used linear mixed model (LMM) to account for the autocorrelation within multiple linear profiles. Nonlinear profile monitoring is also investigated by Jin and Shi [23], Walker and Wright [24], Ding et al. [25], Williams et al. [26], Moguerza et al. [27], Vaghefi et al. [28], and Jensen and Birch [29]. In all of the proposed methods in phase II in the literature, the performance of the methods is only investigated under step shift. In this type of shift, the parameter changes in specific time and remains fixed up to the time the assignable cause(s) detected and removed. However, sometimes it is possible to encounter with a trend shift referred to as drift shift in the literature. In this shift, the parameter will change with a trend pattern. Drifts may be due to causes such as gradual deterioration of equipment, catalyst aging, waste accumulation, or human causes, such as operator fatigue or close supervision (Reynolds et al., [30]). A lot of authors including Bissell [31], Davis and Woodall [32], Gan [33], and Aerne et al. [34] evaluated the performance of different control charts when there is a drift in the process mean. Reynolds and Stoumbos [30] considered the joint monitoring the mean and the standard deviation of the process under drifts. They showed that the combinations of the two EWMA control charts for monitoring both mean and variance have better performance than I/MR control charts in detecting slow-rate and moderate-rate drifts. However, drift shift can occur when the quality of process is characterized by a function between two variables.

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In this paper, we consider phase II monitoring of simple linear profiles and investigate the performance of more popular methods including T^2 and EWMA-R by Kang and Albin [2] and EWMA-3 by Kim et al. [9] under a linear trend shift with positive slope. The paper is outlined as follows: in section 2, the model, assumptions, and the type of shift considered in this paper are discussed. The performance of three methods including T^2, EWMA-R, and EWMA-3 is evaluated under drift shift in section 3. Our concluding remarks and some future researches are given in the final section.

II. MODEL, ASSUMPTIONS, AND TYPE OF SHIFT

We assume that for the *j*th sample collected over time we have observations (x_i, y_{ij}), $i = 1, 2,\ldots,n$. It is further assumed that when the process is under statistical control, the relationship between response and independent variable can be modeled as:

$$y_{ij} = A_0 + A_1 x_i + \varepsilon_{ij}, \tag{1}$$

where ε_{ij}'s are independent, identically distributed normal random variables with mean zero and variance σ^2. It is assumed that the *x*-values are fixed and constant from profile to profile. In this paper, we consider phase II analyses and assume that the parameters A_0, A_1, and σ^2 are known values. Kim et al. [9] coded the *x*-values so that the average coded value is zero. Then, they used three EWMA control charts to monitor the parameters of the following transformed model:

$$y_{ij} = B_0 + B_1 x_i + \varepsilon_{ij}, \tag{2}$$

where $B_0 = A_0 + A_1\bar{X}$, $B_1 = A_1$, and $X'_i = (X_i - \bar{X})$.

The drift in the regression parameters is defined as follows. Let $A_0(t), A_1(t), B_1(t)$ and $\sigma^2(t)$ be the values of the regression parameters, *t* unit time after an assignable cause starts a drift in the process. The drift is assumed to occur at a constant rate.

$$\begin{aligned} A_0(t) &= A_0 + r_{a0}t, \\ A_1(t) &= A_1 + r_{a1}t, \\ B_1(t) &= B_1 + r_{b1}t, \\ \sigma(t) &= \sigma + r_\sigma t, \end{aligned} \tag{3}$$

where r_{a0}, r_{a1}, r_{b1}, and r_σ are rates of drift for the parameters A_0, A_1, B_1, and σ per unit time, respectively.

III. EVALUTION OF CONTROL CHARTS UNDER LINEAR DRIFT

In this section, we compare The ARL performance of three methods in the literature including the T^2 and EWMA-R methods by Kang and Albin [2] and the EWMA-3 method by Kim et al. [9] under drift. We consider the same example first proposed by Kang and Albin [2] and then used by Kim et al. [9] for step shifts. The model is $y = 3 + 2x + \varepsilon$ where ε follows a standardized normal distribution with mean zero and variance 1. *X* values are equal to 2, 4, 6, and 8. In their researches, the parameters of the control charts are designed to have the same overall in-control ARL of 200. Since the parameters of control charts are determined under in-control conditions and they do not depend on the type of shifts, the parameters of the control charts under drift are the same as the parameters determined under step shift. We consider positive constant rate from very small to very large size. We use 10000 simulation runs to calculate the out-of-control ARL under drift in the intercept (A_0), the slope in both equations 1 and 2 (A_1 and B_1) and the standard deviation (σ). These parameters are the same as ones considered by Kim et al. [9]. The methods with the best performance in each considered rate are highlighted with color grey.

The out-of-control ARL values under drift shifts in the intercept are summarized in Tables 1a and 1b. As it is shown in Table 1a and 1b, EWMA-3 method performs uniformly better than the other two methods in very small and small shift rates. However, in large shifts, the performance of T^2 method is the best. As the magnitude of rates increases, the performance of the three methods gets close to each other's and finally leads to the same performance.

Table 1a: Drift shift in the intercept (A_0)

	High small to small rate									
Method \ Rate	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009	0.01
T^2	154	122.4	103.1	90.6	80.8	72.1	67	62.3	56.9	54.1
EWMA-R	124.9	90.7	74	63.3	55.4	49.9	45.8	42.3	39.5	36.7
EWMA-3	48.5	45.3	41.3	38.2	36.1	33.6	31.8	30.2	28.5	27.4

Table 1b: Drift shift in the intercept (A_0)

	small to medium rate					Medium to large rate					Large to very large rate					
Method \ Rate	0.02	0.04	0.06	0.08	0.1	0.2	0.4	0.6	0.8	1	1.5	2	2.5	3	3.5	4
T^2	34.7	21.5	16.2	13.1	11.1	6.6	4	2.9	2.4	2	1.5	1.2	1	1	1	1
EWMA-R	23.7	15.3	12	10	8.7	5.8	3.9	3.1	2.7	2.4	2	1.9	1.6	1.2	1	1
EWMA-3	20	13.8	11	9.4	8.3	5.6	3.8	3.1	2.7	2.3	2	1.8	1.5	1.2	1	1

Table 2a: Drift shift in the slope (A_1)

	High small to small rate									
Method \ Rate	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009	0.01
T2	77	51.1	39.4	32.7	28.1	24.8	22.1	20.1	18.6	17.3
EWMA-R	55.4	36.5	28.4	23.5	20.5	18.3	16.6	15.3	14.2	13.1
EWMA-3	33.4	25.7	21.3	18.4	16.5	15	13.8	12.9	12.1	11.4

Table 2b: Drift shift in the slope (A_1)

	small to medium rate					Medium to large rate					Large to very large rate					
Method \ Rate	0.02	0.04	0.06	0.08	0.1	0.2	0.4	0.6	0.8	1	1.5	2	2.5	3	3.5	4
T^2	10.4	6.2	4.6	3.7	3.1	1.9	1.1	1	1	1	1	1	1	1	1	1
EWMA-R	8.7	5.7	4.5	3.9	3.4	2.3	1.8	1.2	1	1	1	1	1	1	1	1
EWMA-3	7.8	5.4	4.3	3.7	3.3	2.2	1.8	1.1	1	1	1	1	1	1	1	1

The out-of-control ARL for drift shift in the slope in equation (1) is given in Tables 2a and 2b. The results show that the performance of EWMA-3 is better than the other methods under high small to small trend shifts in the slope. However, in medium to large shifts, T^2 method performs the best. The performance of the three methods in large to very large shifts is the same.

ARL_1 for drift in the standard deviation is also summarized in Tables 3a and 3b. As it is shown in these tables, EWMA-3 method performs better than the EWMA-R and T^2 methods under very small, small, and medium rates. On the other hand, from medium to very large shifts, the performance of EWMA-R method is the best.

Table 3a: Drift shift in the standard deviation (σ)

	High small to small rate									
Method \ Rate	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009	0.01
T2	96.7	72.2	59.6	52.3	46.3	42.4	39.1	36.5	34.3	32.4
EWMA-R	92.5	68.8	56.8	48.7	43.8	39.6	36.6	33.9	31.7	29.9
EWMA-3	41.3	35.6	31.8	29.6	27.5	25.7	24.2	23.1	22.3	21.3

Table 3b: Drift shift in the standard deviation (σ)

	small to medium rate					Medium to large rate					Large to very large rate					
Method \ Rate	0.02	0.04	0.06	0.08	0.1	0.2	0.4	0.6	0.8	1	1.5	2	2.5	3	3.5	4
T^2	21.6	14.5	11.3	9.5	8.3	5.4	3.6	2.9	2.5	2.2	1.8	1.5	1.4	1.3	1.3	1.2
EWMA-R	20	13.2	10.3	8.6	7.5	4.9	3.2	2.5	2.1	1.9	1.5	1.3	1.2	1.1	1.1	1.1
EWMA-3	15.6	11.3	9.2	8	7.1	5.1	3.6	2.9	2.5	2.3	1.9	1.6	1.5	1.3	1.2	1.2

Table 4a: Drift shift in the slope (B_1)

	High small to small rate									
Method \ Rate	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009	0.01
T^2	118.9	85.1	68.1	57.9	50.3	44.9	40.8	37.2	34.7	32.2
EWMA-R	127.1	95.9	76.6	65.2	56.7	50.8	46.1	42.1	38.8	36.6
EWMA-3	38.5	31	26.6	23.6	21.1	19.5	18.1	16.9	16.1	15.1

Table 4b: Drift shift in the slope (B_1)

	small to medium rate					Medium to large rate					Large to very large rate					
Method \ Rate	0.02	0.04	0.06	0.08	0.1	0.2	0.4	0.6	0.8	1	1.5	2	2.5	3	3.5	4
T^2	19.9	12	8.9	7.2	6.1	6.3	2.2	1.7	1.3	1.1	1	1	1	1	1	1
EWMA-R	22.4	13.5	9.9	8	6.8	4	2.4	1.8	1.4	1.2	1	1	1	1	1	1
EWMA-3	10.5	7.2	5.8	4.9	4.4	3	2.1	1.8	1.5	1.2	1	1	1	1	1	1

The results of out-of-control ARL under drift in the slope in equation (2) are summarized in Tables 4a and 4b. The results are similar to the ones for the slope in equation (1). In other words, in very small to medium size rates, EWMA-3 is the best. In large shifts, the T^2 method performs better than the other two methods. However, as the magnitude of shifts increases, the performance of three methods will be the same.

IV. CONCLUSIONS AND FUTURE RESEARCHES

In this paper, we investigated the performance of the more popular methods including T^2, EWMA-R, and EWMA-3 under a linear trend shift with positive rates in phase II monitoring of simple linear profiles. The results show that the EWMA-3 method performs better than the other methods under very small to medium shifts. In medium to large rates under shifts in the intercept and slope in both main and transformed model, the T^2 method roughly performs better than the other methods. However, in standard deviation shift, EWMA-R performs the best. In addition, in very large shifts, the performance of three methods is approximately the same.

As a future research, one can investigate the performance of the methods under drift shift with negative rates or study the performance of the other methods in the literature such as SLRT by Zou at al. [12], and MCUSUM/χ^2 by Noorossana and Amiri [10]. Furthermore, one can propose some new methods such as cuscore statistics to detect drift.

REFERENCES

[1] Stover, F.S., R.V. Brill, "Statistical Quality Control Applied to Ion Chromatography Calibrations", *Journal of Chromatography A*, Vol. 804, No. 1-2, pp37–43, 1998.

[2] Kang, L., S.L. Albin, "On-Line Monitoring When the Process Yields a Linear Profile", *Journal of Quality Technology*, Vol. 32, No. 4, pp418-426, 2000.

[3] Mahmoud, M.A., W.H. Woodall, "Phase I Analysis of Linear Profiles With Calibration Applications", *Technometrics*, Vol. 46, No. 4, pp380-391, 2004.

[4] Woodall, W.H., D.J. Spitzner, D.C. Montgomery, S. Gupta, "Using Control Charts to Monitor Process and Product Quality Profiles", *Journal of Quality Technology*, Vol. 36, No. 3, pp309-320, 2004.

[5] Wang, K., F. Tsung, "Using Profile Monitoring Techniques for a Data-Rich Environment with Huge Sample Size", *Quality and Reliability Engineering International*, Vol. 21, No. 7, pp677-688, 2005.

[6] Woodall, W.H., "Current Research on Profile Monitoring", *Revista Producão*, Vol.17, No. 3, pp420-425, 2007.

[7] Zou, C., F. Tsung, Z. Wang, "Monitoring General Linear Profiles Using Multivariate Exponentially Weighted Moving Average Schemes", *Technometrics*, Vol. 49, No. 4, pp395-408, 2007.

[8] Kazemzadeh, R.B., R. Noorossana, A. Amiri, "An Application of Phase I Polynomial Profiles Monitoring in Automotive Industry", *Proceedings of the 38th International Conference on Computers and Industrial Engineering, Beijing, China*, 31 October-2 November, 2008a.

[9] Kim, K., M.A. Mahmoud, W.H. Woodall, "On The Monitoring of Linear Profiles", *Journal of Quality Technology*, Vol. 35, No. 3, pp317-328, 2003.

[10] Noorossana, R., A. Amiri, “Enhancement of Linear Profiles Monitoring in Phase II”, *AmirKabir Journal of Science and Technology* (in Farsi), Vol. 18, No. 66-B, pp19-27, 2007.

[11] Mahmoud, M.A., P.A. Parker, W.H. Woodall, D.M. Hawkins, "A Change Point Method for Linear Profile Data", *Quality and Reliability Engineering International*, Vol. 23, No. 2, pp247-268, 2007.

[12] Zou, C., Y. Zhang, Z. Wang, "Control Chart Based on Change-Point Model for Monitoring Linear Profiles", *IIE Transactions*, Vol. 38, No. 12, pp1093-1103, 2006.

[13] Gupta, S., D.C. Montgomery, D.C. Woodall, "Performance Evaluation of Two Methods for Online Monitoring of Linear Calibration Profiles", *International Journal of Production Research*, Vol. 44, No. 10, pp1927-1942, 2006.

[14] Croarkin, C., Varner, R., “Measurement Assurance for Dimensional Measurements on Integrated-Circuit Photomasks”, *NBS Technical Note 1164, U.S. Department of Commerce, Washington, D.C.*, 1982.

[15] Noorossana, R., A. Amiri, P. Soleimani, "On the Monitoring of Autocorrelated Linear Profiles", *Communications in Statistics-Theory and Methods*, Vol. 37, No. 3, pp425-442, 2008a.

[16] Noorossana, R., S.A.Vaghefi, A. Amiri, "The Effect of Non-Normality on Monitoring Linear Profiles", *Proceedings of the 2nd International Industrial Engineering Conference, Riyadh, Saudi Arabi*, 2004.

[17] Noorossana, R., A. Vaghefi, M. Dorri, "The Effect of Non-Normality on Performance of Linear Profile Monitoring", *Proceedings of the 2008 IEEE Industrial Engineering and Engineering Management, Singapore*, December, pp8-11, 2008b

[18] Kazemzadeh, R.B., R. Noorossana, A. Amiri, "Phase I Monitoring of Polynomial Profiles", *Communications in Statistics-Theory and Methods*, Vol. 37, No. 10, pp1671-1686, 2008b.

[19] Kazemzadeh, R.B., R. Noorossana, A. Amiri, "Monitoring Polynomial Profiles in Quality Control Applications", *The International Journal of*

Advanced Manufacturing Technology, [online], DOI 10.1007/s00170-008-1633-z, 2008c.

[20] Kazemzadeh, R.B., R. Noorossana, A. Amiri, "Statistical Monitoring of Autocorrelated Polynomial Profiles", *Proceedings of the 9th Islamic Countries Conference on Statistical Sciences, Shah Alam, Malaysia,* December, pp12-14, 2007.

[21] Kazemzadeh, R.B., R. Noorossana, A. Amiri, "Phase II Monitoring of Autocorrelated Polynomial Profiles in AR(1) Processes", To appear in the *International Journal of Science and Technology, Scientia Iranica,* 2008d.

[22] Jensen, W.A., J.B. Birch, W.H. Woodall, "Monitoring Correlation Within Linear Profiles Using Mixed Models" *Journal of Quality Technology,* V0l. 40, No. 2, pp167-183, 2008.

[23] Jin, J., J. Shi, "Feature-Preserving Data Compression of Stamping Tonnage Information Using Wavelets", *Technometrics,* Vol.41, No. 4, pp327-339, 1999.

[24] Walker, E., S. Wright, "Comparing Curves Using Additive Models", *Journal of Quality Technology,* Vol. 34, No. 1, pp118-129, 2002.

[25] Ding, Y., L. Zeng, S. Zhou, "Phase I Analysis for Monitoring Nonlinear Profiles in Manufacturing Processes", *Journal of Quality Technology,* Vol. 38, No. 3, pp199-216, 2006.

[26] Williams, J.D., W.H. Woodall, J.B. Birch, "Statistical Monitoring of Nonlinear Product and Process Quality Profiles", *Quality and Reliability Engineering International*, Vol. 23, No. 8, pp925-941, 2007.

[27] Moguerza, J.M., A. Muñoz, S. Psarakis, "Monitoring Nonlinear Profiles Using Support Vector Machines", *Lecture Notes in Computer Science,* No. 4789, pp574-583, 2007.

[28] Vaghefi, A., S.D. Tajbakhsh, R. Noorossana, "Phase II Monitoring of Nonlinear Profiles", To appear in *Communications in Statistics-Theory and Methods*, 2009.

[29] Jensen, W. A., J. B. Birch, "Profile Monitoring via Nonlinear Mixed Model", To appear in *Journal of Quality Technolog*, 2009.

[30] Reynolds M. R., Z. G. Stoumbos, "Individuals Control Schemes for Monitoring the Mean and Variance of Processes Subject to Drifts", *Stochastic Analysis and Applications,* Vol.19, No. 5, pp863-892, 2001.

[31] Bissell, A. F., "The Performance of Control Charts and CUSUMs Under Linear Trend", *Applied Statistics*, Vol. 33, pp145-151, 1984.

[32] Davis R. B., W. H. Woodall, "Performance of the Control Chart Trend Rule Under Linear Shift", *Journal of Quality Technology*, Vol. 20, pp260–262, 1988.

[33] Gan, F. F., "EWMA Control Chart Under Linear Drift", *Journal of Statistical Computation and Simulation,* Vol. 38, pp181-200, 1991.

[34] Aerne, L. A., C. W. Champ, S. E. Rigdon, "Evaluation of Control Charts Under Linear Trend", *Communications in Statistics-Theory and Methods*, Vol. 20, pp3341–3349, 1991.