Asymmetric Taxation in a Competition Between Nonprofit and For-Profit Firms

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Abstract—**This paper considers a Cournot competition between a nonprofit firm and a for-profit firm in a homogeneous goods market, with uncertain demand. Given an asymmetric tax schedule, we compute explicitly the Bayesian-Nash equilibrium. Furthermore, we analize the effects of the tax rate and the degree of altruistic preference on market equilibrium outcomes.**

Keywords: Industrial Organization, nonprofit, asymmetric taxation, stochastic demand, Bayesian-Nash equilibrium

1 Introduction

In some industries, such as education and health care, both nonprofit and for-profit firms coexist and compete against each other. A for-profit firm is subject to profit taxation whereas a nonprofit firm receives benefits from tax exemption. However, nonprofit firms are not allowed to raise capital through equity financing. They also face non-distribution constraints. In the United States, tax practitioners from public accounting firms decide whether a nonprofit hospital should maintain tax exemptions. Reported profits and charitable care are the two major determinants of the final decision (see [9]). Due to competition and expected future profits, some nonprofit hospitals were converted to for-profit hospitals (see [1]).

In this paper, we closely follow Lien [2], by considering a competition between a nonprofit firm (hospital) and a for-profit firm (hospital). It is therefore reasonable to include profit as one of the objectives for the nonprofit hospital. Of course, there are other (altruistic) objectives that the hospital has to account for to justify its nonprofit status. Sansing [5] constructs an analytical model for competition between a nonprofit firm and a for-profit firm taking into account different production objective functions and different tax treatment. The model is then applied to evaluate the potential of joint ventures between the two firms. Specifically, Sansing [5] assumes a linear profit tax with symmetry between gains and losses. If a firm earns profits, it incurs a tax proportional to its profits. On the other hand, if there is a loss, the firm receives a subsidy (i.e., a negative income tax). He also assumes that the objective of the nonprofit hospital can be written as a weighted sum of its own profit and consumer surplus. The Cournot-Nash equilibrium is then derived. The symmetric taxation assumption, however, eliminates the effects of the tax rate on the firms production decision. Consequently, the equilibrium is independent of the tax rate despite that tax being recognized as an important element in the model and the key benefit of nonprofit status. In reality, the tax system is asymmetric such that the firm pays no tax and receives no subsides when incurring losses. Lien [2] incorporates the asymmetric nature of profit tax and a stochastic demand schedule into the Sansing framework. Specifically, instead of a subsidy, there is no subsidy or tax when a for-profit firm incurs a loss. He constructed the Cournot-Nash equilibrium and demonstrated the effect of profit tax on the market equilibrium. In contrast to Lien [2], we are able to compute explicitly the Bayesian-Nash equilibrium. This is due to the fact that we consider the demand following a particular probability distribution. However, again in contrast to Lien [2], the mean of the random variable that we consider does not have to be zero.

The paper is organized as follows In the next section, we have describe the benchmark model. Section 3 incorporates demand uncertainty into the asymmetric taxation framework and presents the objective functions for each firm. Optimal production decisions and the resulting Bayesian-Nash equilibrium for the modified framework are computed in Section 4.

2 The Benchmark model

Consider two firms competing in a homogeneous product market. Firm 1 is for profit and firm F_2 is nonprofit. The inverse demand function is specified as follows:

$$p = \alpha - q,$$

where p is the market price and q is the quantity demanded. Firms have the same constant marginal cost c. We consider from now on prices net of marginal costs. This is without loss of generality since if marginal cost is positive, we may replace α by $\alpha - c$. Let q_i denote the

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production level of firm F_i. The profit for firm F_i is then

$$\pi_i = (\alpha - q_1 - q_2)q_i.$$

firm F_1 is for profit and its profit is subject to taxation. Let t denote the profit tax rate. The firm incurs a tax of $t\pi_1$ $(0 < t < 1)$. Note that this tax structure provides the firm a subsidy when it incurs a loss. The after-tax profit is then $\pi_1^a = (1-t)\pi_1$. Firm F_1 chooses the optimal production level to maximize its after-tax profit. However, maximizing a π_1^a is equivalent to maximizing π_1. The tax rate has no effect on the firm's production decision. On the other hand, firm F_2 is a nonprofit organization. Therefore, in choosing the optimal production level, it must take into account an altruistic objective. Sansing (2000) adopts the consumer surplus in his study. Due to its not-for-profit nature, firm F_2 is not subject to any profit taxation. For a given demand quantity, q, the consumer surplus is measured as follows:

$$CS = \int_0^q (\alpha - z)dz - (\alpha - q)q = \frac{q^2}{2}$$

Let w denote the importance of the consumer surplus for firm F_2 relative to its profit. The firm's objective function (or utility function) is written as $W = \pi_2 + wCS$, or

$$W = (\alpha - q_1 - q_2)q_2 + \frac{w}{2}(q_1 + q_2)^2,$$

where w is the weight assigned to the consumer surplus (i.e., the firm's altruistic preference). Firm F_2 chooses its optimal production level to maximize W. We consider a Cournot-Nash equilibrium in which each firm chooses its optimal production level assuming the other firm maintains its current production level. To maximize π_1^a, the optimal production level, q_1^*, must satisfy the following first order condition:

$$\alpha - (2q_1 + q_2) = 0 \tag{1}$$

whereas the second order condition obviously holds. Similarly, to maximize W, the optimal production level of firm F_2, q_2^*, must satisfy the following equation:

$$\alpha - (1-w)q_1 - (2-w)q_2 = 0 \tag{2}$$

By solving equations (1) and (2) simultaneously, we have

$$q_1^* = \frac{(1-w)\alpha}{3-w}, \tag{3}$$

$$q_2^* = \frac{(1+w)\alpha}{3-w}. \tag{4}$$

Non-negative outputs imply $w \leq 1$. That is, the nonprofit firm cannot weight the C.S. more than of its own profit. Moreover $\partial q_1^*/\partial w = (-2\alpha)/(w-3)^2 < 0$ and $\partial q_2^*/\partial w > 0$. As the nonprofit firm becomes more altruistic, it expands the production level to enhance consumer surplus.

3 Demand uncertainty and asymmetric taxation

Suppose that we replace the symmetric taxation with a more realistic asymmetric taxation in the framework such that the firm incurs a tax of $t\pi_1$ $(0 < t < 1)$ only if it earns a profit, i.e., $\pi_1 > 0$. No tax is imposed when the firm incurs a loss, i.e., $\pi_1 < 0$. The after-tax profit is then

$$\pi_1^a = \pi_1 - t\max\{\pi_1, 0\}. \tag{5}$$

Firm F_1 chooses the optimal production level to maximize its after-tax profit. Within the current demand and cost structure, the firm will not incur any loss. Consequently, $\pi_1^a = (1-t)\pi_1$. Maximizing π_1^a is equivalent to maximizing π_1. Once again, the tax rate has no effect on the firm's production decision. To highlight the effect of the tax rate, either the demand function or the cost function must be modified. In this paper, we consider a stochastic inverse demand function such that

$$p = \alpha + \Delta - q \tag{6}$$

where Δ is a random variable, representing the demand shock. We consider that

$$\Delta = \begin{cases} \varepsilon, & \text{with probability } \phi \\ -\varepsilon, & \text{with probability } 1-\phi \end{cases}.$$

Both firms make their production decisions prior to the realization of the demand shock. In what follows, we restrict the parameters of the model to satisfy the following assumption:

Assumption 1. $\alpha - \varepsilon < q_1 + q_2 \leq \alpha + \varepsilon$.

This assumption requires that the aggregate quantity is grater than the lowest possible demand value but does not exceed the highest demand value. Given the stochastic demand, the profit for firm F_i is also stochastic such that

$$\pi_i = (\alpha + \Delta - q_1 - q_2)q_i, \tag{7}$$

with $i = 1, 2$. Under asymmetric taxation, the after-tax profit for firm F_1 is described in (5). Firm F_1 chooses the optimal production level to maximize its expected profit, $E(\pi_1^a)$, where the expectation is taken over Δ. Note that $\pi_1 > 0$ is equivalent to the condition that $p > 0$ or $\Delta > -\alpha + q_1 + q_2$. So, the expected after tax profit for firm F_1 is given by

$$E(\pi_1^a) = (\alpha + \varepsilon(2\phi - 1) - q_1 - q_2 - t(\alpha + \varepsilon - q_1 - q_2)\phi)q_1.$$

On the other hand, firm F_2 attempts to maximize the expectation of a weighted sum of its own profit and consumer surplus. For a given demand quantity, q, the expected consumer surplus is:

$$E(CS) = E\left(\int_0^q (\alpha + \varepsilon - z)dz - (\alpha + \varepsilon - q)q\right) = \frac{q^2}{2}.$$

As a result, the objective function of firm F_2 is $E(W) = E(\pi_2) + wE(CS)$, or

$$E(W) = (\alpha - q_1 - q_2 + \varepsilon(2\phi - 1))q_2 + \frac{w}{2}(q_1 + q_2)^2.$$

4 Production decisions and market equilibrium

We now characterize each firm's production decision and the resulting Bayesian-Nash equilibrium. To maximize $E(\pi_1^a)$, the optimal production level, q_1^*, must solve the following equation:

$$\frac{\partial E(\pi_1^a)}{\partial q_1} = \alpha + \varepsilon(2\phi - 1) - 2q_1 - q_2 - t(\alpha + \varepsilon - 2q_1 - q_2) = 0. \quad (8)$$

Furthermore, we have that

$$\frac{\partial^2 E(\pi_1^a)}{\partial^2 q_1} = -2 + 2t < 0.$$

Thus, the second-order condition for maximization is always satisfied. To maximize $E(W)$, the optimal production level of firm F_2, q_2^*, must satisfy the following equation:

$$\frac{\partial E(W)}{\partial q_2} = \alpha + \varepsilon(2\phi - 1) - q_1 - 2q_2 + w(q_1 + q_2) = 0. \quad (9)$$

The Bayesian-Nash equilibrium is derived by solving equations (8) and (9) simultaneously. So,

$$q_1^* = \frac{(1 - w)(\alpha + \varepsilon(2\phi - 1))}{(3 - w)(1 - t\phi)} - \frac{t((1 - w)(\alpha + \varepsilon) + 2\varepsilon(1 - \phi))\phi}{(3 - w)(1 - t\phi)} \quad (10)$$

and

$$q_2^* = \frac{(1 + w)(\alpha + \varepsilon(2\phi - 1))}{(3 - w)(1 - t\phi)} - \frac{t((1 + w)(\alpha + \varepsilon) - 4\varepsilon(1 - \phi))\phi}{(3 - w)(1 - t\phi)}. \quad (11)$$

Therefore, the aggregate quantity is given by

$$q_1^* + q_2^* = \frac{2(\alpha + \varepsilon(2\phi - 1) - t(\alpha + \varepsilon\phi)\phi)}{(3 - w)(1 - t\phi)}. \quad (12)$$

5 Comparative static analysis

In this section, we evaluate the effects of the tax rate and the preference of consumer surplus on the market equilibrium. Applying comparative static analysis, we have

$$\frac{\partial q_1^*}{\partial t} = \frac{2\varepsilon\phi(2 - w)(\phi - 1)}{(3 - w)(1 - t\phi)^2} \leq 0.$$

Firm F_1 produces less when the profit tax rate increases. We conclude that the recognition of asymmetric tax treatment leads to a smaller output level for the for-profit firm. Turning to firm F_2, we derive

$$\frac{\partial q_2^*}{\partial t} = \frac{2\varepsilon\phi(1 - w)(1 - \phi)}{(3 - w)(1 - t\phi)^2} \geq 0.$$

As the profit tax rate increases, the nonprofit firm increases its output. On the other hand,

$$\frac{\partial(q_1^* + q_2^*)}{\partial t} = \frac{2\varepsilon\phi(\phi - 1)}{(3 - w)(1 - t\phi)^2} \leq 0.$$

Regardless of firm F_2's production decision, the total output decreases with an increasing tax rate, leading to an increase in the market price. Theorem 1 summarizes the above results.

Theorem 1. As the profit tax rate increases, the for-profit firm reduces its production whereas the nonprofit firm increases its production. The overall effect is a reduction in the total production and, therefore, an increase in market price.

Now, we are going to analyze the effect of the preference for consumer surplus. From equality (10), we get that

$$\frac{\partial q_1^*}{\partial w} = \frac{2(\alpha + \varepsilon(2\phi - 1) - t\phi(\alpha + \varepsilon\phi))}{(3 - w)^2(t\phi - 1)} < 0.$$

Hence, when firm F_2 cares more about consumer surplus, firm F_1 reduces its production level. From equality (11), we get that

$$\frac{\partial q_2^*}{\partial w} = \frac{4(\alpha + \varepsilon(2\phi - 1) - t\phi(\alpha + \varepsilon\phi))}{(3 - w)^2(1 - t\phi)} > 0.$$

Thus, firm F_2 expands its production when the consumer surplus becomes more important to its operations. From equality (12), we get that

$$\frac{\partial(q_1^* + q_2^*)}{\partial w} = \frac{2(\alpha + \varepsilon(2\phi - 1) - t\phi(\alpha + \varepsilon\phi))}{(3 - w)^2(1 - t\phi)} > 0.$$

That is, the increase in firm F_2's production more than offsets the reduction in firm F_1's production. These results are summarized in the following theorem.

Theorem 2. If the nonprofit firm values the consumer surplus higher, it expands its production level. Meanwhile, the for-profit firm produces less. The overall effect is an increase in the total production and, therefore, a reduction in the market price.

6 Conclusions

This paper addressed the question of asymmetric tax schedule in a Cournot competition between a nonprofit firm and a for-profit firm, with uncertain demand. We computed explicitly the Bayesian-Nash equilibrium, and we analyzed the effects of the tax rate and the degree of altruistic preference. We proved that as the tax rate increases, the for-profit firm reduces its production, whereas the nonprofit firm increases its production. The overall effect is a reduction in the total production and, therefore, an increase in market price. We also proved that if the nonprofit firm values the consumer surplus higher, it expands its production level, and the for-profit firm produces less. The overall effect is an increase in the total production and, therefore, a reduction in the market price.

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