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Electromagnetic Scattering of Plane Wave from a Finite Array of Partially-Shielded Dielectric Cylinders

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Abstract— **The rigorous Method of Regularization is applied to the problem of electromagnetic scattering of plane waves by a finite array of partially-shielded circular cylinders. A set of ill-posed dual series equations is resulted from imposing the mixed boundary conditions on the surface of each of the cylinders. Each of the M sets of dual series equations are treated separately using the Method of Regularization and transformed analytically to an infinite system of linear algebraic equations of the second kind. These M systems of equations are then solved simultaneously by the truncation method. The computed solution has properties of guaranteed accuracy and fast convergence.**

Keywords: ***electromagnetic scattering, finite array, partially-shielded dielectric cylinders, mixed boundary conditions, Method of regularization***

1 Introduction

The problem of electromagnetic (EM) scattering of an incident plane wave from an array of parallel circular cylinders has been studied extensively over the years. Various techniques have been developed to solve for the scattering problem. A comparison of some of the basic treatments to the multiple scattering problem is given in [1]. However, in most of the previous work, substantial assumptions have been made on the cylinders. The cylinders are assumed to either have the same size or are made of the same material. A few of the papers have focused on the scattering problem for a combination of dielectric and conducting circular cylinders but the surface of each of these cylinders is taken to be either purely dielectric or purely perfectly conducting (PEC). The scattering problem from a finite array of partially-shielded dielectric cylinders has not been considered before.

This paper demonstrates the feasibility of using the mathematically rigorous Method of Regularization (MoR) in solving the scattering problem by a finite array of partially-shielded dielectric cylinders. The MoR has been used extensively in the frequency domain solution of scattering by single canonical structure, such as the open spheroidal shell, toroidal shell and slotted cylinder [2, 3]. In contrast to purely-numerical methods, such as the Method of Moments applied to the electrical field integral equation, the computed solution converges to the exact solution as the truncation number increases. The present paper extends the work by investigating the scattering problem from a finite array of dielectric circular cylinders of arbitrary radii and positions, where each of these cylinders is partially-shielded with a PEC strip of arbitrary size.

The analysis starts by representing the scattered and transmitted field components from each of the M cylinders due to the incident plane wave by an infinite series of cylindrical harmonic functions with unknown coefficients. By enforcing the mixed boundary conditions on the surface of each cylinder, M sets of dual series equations (DSEs) are derived. The values of the unknown coefficients can be derived from these M sets of DSEs. A direct matrix inversion of these ill-conditioned DSEs does not guarantee a convergent solution. The MoR analytically converts these first kind singular series equations to a second kind Fredholm matrix equation. The system is then solved numerically using the truncation method. Only the detailed derivation for the TM case is given as the result can be easily extended to the TE case.

The solution computed in this paper is semi-analytic and the effects of multiple scattering between different cylinders are included in the analysis. Some numerical examples are given. The near field is calculated to prove the satisfaction of the boundary conditions on each of the surface of the cylinders.

2 Problem Formulation

The geometry of the EM scattering problem considered is as shown in Fig. 1. M infinitely-long circular cylinders are illuminated by a time harmonic plane wave that impinges normally on the z-axis and makes an angle of ϕ_{inc} with respect to the x-axis of the global coordinate system. The cylinders are assumed to be parallel to each other and to the z-axis of the global coordinate system. The gth cylinder is positioned with center located at (r_g, φ_g) of the global polar coordinates (ρ, θ). The value of its

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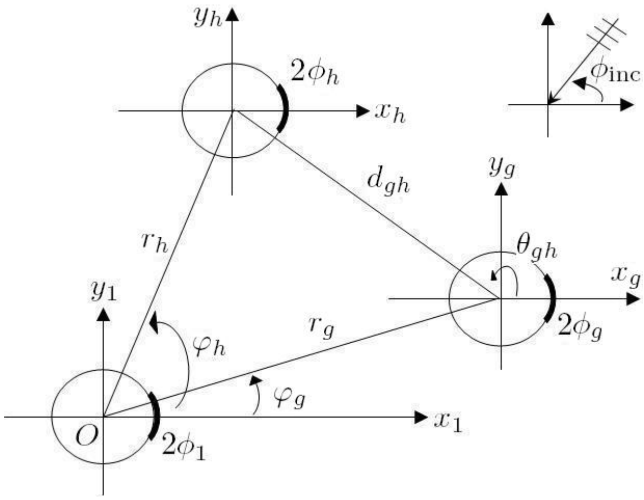

Figure 1: Cross-sectional view of the cylinders

radius is denoted as R_g. It is filled with homogeneous dielectric material with relative permittivity ϵ_g and relative permeability μ_g. An infinitely-thin, PEC strip of angular size $2\phi_g$ is placed on the surface of each of the gth cylinder as shown. The surrounding medium is taken to be free space with permittivity ϵ_0 and permeability μ_0. Time dependence of $e^{-j\omega t}$ is assumed and omitted everywhere throughout this paper.

The time harmonic Maxwells equations are solved in polar coordinates. For a TM polarized incident wave, the governing equation for the electric field component is a Helmholtz equation in polar coordinates

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}U\right)+\frac{1}{\rho^2}\frac{\partial^2}{\partial\theta^2}U+k^2U=0\,, \qquad (1)$$

where U denote the z-component of the electric field.

From Eqn.(1), the series representations of the scattered and transmitted fields for each of the cylinders, with respect to the gth coordinate system (ρ_g,θ_g), which is centered at (r_g,φ_g), are obtained as

$$U_{\text{sc}}^{(g)}(\rho_g,\theta_g)=E_0\sum_{n\in\mathbb{Z}}a_n^{(g)}H_n(k_0\rho_g)e^{jn\theta_g}\,, \qquad (2)$$

$$U_{\text{tr}}^{(g)}(\rho_g,\theta_g)=E_0\sum_{n\in\mathbb{Z}}b_n^{(g)}J_n(k_g\rho_g)e^{jn\theta_g}\,, \qquad (3)$$

where E_0 is the amplitude of the incident electric field component, $J_n(z)$ and $H_n(z)$ are respectively the Bessel function and Hankel function of the first kind (with the superscript omitted for simplicity), of order n and with argument z, $k_0=\omega\sqrt{\epsilon_0\mu_0}=2\pi/\lambda$ is the wavenumber of the free space and $k_g=k_0\sqrt{\epsilon_g\mu_g}$ is the wave number of the dielectric material inside the gth cylinder. Without loss of generality, we set $E_0=1$.

Here, $U_{\text{sc}}^{(g)}$ denotes the z-component of the scattered electric field resulting from the gth cylinder and $U_{\text{tr}}^{(g)}$ denotes the z-component of the transmitted electric field inside the gth cylinder. $a_n^{(g)}$ and $b_n^{(g)}$ are the unknown coefficients to be determined by imposing the mixed boundary conditions on each of the cylinder surfaces. It is worth noting that the expressions above satisfy the Sommerfeld radiation condition. The Meixner finite energy condition must be imposed to guarantee a unique solution. It places an additional constraint on the unknown coefficients, which after some rescaling lie in ℓ_2.

The plane wave impinging normally on the z-axis, with source lying outside of the scatterer, has the form

$$\mathbf{U}_{\text{inc}}(\rho,\theta)=E_0e^{-jk_0\rho\cos(\theta-\phi_{\text{inc}})}\,, \qquad (4)$$

in the global coordinate system. In terms of the gth local coordinate system, the incident plane wave on the gth cylinder can be expressed as

$$U_{\text{inc}}^{(g)}(\rho_g,\theta_g)=e^{-jk_0r_g\cos(\varphi_g-\phi_{\text{inc}})}\times e^{-jk_0\rho_g\cos(\theta_g-\phi_{\text{inc}})}$$
$$=\sum_{n\in\mathbb{Z}}Z_n^{(g)}e^{jn\theta_g}\,, \qquad (5)$$

where $Z_n^{(g)}=e^{-jk_0r_g\cos(\varphi_g-\phi_{\text{inc}})}(-j)^nJ_n(k_0\rho_g)e^{-jn\phi_{\text{inc}}}$.

The mixed boundary conditions on the surface of the gth cylinder are given by

$$U_{\text{inc}}^{(g)}+\sum_{h=1}^{M}U_{\text{sc}}^{(h)}=0\,, \qquad (6)$$

$$U_{\text{tr}}^{(g)}=0\,, \qquad (7)$$

for $\rho_g=R_g,|\theta_g|\le\phi_g$; that is, the tangential components of total electric fields vanish on the PEC strip, and

$$U_{\text{inc}}^{(g)}+\sum_{h=1}^{M}U_{\text{sc}}^{(h)}=U_{\text{tr}}^{(g)}\,, \qquad (8)$$

$$\frac{1}{\mu_0}\frac{\partial}{\partial\rho}\left[U_{\text{inc}}^{(g)}+\sum_{h=1}^{M}U_{\text{sc}}^{(h)}\right]=\frac{1}{\mu_g}\frac{\partial}{\partial\rho}U_{\text{tr}}^{(g)}\,, \qquad (9)$$

for $\rho_g=R_g,|\theta_g|>\phi_g$; that is, the tangential fields are continuous across the dielectric interface.

Both the scattered field and the transmitted field associated with each of the cylinders are based on its local coordinate system. However, to enforce the mixed boundary conditions on the gth cylinder, taking into account the interaction between the M cylinders, the scattered fields of all the other cylinders need to be expressed in terms of the gth coordinate system. The additional theorem for Bessel functions is used to transfer from one local coordinate system to another. The transformation from (ρ_h,θ_h) to (ρ_g,θ_g) of the Hankel function is given by

$$H_n(k\rho_h)\,e^{jn\theta_h}=\sum_{m\in\mathbb{Z}}J_m(k\rho_g)H_{m-n}(kd_{g,h})$$
$$\times\, e^{-j(m-n)\theta_{g,h}}e^{jm\theta_g}\,. \qquad (10)$$

Here, $d_{g,h} = \sqrt{r_g^2 + r_h^2 - 2r_g r_h \cos(\varphi_g - \varphi_h)}$ and $\theta_{g,h} = \pm\cos^{-1}\left(\frac{r_h \cos\varphi_h - r_g \cos\varphi_g}{d_{g,h}}\right)$, where '$-$' is taken when $r_h \sin\varphi_h < r_g \sin\varphi_g$.

3 Solution Method

We now have $2M$ sequences of unknown coefficients $a_n^{(g)}$ and $b_n^{(g)}$ $(g = 1, \ldots, M)$ to be determined from imposing the mixed boundary conditions on each of the cylinders. Due to the continuity condition, by matching the fields across each of the surfaces, everyone of the unknown $b_n^{(g)}$ can be expressed as a linear combination of all the $a_n^{(h)}$ for $h = 1, \ldots, M$. The mixed boundary conditions on the gth cylinder lead to the following DSEs

$$0 = \sum_{n\in\mathbb{Z}} e^{jn\theta_g} \left[c_n^{(g)} + Z_n^{(g)} + \sum_{h\neq g}\sum_{m\in\mathbb{Z}} c_m^{(h)} A_{n,m}^{(g,h)} \right], \quad (11)$$

for $|\theta_g| \leq \phi_g$, and

$$0 = \sum_{n\in\mathbb{Z}} e^{jn\theta_g} \left[c_n^{(g)} p_n^{(g)} + d_n^{(g)} + q_n^{(g)} \sum_{h\neq g}\sum_{m\in\mathbb{Z}} c_m^{(h)} A_{n,m}^{(g,h)} \right], \quad (12)$$

for $|\theta_g| > \phi_g$. $\sum_{h\neq g}$ denotes the summation for $h = 1, \ldots, g-1, g+1, \ldots, M$. The notations used above are

$$c_n^{(g)} = a_n^{(g)} H_n(k_0 R_g), \quad (13)$$

$$d_n^{(g)} = \frac{k_0}{\mu_0} Z_n'^{(g)} - \frac{k_g}{\mu_g} Z_n^{(g)} \frac{J_n'(k_g R_g)}{J_n(k_g R_g)}, \quad (14)$$

$$p_n^{(g)} = \frac{k_0}{\mu_0} \frac{H_n'(k_0 R_g)}{H_n(k_0 R_g)} - \frac{k_g}{\mu_g} \frac{J_n'(k_g R_g)}{J_n(k_g R_g)}, \quad (15)$$

$$q_n^{(g)} = \frac{k_0}{\mu_0} \frac{J_n'(k_0 R_g)}{J_n(k_0 R_g)} - \frac{k_g}{\mu_g} \frac{J_n'(k_g R_g)}{J_n(k_g R_g)}, \quad (16)$$

$$A_{n,m}^{(g,h)} = \frac{H_{n-m}(k_0 d_{g,h}) J_n(k_0 R_g)}{H_m(k_0 R_h)} e^{j(m-n)\theta_{g,h}}. \quad (17)$$

By examining the asymptotic behaviours of the Bessel function and the Hankel function, the following parameters for $n \neq 0$ are introduced

$$r_n^{(g)} = 1 + \frac{R_g \mu_0 \mu_g}{\mu_0 + \mu_g} \frac{p_n^{(g)}}{|n|}, \quad (18)$$

$$s_n^{(g)} = \frac{q_n^{(g)}}{|n|}. \quad (19)$$

They are asymptotically small as $n \to \infty$, both having the order of $O\left(\frac{k_g^2 R_g^2}{n^2}\right)$. With the introduction of the asymptotically small parameters, the gth set of the DSEs now takes the following form

$$\begin{aligned} 0 =& c_0^{(g)} + Z_0^{(g)} + \sum_{h\neq g}\sum_{m\in\mathbb{Z}} c_m^{(h)} A_{0,m}^{(g,h)} \\ &+ \sum_{n\neq 0} e^{jn\theta_g} \left[c_n^{(g)} + Z_n^{(g)} + \sum_{h\neq g}\sum_{m\in\mathbb{Z}} c_m^{(h)} A_{n,m}^{(g,h)} \right] \end{aligned} \quad (20)$$

for $|\theta_g| \leq \phi_g$, and

$$\begin{aligned} 0 =& \kappa_g \left[p_0^{(g)} c_0^{(g)} + d_0^{(g)} + q_0^{(g)} \sum_{h\neq g}\sum_{m\in\mathbb{Z}} c_m^{(h)} A_{0,m}^{(g,h)} \right] \\ &+ \sum_{n\neq 0} e^{jn\theta_g} |n| \left(c_n^{(g)} \left[1 - r_n^{(g)}\right] + \kappa_g \frac{d_n^{(g)}}{|n|} \right. \\ &\left. + \kappa_g s_n^{(g)} \sum_{h\neq g}\sum_{m\in\mathbb{Z}} c_m^{(h)} A_{n,m}^{(g,h)} \right) \end{aligned} \quad (21)$$

for $|\theta_g| > \phi_g$, where $\kappa_g = -\frac{R_g \mu_0 \mu_g}{\mu_0 + \mu_g}$.

The Meixner finite energy condition constrains the rescaled unknown $c_n^{(g)}$ so that $\sum_{n\in\mathbb{Z}} n|c_n^{(g)}|^2 < \infty$, for all $g = 1, \ldots, M$. Hence, the operations of term-by-term integration and differentiation in the regularization process that follows are justified and valid.

It may be shown with the ideas of [2], that the gth set of DSEs above can be transformed to two connected infinite systems of linear algebraic equations (ISLAEs) in terms of $x_n^{(g)} = c_n^{(g)} + c_{-n}^{(g)}$ and $y_n^{(g)} = c_n^{(g)} - c_{-n}^{(g)}$, by the MoR. The ISLAE for $x_n^{(g)}$ has the form of

$$\begin{aligned} &\sqrt{m} x_m^{(g)} \left[1 - r_m^{(g)}\right] + \sum_{n\in\mathbb{N}} \sqrt{n} x_n^{(g)} r_n^{(g)} T_{m,n}^{(g)} \\ &= 2\kappa_g \left[p_0^{(g)} Z_0^{(g)} - d_0^{(g)}\right] \tau_m^{(g)} - \kappa_g \sqrt{m} f_m^{(g)} \\ &+ \sum_{n\in\mathbb{N}} \sqrt{n} \left[\kappa_g f_n^{(g)} - e_n^{(g)}\right] T_{m,n}^{(g)} - \sum_{h\neq g} c_0^{(h)} H_m^{(g,h)} \\ &+ \sum_{h\neq g}\sum_{n\in\mathbb{N}} \left[\sqrt{n} x_n^{(h)} U_{m,n}^{(g,h)} + \sqrt{n} y_n^{(h)} V_{m,n}^{(g,h)}\right] \end{aligned} \quad (22)$$

where $\hat{P}_n^{(\alpha,\beta)}(z)$ is the normalized Jacobi polynomial of degree n, argument z. The notations adopted are detailed in the Appendix. They are parameters that can be explicitly calculated in terms of the geometry and physical quantities of the problem.

The ISLAE for $y_n^{(g)}$ is similar to that of $x_n^{(g)}$ and is omitted here for reason of space. Together with the two ISLAEs obtained, an expression for $c_0^{(g)}$ is derived, which has been isolated from the ISLAEs for $x_n^{(g)}$ and $y_n^{(g)}$,

$$c_0^{(g)} = -\sum_{h\neq g} c_0^{(h)} a_{g,h} + \Psi_g. \quad (23)$$

This expression is obtained by making use of the fact that $x_n^{(g)}, y_n^{(g)}$ belong to ℓ_2 and the continuity at $z = z_g$. The expression Ψ_g is given in terms of all the unknowns $x_n^{(h)}$ and $y_n^{(h)}$ as follows. Other notations adopted are parameters that can be calculated explicitly (see Appendix).

$$\Psi_g = \lambda_g + \sum_{h=1}^{M} \sum_{n\in\mathbb{N}} \left[\sqrt{n} x_n^{(h)} F_n^{(g,h)} + \sqrt{n} y_n^{(h)} G_n^{(g,h)} \right]. \quad (24)$$

For the scattering problem involving only single cylinder, the ISLAEs for $x_n^{(1)}$ and $y_n^{(1)}$ are disjoint. They can readily and separately be solved by matrix inversion. When there are M cylinders involved, the resulting $2M$ systems are all connected to one another. Some manipulations are required before the systems can be solved numerically by truncation method. All the unknowns $c_0^{(g)}$ ($g = 1, \ldots, M$) need to be eliminated from each of the $2M$ systems. This is achieved by solving the $2M$ expressions of the form of Eqn.(23) by Cramer's rule. An explicit expression for the $c_0^{(g)}$ that is independent of all the other $c_0^{(h)}$ (for $h = 1, \ldots, i-1, i+1, \ldots, M$) is obtained as

$$c_0^{(g)} = \sum_{h=1}^{M} \Psi_g \Phi_{g,h}\,, \quad (25)$$

where the coefficients can be computed from

$$\Phi_{g,h} = \frac{\begin{vmatrix} a_{1,1} & \cdots & a_{1,h-1} & a_{1,h+1} & \cdots & a_{1,M} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{g-1,1} & \cdots & a_{g-1,h-1} & a_{g-1,h+1} & \cdots & a_{g-1,M} \\ a_{g+1,1} & \cdots & a_{g+1,h-1} & a_{g+1,h+1} & \cdots & a_{g+1,M} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{M,1} & \cdots & a_{M,h-1} & a_{M,h+1} & \cdots & a_{M,M} \end{vmatrix}}{\begin{vmatrix} a_{1,1} & \cdots & a_{1,M} \\ \vdots & \vdots & \vdots \\ a_{M,1} & \cdots & a_{M,M} \end{vmatrix}} \quad (26)$$

Once the values for the unknowns $x_n^{(g)}$ and $y_n^{(g)}$ ($g = 1, \ldots, M$) are computed, all the unknowns $c_0^{(g)}$ can be readily calculated, for $g = 1, \ldots, M$. These expressions for $c_0^{(g)}$, $g = 1, \ldots, M$, in the form of Eqn.(25) are then substituted back into each of the $2M$ infinite systems. For the unknown $x_n^{(g)}$, the following ISLAE is obtained

$$K_m^{(g)} = \sqrt{m} x_m^{(g)} \left[1 - r_m^{(g)} \right] + \sum_{h=1}^{M} \sum_{n\in\mathbb{N}} \left[\sqrt{n} x_n^{(h)} I_{m,n}^{(g,h)} + \sqrt{n} y_n^{(h)} J_{m,n}^{(g,h)} \right]. \quad (27)$$

The expressions for $I_{m,n}^{(g,h)}, J_{m,n}^{(g,h)}$ and $K_{m,n}^{(g)}$ are given in the Appendix. A similar ISLAE can be obtained for $y_n^{(g)}$. These ISLAEs can now be written as a single matrix equation in the operator form of $(I + H)\vec{x} = \vec{b}$. By truncating the infinite system of equations to N_{trunc}, the problem is solved by taking matrix inversion. Here, I is the $(2N_{\text{trunc}}M \times 2N_{\text{trunc}}M)$ identity matrix, H is a compact $2M \times 2M$ block matrix in ℓ_2 where each of these block matrices is of size $N_{\text{trunc}} \times N_{\text{trunc}}$, $\vec{b}$ is a known vector and $\vec{x}$ is the vector consisting of the unknown coefficients $x_n^{(g)}$ and $y_n^{(g)}$. It is worth noting that this is a Fredholm matrix equation of the second kind. By increasing N_{trunc}, the accuracy of the computed solution can be improved.

4 Numerical Result

To check the adequacy and the accuracy of the proposed method, the scattering problem of five PEC cylinders as described in [1] is solved. Five PEC cylinders of the same size ($R_g = 0.1\lambda$) are considered. With the center of the first cylinder located at the origin, the centers of the remaining four cylinders are located 0.5λ away. That is, at the points $(0.5\lambda, 0)$, $(0.5\lambda, \pi/2)$, $(0.5\lambda, \pi)$ and $(0.5\lambda, -\pi/2)$ respectively. In Fig.2, the scattering cross section $\sigma(\theta)$ of this problem is computed by making use of the asymptotic expression of the Hankel function and the far-field approximations $\rho_g \cong \rho - r_g \cos(\varphi_g - \theta)$, as well as $\theta_g \cong \theta$. The result is compared with that in [1] and good agreement is seen with the results calculated using the hybrid exact-method of moments technique, the iterative technique and the boundary value solution method.

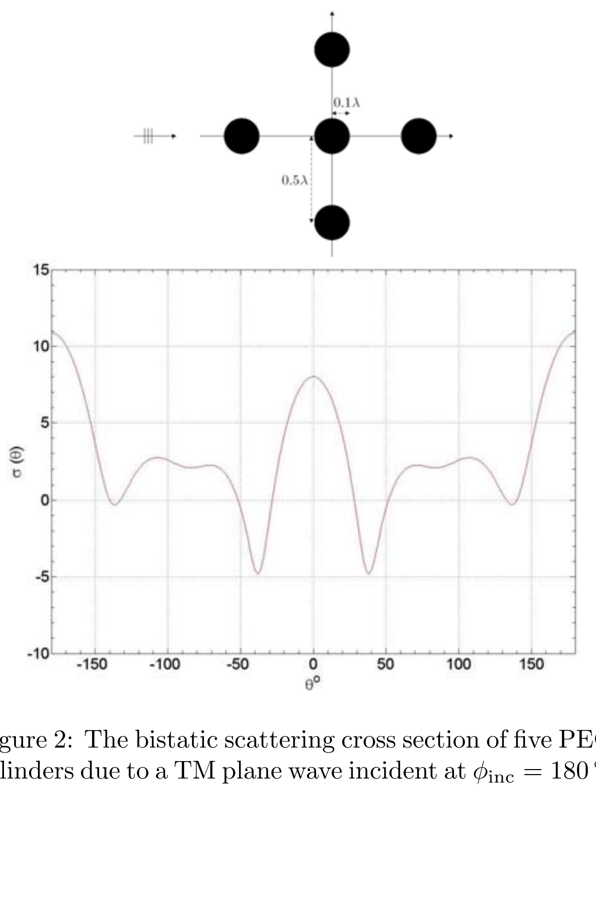

Figure 2: The bistatic scattering cross section of five PEC cylinders due to a TM plane wave incident at $\phi_{\text{inc}} = 180\,^{\circ}$.

As another method to check the validity of this method for slotted cylinders (when $\phi_g \neq \pi$), the computed values of the M coefficients $c_n^{(g)}$, $(g = 1, \ldots, M)$ have been substituted back to Eqn.(11) and Eqn.(12) to check the validity of boundary conditions. Consider the scattering of a TM plane wave incident on a pair of cylinders with radius $R_1 = 2.5\lambda$ and $R_2 = 3.75\lambda$. Both cylinders are filled with dielectric material with $\epsilon_{1,2} = 2$ and attached with a PEC strip of size $\phi_1 = 90^\circ$, $\phi_2 = 60^\circ$ respectively. The centers of the cylinders are located at $(0, 0)$ and $(10, \pi/4)$. Fig.3 and Fig.4 confirm the mixed boundary conditions given in Eqn.(6)-Eqn.(9) on the surface of the first cylinder. In Fig.3, the computed values of the transmitted electric field inside the first cylinder is compared with the computed values of the superposition of all the scattered electric fields and the incident field, on the surface of the first cylinder. It can be seen that the total electric field is continuous across the surface and vanishes on the PEC strip. The difference between the computed values of the transmitted tangential magnetic field inside the first cylinder and that of the total tangential magnetic field outside the cylinder is displayed in Fig.4 and Eqn.(9) is verified. Similar figures can be obtained on the surface of the second cylinder but is omitted here. The near field distribution resulting from the incidence wave for such structures is shown in Fig.5.

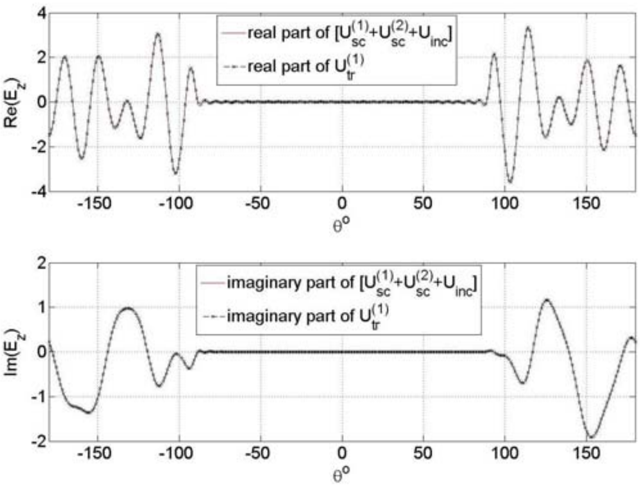

Figure 3: Continuity of the tangential electric field across the surface of the first cylinder. Here, $\phi_{\rm inc} = 180^\circ$, $R_1 = 2.5\lambda$, $R_2 = 3.75\lambda$, $\epsilon_{1,2} = 2$, $\phi_1 = 90^\circ$, $\phi_2 = 60^\circ$, $r_1 = 0$, $r_2 = 10\lambda$, $\varphi_1 = 0$, $\varphi_2 = \pi/4$.

After the problem formulation and the validation of the method, the solution can now be computed numerically and applied to actual analyses. However, there are a number of parameters to choose from. Due to the limit of space, it is not feasible to present a full range of features of the structures. Fig.6 illustrates the near field distribution of a TM plane wave incident at $\phi_{\rm inc} = 180^\circ$ on

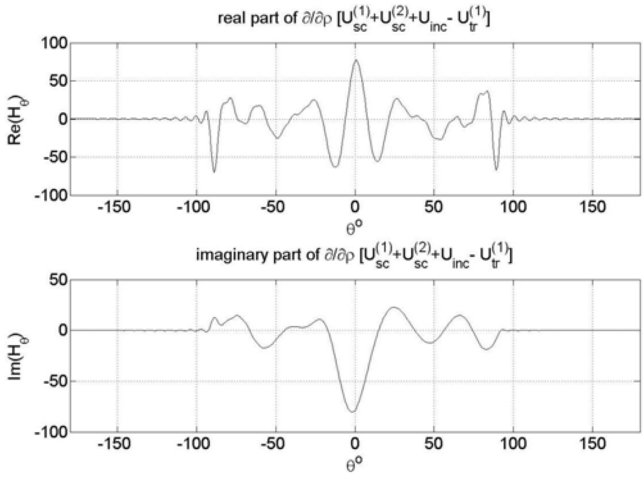

Figure 4: Continuity of tangential magnetic field across the dielectric surface of the first cylinder. The characteristics of structures are described in Fig.3.

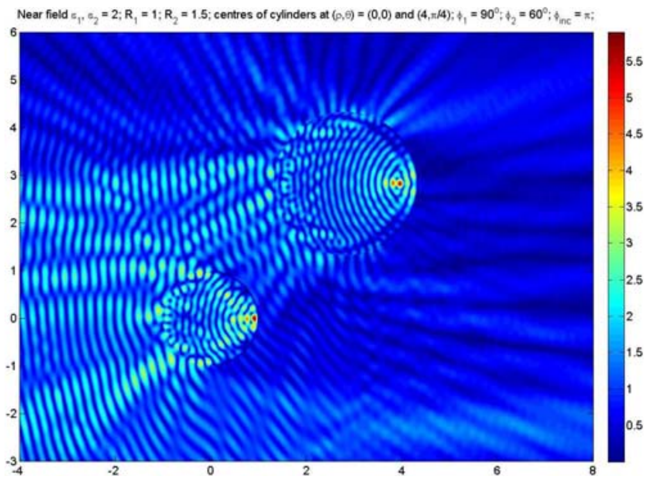

Figure 5: The near field distribution of a TM plane wave incident on the two cylinders described in Fig.3.

a sawtooth-shaped PEC surface formed by joining slotted cylinders. Consider such a surface constructed using three cylinders of the same size, where $R_g = 2.5\lambda$ and $\epsilon_g = 1$ (for $g = 1, 2, 3$).

5 Conclusion

A rigorous and accurate technique for the problem of two-dimensional scattering from a finite array of partially-shielded circular-cylinders has been presented. The technique is based on the MoR, and provide us with a convergent solution to the scattering problem. The dielectric constant, strip width, radius and location of each of the cylinders can be varied to model for a range of scattering problem, including a periodic array of fully- (or partially-) shielded cylinders.

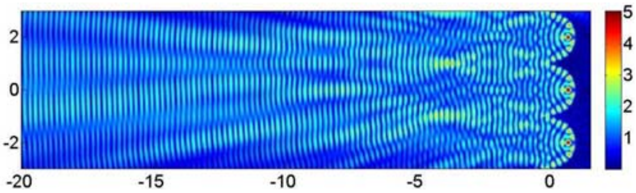

Figure 6: The near field distribution of a TM plane wave incident on a sawtooth-shaped surfaced, where $\phi_{\mathrm{inc}} = 180^\circ$, $R_{1,2,3} = 2.5\lambda$, $\epsilon_{1,2,3} = 1$, $\phi_{1,2,3} = 90^\circ$, $r_1 = 0$, $r_{2,3} = 5\lambda$, $\varphi_1 = 0$, $\varphi_2 = \pi/2$, $\varphi_3 = -\pi/2$.

6 Appendix

$$z_g = \cos(\phi_g) \tag{28}$$

$$T^{(g)}_{m,n} = \hat{Q}^{(0,1)}_{m-1,n-1}(z_g) - \frac{\kappa_g p_0^{(g)}(1+z_g)^2}{\kappa_g p_0^{(g)} \ln\left(\frac{1-z_g}{2}\right) - 1} \times \frac{\hat{P}^{(0,1)}_{m-1}(z_g)}{m}\frac{\hat{P}^{(0,1)}_{n-1}(z_g)}{n} \tag{29}$$

$$\hat{Q}^{(\alpha,\beta)}_{m,n}(z) = \int_z^1 (1-t)^\alpha (1+t)^\beta \hat{P}^{(\alpha,\beta)}_m(t)\hat{P}^{(\alpha,\beta)}_n(t)\mathrm{d}t \tag{30}$$

$$\tau^{(g)}_n = \frac{(1+z_g)}{\sqrt{2}\left[\kappa_g p_0^{(g)} \ln\left(\frac{1-z_g}{2}\right) - 1\right]}\frac{\hat{P}^{(0,1)}_{n-1}(z_g)}{n} \tag{31}$$

$$f^{(g)}_n = \frac{1}{n}\left(d^{(g)}_n + d^{(g)}_{-n}\right) \tag{32}$$

$$e^{(g)}_n = Z^{(g)}_n + Z^{(g)}_{-n} \tag{33}$$

$$H^{(g,h)}_n = 2\kappa_g\left[q_0^{(g)} - p_0^{(g)}\right]\tau^{(g)}_n A^{(g,h)}_{0,0} + \kappa_g s^{(g)}_n B^{(g,h)}_{n,0} + \sum_{m\in\mathbb{N}}\left[1-\kappa_g s^{(g)}_m\right] B^{(g,h)}_{m,0} T^{(g)}_{n,m} \tag{34}$$

$$U^{(g,h)}_{m,n} = \kappa_g\left[p_0^{(g)} - q_0^{(g)}\right]\tau^{(g)}_m B^{(g,h)}_{0,n} - \kappa_g s^{(g)}_m B^{(g,h)}_{m,n} - \sum_{k\in\mathbb{N}}\left[1-\kappa_g s^{(g)}_k\right] T^{(g)}_{m,k} B^{(g,h)}_{k,n} \tag{35}$$

$$V^{(g,h)}_{m,n} = \kappa_g\left[p_0^{(g)} - q_0^{(g)}\right]\tau^{(g)}_m C^{(g,h)}_{0,n} - \kappa_g s^{(g)}_m C^{(g,h)}_{m,n} - \sum_{k\in\mathbb{N}}\left[1-\kappa_g s^{(g)}_k\right] T^{(g)}_{m,k} C^{(g,h)}_{k,n} \tag{36}$$

$$B^{(g,h)}_{m,n} = \frac{1}{2}\sqrt{\frac{m}{n}}\left[A^{(g,h)}_{m,n} + A^{(g,h)}_{-m,n} + A^{(g,h)}_{m,-n} + A^{(g,h)}_{-m,-n}\right] \tag{37}$$

$$C^{(g,h)}_{m,n} = \frac{1}{2}\sqrt{\frac{m}{n}}\left[A^{(g,h)}_{m,n} + A^{(g,h)}_{-m,n} - A^{(g,h)}_{m,-n} - A^{(g,h)}_{-m,-n}\right] \tag{38}$$

$$\gamma_g = \frac{1 - \kappa_g q_0^{(g)} \ln\left(\frac{1-z_g}{2}\right)}{\kappa_g p_0^{(g)} \ln\left(\frac{1-z_g}{2}\right) - 1} \tag{39}$$

$$\lambda_g = \frac{Z_0^{(g)} - \kappa_g d_0^{(g)} \ln\left(\frac{1-z_g}{2}\right)}{\kappa_g p_0^{(g)} \ln\left(\frac{1-z_g}{2}\right) - 1} - \sum_{n\in\mathbb{N}}\sqrt{n}\left[\kappa_g f^{(g)}_n - e^{(g)}_n\right]\tau^{(g)}_n \tag{40}$$

$$a_{g,h} = \begin{cases} 1 & ,g=h \\ -\left[\gamma_g A^{(g,h)}_{0,0} + \sum_{n\in\mathbb{N}} \tau^{(g)}_n B^{(g,h)}_{n,0}\right] & ,g\neq h \end{cases} \tag{41}$$

$$F^{(g,h)}_n = \begin{cases} \tau^{(g)}_n r^{(g)}_n & ,g=h \\ \frac{1}{2}\gamma_g B^{(g,h)}_{0,n} & ,g\neq h \\ +\sum_{m\in\mathbb{N}}\left[1-\kappa_g s^{(g)}_m\right]\tau^{(g)}_m B^{(g,h)}_{m,n} & \end{cases} \tag{42}$$

$$G^{(g,h)}_n = \begin{cases} 0 & ,g=h \\ \frac{1}{2}\gamma_g C^{(g,h)}_{0,n} & ,g\neq h \\ +\sum_{m\in\mathbb{N}}\left[1-\kappa_g s^{(g)}_m\right]\tau^{(g)}_m C^{(g,h)}_{m,n} & \end{cases} \tag{43}$$

$$I^{(g,h)}_{m,n} = \begin{cases} r^{(g)}_n T^{(g)}_{m,n} + \sum_{q=1}^M F^{(q,g)}_n R^{(g,q)}_m & ,g=h \\ \kappa_g s^{(g)}_m B^{(g,h)}_{m,n} + \sum_{q=1}^M F^{(q,h)}_n R^{(g,q)}_m & ,g\neq h \\ +\sum_{k\in\mathbb{N}}\left[1-\kappa_g s^{(g)}_k\right] T^{(g)}_{m,k} B^{(g,h)}_{k,n} & \\ +\kappa_g\left[q_0^{(g)} - p_0^{(g)}\right]\tau^{(g)}_m B^{(g,h)}_{0,n} & \end{cases} \tag{44}$$

$$J^{(g,h)}_{m,n} = \begin{cases} \sum_{q=1}^M G^{(q,g)}_n R^{(g,q)}_m & ,g=h \\ \kappa_g s^{(g)}_m C^{(g,h)}_{m,n} + \sum_{q=1}^M G^{(q,h)}_n R^{(g,q)}_m & ,g\neq h \\ +\sum_{k\in\mathbb{N}}\left[1-\kappa_g s^{(g)}_k\right] T^{(g)}_{m,k} C^{(g,h)}_{k,n} & \\ +\kappa_g\left[q_0^{(g)} - p_0^{(g)}\right]\tau^{(g)}_m C^{(g,h)}_{0,n} & \end{cases} \tag{45}$$

$$R^{(g,h)}_n = \sum_{p\neq g} H^{(g,p)}_n \Phi_{p,h} \tag{46}$$

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