Transmit and Receive Power Optimization for Source-Initiated Broadcast in Wireless-Relay Sensor Networks

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Abstract—**Minimization of power consumption is a critical design goal for wireless-relay networks comprised of battery-powered sensor devices. In the paper, we consider a source-initiated (one-to-all) broadcast scenario in wirelessly-relay sensor networks (WSNs) with a high delivery probability requirement. We analyze both the transmission and the reception power to gain insights into the optimization of the overall power consumption of WSNs at network planning stage. Specifically, at the network planning stage, one wishes to maximize the network lifetime of the network through a proper selection of three key design parameters including the transmission range, the sleep period, and the broadcast probability. As a rule of thumb, the optimal number of nodes awaken to broadcast messages within a transmission range is constant with a given network size and path loss exponent for a target delivery probability requirement. Given the optimal settings above, we also present the trade-off between the delay requirement and power consumption. Our results can serve as general guidelines at the network planning stage.**

1 Introduction

A wireless-relay sensor network (WSN) [1] is a collection of sensor nodes with the ability to transmit and receive wirelessly. To serve the underlying application, nodes may wish to communicate with each other. Whenever necessary, nodes serve as relays to transport messages for others. For a WSN with battery-powered sensors, the ideal use of power is clearly of great significance. For many sensor applications, the node energy used for sensing is minuscule compared to that used for wireless communication [2]. As a result, the way nodes communicate with each other shall be carefully designed.

Broadcast is a fundamental communication primitive of wireless radio networks; any transmission is inherent a broadcast within its RF range. It is essential for the route discovery process, query broadcast, and topology managements [3–5] in WSNs. Flooding is an intuitive idea for broadcasting in wireless networks. To broadcast self-information to all nodes in the network, many broadcasting protocols [6–10] are proposed based on the flooding techniques. In order to prolong the network lifetime, some of the protocols [9, 10] are proposed to reduce the transmission energy by a probabilistic flooding approach.

In a number of recent studies [11, 12], many protocols have been proposed for power consumption minimization on source-initiated broadcast to prolong the network lifetime of WSNs. However, analysis for specific protocols is often complicated and is not easy to get general design concepts. In this paper, instead of case-by-case analysis, we seek insight into the optimization among three key design parameters: the transmission range, the broadcast probability, and the sleep period. We analyze both the transmission and reception power to gain insights into the optimization of the overall power consumption of a WSN.

Specifically, at the network planning stage, one wishes to maximize the network lifetime by a proper selection of the sleep period, the transmission range, and the broadcast probability. One can suppress the transmission power by employing a smaller transmission range and a lower broadcast probability; however, in order to meet the delivery time requirement, one must then use a shorter sleep period. This shall cause the reception power to increase. The optimal trade-off among these three design parameters depends on the application parameters and energy parameters such as message origination rate, reception mode power, path loss exponent, and delivery time requirement.

In this paper, we propose a "random broadcast" wirelessly relay network as a framework for our analysis. We derive the optimal values of them in various settings. Given the optimal solutions, we investigate the tradeoffs between power consumption and delay requirement. As a rule of thumb, the optimal number of nodes awaken to broadcast messages within a transmission range is constant with a given network size and path loss exponent for a target delivery probability requirement. If one can abstract the three key design parameters of a specific routing protocol, the results derived by the proposed framework can be applied. The results derived by the proposed framework can serve as design guidelines.

The rest of this paper is organized as follows. In Section 2, we present the framework of random broadcast WSNs. In Section 3, we analyze the power consumption

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as a function of application parameters. In Section 4, we derive optimal trade-offs among sleep period, transmission range, and broadcast probability with a given delivery time requirement. Concluding remarks are given in Section 5.

2 A random broadcast WSN

Many protocols have been proposed for power consumption minimization in wireless sensor networks to prolong the network lifetime of WSNs. However, analysis for specific protocols is often complicated and does not provide general design concepts. In this paper, instead of case-by-case analysis, we seek insight into the optimization among three key design parameters: transmission range, broadcast probability, and sleep period. We propose the following "random broadcast" wirelessly relay network as a framework for our analysis.

In a random broadcast network, all sensors are synchronized by a common clock. Time is partitioned into contiguous same-sized epoches of duration T_s. This duration is called the sleep period. Nodes carry out communication at the start of the sleep periods. For the rest of the period, the transceivers are shutdown to save power.

When a node received a message at the first time, the node would unilaterally determine whether or not it shall broadcast the message to its neighborhood. It is certainly possible for a node to detect the same message more than once, but nodes in the network are possible forwarding a message that is received at the first time only. We assume that the source node will broadcast a message with probability one and broadcast only once.

Clearly, one can directly investigate the effect of parameters such as sleep period and the transmission range in this framework. What may not be immediately apparent is that, due to the fact that any node that broadcasts messages can be thought of as a volunteered center node, the solution for the optimal broadcast probability hints at the corresponding choice of optimal degrees for a hierarchical topology.

3 Power consumption for a random broadcast WSN

The power consumption of the whole network will depend on the application parameters, energy parameters, and the free design parameters selected. If there is no message in the network, nodes in the network will wake up periodically with period T_s and consume energy E_r to listen to the messages. We refer to such energy consumption as the energy consumption for silence (ECS). This function is usually a simple function of sleep mechanism.

On the other hand, the additional energy consumed over silence (AECOS) is defined as the energy used to broadcast the messages on the top of the energy consumed for silence. According to this definition, if there is no traffic in the network, the average AECOS of the network is zero. To minimize the energy consumption of the network, the ECS and AECOS must be balanced carefully.

3.1 A one-dimensional random broadcast WSN

To facilitate analysis, we restrict our attention to the following one-dimensional uniform placed random broadcast network. The nodes are placed on a one-dimensional line with unit distance spacing. The nodes are labelled as node 0 to node N from left to right. Without loss of generality, we assume that node 0 generates a message to be broadcasted over N nodes. The following is the notation used in the paper.

- N: number of nodes in the network.
- T_s: as defined in section 2, the sleep period of the network. The period between nodes wake up in the network.
- T_m: the message origination period. The period that the source node broadcasts its own message to all the other nodes in the network.
- D: the transmission range.
- p: the broadcast probability per node in a sleep period.
- α: the path loss exponent.
- E_t: the energy consumption of a node to transmit a message. It is a function of the transmission range D and the path loss exponent α. For a path loss exponent α, E_t is often modelled as $E_t^1 D^\alpha$ where E_t^1 stands for the transmit energy when the reach D is set to 1.
- E_r: the energy consumption of a node to listen to the traffic in a sleep period.

For a 1-D uniform random broadcast WSN, all the nodes are assumed to use the same broadcast probability and transmission range.

We note that the model above indeed does not fully model the nature of wireless transmission, such as a gradual increase of error probability in transmission range instead of a sudden jump. Nevertheless, our simplification is not expected to affect the insights produced out of our work.

3.2 Mean AECOS for a random broadcast WSN

Here we derive the probabilistic property of energy consumption for a source-initiated message broadcast scenario in a 1-D uniform placed WSN with a high delivery probability requirement.

Define the random variable J as the number of time epoches it takes for a message originated from a node to reach all the other nodes in the network and the random variable W_k as the AECOS during sleep epoch k. Our first result is the expected value of AECOS incurred by a message propagating to the whole network. In the following derivation, we will omit the difference of energy consumption at first time epoch compared with other time epoches.

Theorem 1. *The expected value of AECOS incurred by a message propagating to the whole network is $E[J]E[W]$ where $E[W]$ stands for the expected value of AECOS during a time epoch.*

Proof. From the discussion above, as long as a transmission is to take place from some node n in an epoch k, the incurred AECOS during the epoch is independent of either n or k. Thus the expected value of AECOS is the sum of expected value of AECOS consumed in all epoches:

$$\begin{aligned} E[\text{AECOS}] &= E\left[\sum_{k=1}^{\infty} W_k\right] \\ &= \sum_{k=1}^{\infty} P(J \geq k) E[W_k | J \geq k] \\ &= E[J]E[W]. \end{aligned}$$

□

To gain better insights to the optimization of power consumption, we further approximate the expected number of time epoches that messages broadcasted to the whole network. The expected number of time epoches of a message received by all nodes in the network depends on the number of nodes in network, transmission range, and the broadcast probability selected. Assume a message is broadcasted by the source node at time epoch 0 and then successive D nodes will receive the message. The D nodes will unilaterally determine where or not it shall broadcast the message or not with a broadcast probability p. Therefore, at time epoch 1, the probability that node n will be the farthest node received the message at first time is a geometric distribution, where $D < n \leq 2D$. Note that there is a finite probability that a message does not find a new forwarding node during a sleep epoch. The expected forwarding distance experienced by the message during the time epoch 1, denoted by μ_1, is:

$$\begin{aligned} \mu_1 &= \sum_{d=1}^{D} d(1-p)^{D-d} p \\ &\approx D - \frac{1}{p}(1 - e^{-pD}). \end{aligned} \tag{1}$$

With the requirement that message are received by all nodes in the whole network of a high probability, we further approximate $E[J]$ as follows:

$$E[J] \approx N/\mu_1. \tag{2}$$

The AECOS of the whole network in a time epoch depends on the number of nodes first received the message and the broadcast probability. Intuitively, the expected forwarding distance for each time epoch can be approximated to μ_1. Therefore, we approximate the average AECOS of the whole network in a time epoch as $p\mu_1 E_t$.

The average energy consumption of the whole network will be

$$\frac{NE_r}{T_s} + E[J]\frac{p\mu_1 E_t}{T_m}. \tag{3}$$

We further approximate the probability that a message will be delivered to all nodes in the whole network successfully, the delivery probability, denoted as P_d:

$$P_d = (1 - (1-p)^{\mu_1})^{N/\mu_1}. \tag{4}$$

The approximation is quite intuitive. Specifically, the expected forwarding distance for every time epoches is μ_1, approximately. Therefore, the probability a message forwarded at a time epoch can be approximated to $(1 - (1-p)^{\mu_1})$. Moreover, a message forwarded over the whole network requires average N/μ_1 successive time epoches that some node broadcasts the message. From the above equation, we can observe that in order to delivery the message to the whole network with a high probability, the expected number of nodes broadcast in a transmission range, the pD-product, will be large.

4 Optimization for source-initiated broadcast in WSNs

In this section, we derive the optimal settings of the design parameters, including the transmission range, the sleep period, the broadcast probability for minimizing average energy consumption of the whole network. In a network, the message origination period T_m, the path loss exponent α, and the energy parameters, E_r, E_t^1 are given constants. In order to broadcast the message to whole network with a high probability, we put a constraint on the expected number of nodes broadcast in a transmission range. We assume that the node broadcast in a transmission range equal to a constant c. Moreover,we place a constraint on the delivery time at which a message must be received by all node in the network. Specifically, the mean time for a message to broadcast to the whole network must not exceed T_{max}. The delivery time constraint can be represented as

$$E[J]T_s \leq T_{max}. \tag{5}$$

We assume that nodes have energy-distance relationships of $E_t = E_t^1 D^\alpha$. We shall use the following normalized notation:

- $E_t' \equiv E_t^1/T_m$, the transmit cost, represents the transmit energy normalized by the message origination period given a range of 1; and

- $E_r' \equiv NE_r/T_{max}$, the receive cost, represents the whole network receive energy normalized by the delay requirement of message received by all nodes in the network.

It can be shown that the combination of (p^*, D^*, T_s^*) that minimized power consumption must satisfy $T_s^* = T_{max}/E[J]$. By substituting $T_s = T_{max}/E[J]$, one can express the average total energy consumption rate in (3) as:

$$E[J](\frac{NE_r}{T_{max}} + \frac{p\mu_1 E_t}{T_m}). \tag{6}$$

Henceforth, for the ease of analysis, we approximate $E[J]$ as N/μ_1. The objective function for minimization thus becomes:

$$\begin{aligned} & E[J](\frac{NE_r}{T_{max}} + \frac{p\mu_1 E_t}{T_m}) \\ & \approx N(\frac{NE_r}{T_{max}D}(1 - \frac{1-e^{-pD}}{pD})^{-1} + \frac{pE_t}{T_m}) \\ & = N(\frac{E_r'}{D}(1 - \frac{1-e^{-pD}}{pD})^{-1} + pE_t'D^{\alpha}). \end{aligned} \tag{7}$$

With the constraint on $pD = c$, we can obtained the optimal transmission range D^* by standard calculus. The optimal transmission range will be

$$D^* = \left((\alpha - 1)(c - 1 + e^{-c})\frac{E_t'}{E_r'}\right)^{-1/\alpha}. \tag{8}$$

The optimal broadcast probability p^* and optimal sleep period T_s^* can be obtained by satisfying $p^*D^* = c$ and $T_s^* = T_{max}/E[J]$. Moreover, by substituting the D^* and p^* to equation (4), we can obtain the probability that a message reach all nodes in the network. The delivery probability is a function of pD-product. By choosing the pD-product, we can obtain the approximated delivery probability and the optimal power consumption of the network. As a rule of thumb, the optimal number of nodes awaken to broadcast messages within a transmission range is constant with a given network size and path loss exponent for a target delivery probability requirement.

Given the optimal setting above, we examine the trade-off between delay requirement and power consumption. The power consumption under optimal setting can be found by substituting p^*, D^*, T_s^* into (7). It turns out that the power consumption is exactly proportional to the $\frac{\alpha-1}{\alpha}$-th power of the inverse mean delivery time.

We further analyze the balance between ECS term and AECOS term of the whole network. As the optimal settings are applied, we derive that the ratio between ECS and AECOS is exactly $\alpha - 1$. It does not depend on the energy parameters E_r, E_t^1 and application parameters T_{max}, T_m. The power consumption to listen the messages is at the same order of power consumption to forward the messages, and the ratio is higher than conventional thinking. For power consumption minimization, one should increase the ratio of power consumption to listen for messages in the whole network as the path loss exponent grows.

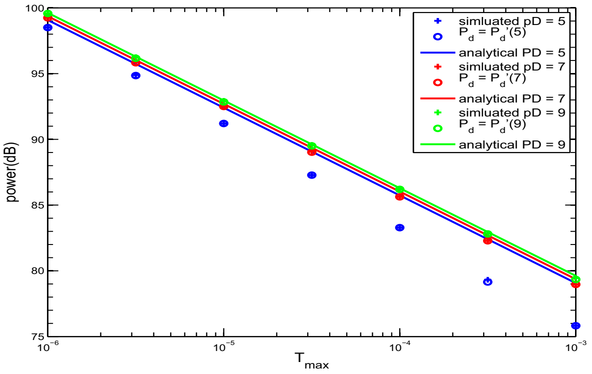

Figure 1: The optimal trade-off between delay requirement and power consumption with various pD-product.

To evaluate the performance of our results, a network consisting of $N = 1000$ nodes was simulated with the following parameter settings: the message origination period $T_m = 1$, the path loss exponent $\alpha = 3$, the transmission energy $E_t^1 = 10$, and receive energy $E_r = 1$. We simulated the network with 1000 message generated. Assume nodes are uniform placed in a 1-D line network with unit length spaced.

We will compare our results with the optimal settings by direct optimization for a delivery probability requirement. We focus on the case that the delivery probability is high. For direct optimization for a delivery probability requirement, we set the requirement on delivery probability the same as our simulated delivery probability $P_d'(pD)$ for power consumption minimization with a given pD-product. In the following, we will shows both analytical and simulated results.

Figure 1 shows the optimal power consumption with a delivery time requirement. Figure 2 shows the delivery probability as a function of delay requirement. From Figure 1, we can observe that our results are approximately optimal with a high delivery probability requirement. From both the simulated and analytical results, the power consumption of the optimal settings grows 2/3-th power of the inverse delay time, approximately. From figure 2, the delivery probability will grow as the number of nodes broadcast in a transmission range increases. This can be verified by equation (4). As a rule of thumb, regardless of how fast one requires a message to travel, the optimal number of nodes awaken in a transmission range is constant for a target delivery probability.

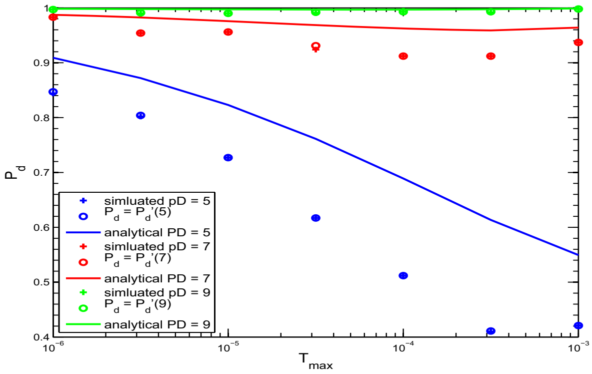

Figure 2: The delivery probability as a function of delay requirement with various pD-product.

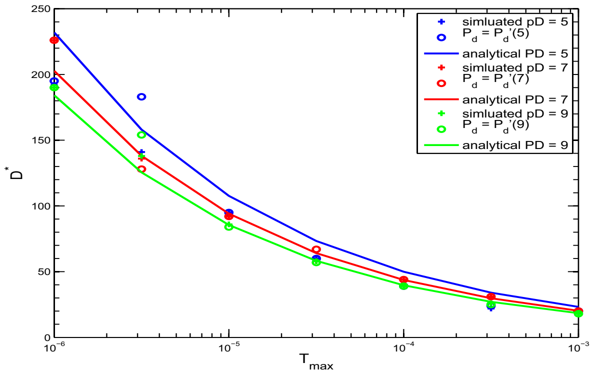

Figure 3: The optimal transmission range as a function of delay requirement.

Figure 3 shows the results for the optimal transmission range D^* as a function of delay requirement T_{max}. Figure 4 shows the optimal broadcast probability p^* as a function of delay requirement. From the figures, we can observe that the shortest hop will not be optimal, as the delay requirement is tightened. As the delay requirement is tightened, the optimal transmission range shall increase and the optimal broadcast probability shall shrink. Moreover, we can observe that the optimal pD is approximately a constant, and that the approximations are good for optimal design parameter settings with a high delivery probability requirement.

To show the power consumption penalty of the use of a non-ideal design parameter setting, we use the simple flooding and gossip routing as examples. Moreover, to ensure the delivery probability that equal to one approximately, we set $pD = 9$ for our simulation. In the simple flooding protocol, there is a node broadcast the message at every sleep period with unit length transmission range. The only free design parameter, the sleep period, will be selected to satisfy the delay requirement. The power consumption for the simple flooding can be easily computed by substituting the selected settings to equation (3). The AECOS is a constant and the ECS is proportional to the inverse delay time. For gossip routing, based on the proposed gossip-based routing in [9] and probabilistic broadcast in [10], we assume nodes in the network have a predetermined transmission range. The free design parameters are the broadcast probability and the sleep period for the gossip routing. We give a constant transmission range $D = 40$ for our simulation. We show the power consumption of the whole network as a function of delay requirement in Figure 5. It turns out that the power consumption penalty grows at a rate of 1/3-th power of inverse delay time for simple flooding, as the delay requirement is tighten. For gossip routing, starting from the optimal setting, the power consumption penalty will increase at a rate the same as simple flooding, when the delay requirement is tightened. On the other hand, as the delay requirement is loosened, the power consumption will be dominated by the AECOS. The power consumption is a constant, approximately. Therefore, the power consumption penalty grows at a rate of 2/3-th power of delay time, as the delay requirement is loosened.

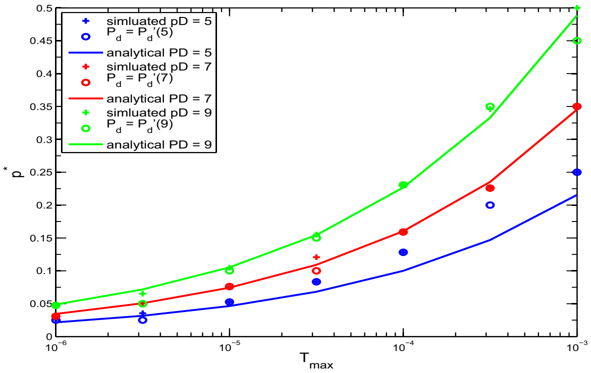

Figure 4: The optimal broadcast probability as a function of delay requirement.

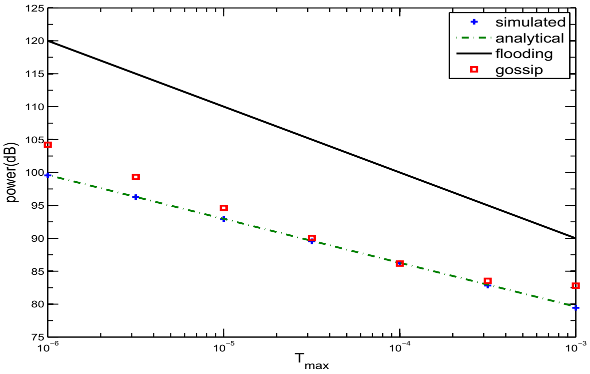

Figure 5: The power consumption comparison as a function of delay requirement.

5 Conclusion

In this paper, we investigate the power consumption for a source-initiated broadcasting in a wirelessly-relay sensor network. In light of the fact that it is typical for a low traffic network to consume most of its energy on detecting the presence of incoming packets, we specifically take into account the reception mode power entailed by the sleep operation.

We investigate the three design parameters: the transmission range, the broadcast probability, and the sleep period for power consumption minimization at the network planning stage. Suppose that the path loss exponent α, the message origination rate, and the energy for message detection are predetermined constants. To achieve the right balance among the message delivery time, the network power consumption, the delivery probability, a designer may manipulate the range of transmission, the sleep period, and the broadcast probability. The broadcast probability hints at the level of hierarchy for an actual WSN.

For a network with high delivery probability requirement, we derive the optimal settings for power consumption minimization with a delay requirement. As a rule of thumb, the optimal number of nodes awaken to broadcast messages within a transmission range is constant with a given network size and path loss exponent for a target delivery probability requirement. Although the optimal values for these three parameters depend on the message origination rate and the delay requirement, the relationship is relatively less sensitive as conventional thinking. Furthermore, given the optimal setting above, the ratio of the power consumption of listening the messages to power consumption in the whole network depends only on path loss exponent. The ratio is $\alpha - 1$. Given the optimal setting, we derive that the network power consumption grows at the $\frac{\alpha-1}{\alpha}$-th power of the factor by which the delay requirement is tightened. Our results can serve as general guidelines at the network planning stage.

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