Proceedings of the World Congress on Engineering 2010 Vol II
WCE 2010, June 30 - July 2, 2010, London, U.K.

Dynamics of Transtropic Elastic Continuums with Non-Homogeneities

Lyazzat B. Atymtayeva , Zhailau K. Masanov, Gulzada S. Myrzakhmetova, and Bagdat E. Yagaliyeva

Abstract— **This paper is devoted to research the dynamic stress-strain behavior of transtropic massif with a few mine tunnels located not too deep from earth surface. Case of propagation and diffraction of elastic SH-waves is considered. Analytical treatment for definition of stresses and strains in condition of anti-plane deformation are received. An influence of physical and mechanical properties of surround massif and falling SH-waves to stress-strain behavior of shallow located horizontal mine tunnel is shown.**

Index Terms—**transtropic massif, shallow located non-homogeneities, mines, tunnels, drifts, diffraction, SH-wave, anti-plane deformation.**

I. Introduction

Development of various areas of mechanical engineering, such as rock mechanics; building of the underground constructions; development of new composite materials and their using at processes of building of engineering constructions; modern problems of geophysics and seismology, and also a number of other scientific and technical tendencies promoted increasing of an urgency of problems of deformable solid dynamics.

Elastic wave diffraction on the various types of non-homogeneities relates to the one of the most difficult and actual applied problems of deformable solid dynamics. In particular, it deals with the research issues of rock mechanics and the building of underground constructions which are usually modeled as non-homogeneities inside the continuum. The continuum, in this case, is represented as a massif with various physical and mechanical properties. Natural, constructional and deformable anisotropy of these kinds of properties is inherent to the majority of materials, including rock mass with its layering and bedding. The account of anisotropy properties at the research of dynamic deformation processes allows more adequate representing the qualitative character of elastic solid stress-strain behavior and its wave-conductive features. Besides it helps receiving more authentic quantitative estimations.

Wave diffraction researches often require using the complex mathematical calculations. Last circumstance throughout long time has not allowed investigating wide spectrum of problems including estimation of dynamic intensity close by non-homogeneities. One of the main objectives is to find not only the formal mathematical solution, but the effectively determination of diffraction strain and stress fields close by non-homogeneities. Both the improvement of scientific and technological advance and development of computer systems help us to select two approaches for successful analysis of elastic wave diffraction problems.

The first approach concerns the development of numerical methods when we need to carry out the task digitization with using of computer application at all stages of task solution. This approach has universal algorithms and provides the investigation of wide classes of relevant problems (see, for instance, [1]). However results of researches in the given direction depend on possibilities of the computer and assume revision of task statements and redesign of algorithms if the border conditions and other initial data would be changed. The second approach is often considered as more successful. It consists of two stages, the first of them is the finding solution with help of analytical methods (variables separation method and its generalizations, methods of the changing forms theory, methods of transformations to the integrated equations after incomplete separation of variables, etc.) and the second one is final stage with using of computer simulation. In this direction the wide classes of problems are already investigated, main monographs ([2]-[9]) are published. This paper is devoted to researches based on the last approach. We try to extent it for analysis of elastic wave diffraction by non-homogeneities located inside the massif with more complicated physical and mechanical properties.

II. Model of Transtropic Continuum with Non-Homogeneities

There is a wide spectrum of continuum models with non-homogeneities, such as holes, mine tunnels, subways, etc. It might be looked at the isotropic (most studied) model and anisotropic model (with the complex physical and mechanical properties) of continuums.

Actually, the real massif is not isotropic. Various ways of massif layers occurrence, methods of their bedding predetermine the anisotropic model of rock mass. The model of transtropic (transversal isotropic) solid with an inclined plane of isotropy, simulating the folded rock layers, is really relevant to anisotropic properties of massif. Anisotropic (transtropic) model of the folded-layered massif with

Manuscript received
The 2010 International Conference of Mechanical Engineering-Geomechanics
Lyazzat B. Atymtayeva with the Kazakh-British Technical University, Almaty, Republic of Kazakhstan (phone: 8-727-272-15-02; fax: 8-727- 250-46-51, e-mail: atymtayeva@gmail.com, l.atymtayeva@kbtu.kz)
Zhailau K. Masanov with the Institute of Mechanics and Engineering Science named after U. A. Zholdasbekov, Almaty, Republic of Kazakhstan (phone: 8-727-291-33-94; fax: 8-727-291-50-60)
Gulzada S. Myrzakhmetova with the Institute of Mechanics and Engineering Science named after U. A. Zholdasbekov, Almaty, Republic of Kazakhstan (phone: 8-727-291-33-94; fax: 8-727-291-50-60, e-mail: mgsnur@mail.ru)
Bagdat E. Yagaliyeva with the Kazakh-British Technical University, Almaty, Republic of Kazakhstan (phone: 8-727-272-15-02; fax: 8-727- 250-46-51, e-mail: bagdat.yag@gmail.com).

inclined plane-parallel layers near to underground construction gives the possibility to subdivide horizontal mine tunnels to drifts, crosscuts and diagonal developments, which depend on their spatial orientation in the massif. The drifts pass across of spreading of plane of isotropy. Crosscuts pass transversely the drifts. The diagonal mines occupy intermediate position between drifts and crosscuts. Such underground constructions are used in the system of preparatory and capital mine tunnels. So, pit-bottom bypass mine tunnels cross the inclined layers of rocks in different directions. For instance, lines of transport tunnels are usually arbitrarily oriented relative to elements of rock superposition.

If capacity of each layer of massif, at least, 10 times less the characteristics of cross-section size of mine tunnel lengthy, such massif could be presented by transversal-isotropic (transtropic) solid with plane of isotropy, coinciding to plane of layers spreading. In the most cases, the underground constructions is modeled as horizontal extent non-homogeneities (for instance, hole, mine tunnel, etc.) in the described massif.

The case of drift mine tunnels, passed in transtropic massif across the inclined plane of isotropy is showed in the given work (Fig. 1).

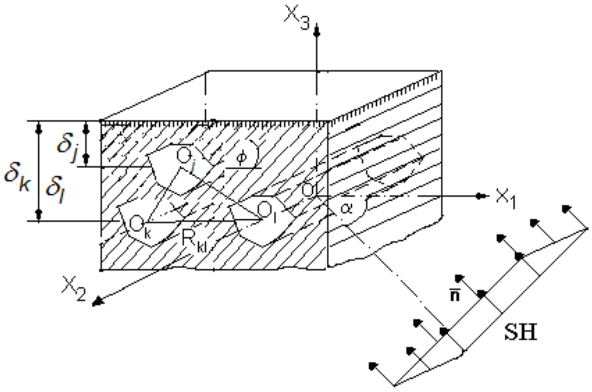

Figure 1.

Underground constructions of different purposes, such as capital, preparatory and treatment coal mine tunnels and collieries, transport and hydro technical tunnels, etc., are usually put in sedimentary rock mass at Earth's crust seismic active zones, and they quite often are exposed to destructive influence of earthquake.

Both according to natural and technogenic conditions and to behavior of seismic influences appearances to the underground constructions it is expedient to allocate two kinds (shallow location and deep location) of constructions. Depending on these kinds of location the statements of theory of seismic stability problems and the approaches to the solutions have the specific features.

Segregation of shallow (by other words, not too deep) located constructions into the special class is connected mainly with nature of seismic stability calculations. It is necessary to take into account not only influence of volume seismic waves, but also impact of superficial ones. Thus it's quite often required to look at the waves reflected from the original ground. So there is appeared a number of the problems caused by dynamic interaction of underground and land constructions. For instance, depth of subway tunnels location might be found out from the point of view not only necessity of their solidity and stability, but also with a glance of admissible size of vibrations level in the buildings and constructions situated close to these tunnels. Apparently, the seismicity specification of shallow located constructions depending on depth of their location plays the important role for the areas which might be characterized by small depth of the earthquakes hypocenters.

Prognostic estimations of tunnels, mines, pipelines, undergrounds behavior at their real building and maintaining that are impacted by the dynamic influences are defined by construction strain-stress condition (SSC) with surrounding rock mass interaction.

In practice of designing and building the capital and preparatory underground mines, rundown subway tunnels, hydro technical transport underpasses with various non-circle profiles are received a wide circulation. Giving the non-circle form to the cross-section contour of an underground construction is connected with a non-uniform folded massif structure, with geological conditions of mountain pressure occurrences, with types and kinds of material of fixturing constructions, and at last, with object matter of structure and its exploitation conditions. Mine non-circle contour (or non-homogeneity) can be described by the next parametrical expression:

$$\omega(\varsigma) = R\left[\varsigma + \sum_{m=1}^{N}\left(d_m \varsigma^{-m}\right)\right] \tag{1}$$

with real coefficients d_m at the case of non-homogeneities symmetric profiles and with complex values of d_m at the case of their asymmetric profiles.

Let's look at a problem about research of the strain-stressed conditions of transtropic massif with an inclined plane of isotropy at stationary diffraction of elastic SH-waves on mutual shallow located holes with various kinds of cross-sections.

In this paper an elastic anisotropic massif is modeled by the transversal-isotropic body with an inclined under corner φ plane of isotropy, related to $Ox_1x_2x_3$ co-ordinates system and contained, in general, M non-circle holes located not too deep from the earth surface. The longitudinal axis of holes is parallel to axis Ox_2 (see fig.1). For *l-th* hole ($l = \overline{1,M}$) contour expression (1) can be rewritten as

$$z_l = \widetilde{\omega}_l(\zeta) = R_l(\zeta + \sum_{m=1}^{N_l} d_{ml}\zeta^{-m}),\, z_l = x_{1l} + ix_{3l},\quad \zeta = \rho_l e^{i\theta_l}\,, \tag{1*}$$

The equations of generalized Hook's law for the transversal-isotropic massif with an inclined plane of isotropy can be written in next form

$$\{\sigma\} = [D]\{\varepsilon\},$$

where

$$\{\sigma\}^T = [\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{13}, \sigma_{23}, \sigma_{12}];$$

$$\{\varepsilon\}^T = [\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}, \varepsilon_{13}, \varepsilon_{23}, \varepsilon_{12}];\ [D] = [b_{ij}], (i, j = \overline{1,6}),$$

Coefficients b_{ij} depend on elastic massif constants - E_k, ν_k, $(k = 1,2)$, G_2 and corner of plane of isotropy φ [10].

III. Stationary diffraction of elastic SH-waves on non-homogeneities in transtropic massif

Let's assume, that in a plane of holes cross-section along a direction that is set by unit vector $\vec{n} = (n_1, n_3)$, there is fallen the flat stationary SH shift wave polarized in parallel to axis Ox_2.

ISBN: 978-988-18210-7-2
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

Expression for this kind of wave is described as

$$u_2 = u_2^* e^{-i\omega t},$$

where

$$u_2^* = V_a^* \exp[i(k_1 x_1 + k_3 x_3)] = V_a^* \exp[i\bar{k}(n_1 x_1 + n_3 x_3)],$$

- wave amplitude, ω - wave frequency, $\bar{k} = (k_1, k_3)$ - wave vector, $k_i = \omega_0 n_i (n_1^2 + 2(b_{46}/b_{66}) n_1 n_3 + (b_{44}/b_{66}) n_3^2)^{-1/2}, i = 1,3$, (n_1, n_3) - directing cosinus and α -corner of falling waves.

Let's assume, that holes are in conditions of antiflat deformation which is characterized by peak components of displacements $u_2(x_1, x_3)$ in reflected wave, related to the linear size of hole R_l. Thus, the movement equations in absence of mass forces will be the following

$$\{\partial_1^2 + 2(b_{46}/b_{66})\partial_1\partial_3 + (b_{44}/b_{66})\partial_3^2 + \omega_0^2\}\, u_2(x_1, x_3) = 0; \qquad (2)$$

$$\omega_0^2 = \omega_1^2 \rho R^2 / b_{66}$$

For statement of edge problem we should define the boundary conditions. At formulation of edge problems in case of shallow located holes it is necessary to look at the flat border condition, in our case, $x_3 = 0$ (we take into account bottom half-space $x_3 < 0$):

$$(\alpha_1 \widetilde{V} + \alpha_2 \sigma_{32})|_{x_3=0} = 0 \qquad (3)$$

This is condition of elastic solid edge support which by means of expressions for σ_{32} from generalized Hook's law equations can be transformed to:

$$(m_0 + m_1\partial_1 + m_2\partial_2)\widetilde{V}|_{x_3=0} = 0 \qquad (4),$$

where $m_0 = \alpha_1$, $m_1 = \alpha_2 b_{46}$, $m_2 = \alpha_2 b_{44}$

In expressions (3), (4) $\widetilde{V}$ is the moving field as superposition of u_2 - moving field of waves reflected from the holes and moving field $u_2^* = V_1^* + V_2^*$, that is sum of V_1^* - moving field in falling wave and V_2^* - moving field in wave reflected from flat massif border in case of holes absence.

if $\alpha_2 \neq 0$,

$$V_2^* = V_a^* \exp[i * (-(k_1 + 2\frac{m_2}{m_1}k_3 - 2i\frac{m_0}{m_1})x_1 + k_3 x_3)], \qquad (5a)$$

if $\alpha_2 = 0$,

$$V_2^* = -V_a^* \exp[i * (-k_1 x_1 + k_3 x_3)], \text{ если } \alpha_2 = 0 \qquad (5b)$$

Assume that the each hole's contour is not supported, and free from loadings, i.e. we will solve the first basic task of solid mechanics. In this case parameter α_1=0.

The boundary conditions on each hole's contour $l\,(l = \overline{1,M})$ can be written as

$$(\sigma_{n2} + \sigma_{n2}^*)_{B_l} = 0,\; (\sigma_{\gamma 2} + \sigma_{\gamma 2}^*)_{B_l} = 0,$$

where σ_{n2}^*, $\sigma_{\gamma 2}^*$ - tangents stresses on the platform with normal (n_1, n_3), caused by a falling shift wave, σ_{n2}^* is directed on normal and $\sigma_{\gamma 2}^*$ is defined by tangent line; σ_{n2}, $\sigma_{\gamma 2}$ - relevant peak stresses components in the reflected wave.

IV. The solution of the first basic task

For integration of differential equations of movement (2) in partial second-order derivatives we consider the affine transformation [5]:

$$x_1 = \xi_1,\; x_3 = -\mu_1\xi_1 + \mu_3\xi_3,\; \mu = \mu_1 + i\mu_3,\; \mu_3 > 0,$$

which transform (2) into Helmholtz equation in coordinates (ξ_1, ξ_3):

$$(\nabla^2_{\xi_1\xi_3} + \omega_0^2)u_2(\xi_1, \xi_3) = 0,$$

where $\nabla^2_{\xi_1\xi_3}$ - Laplace operator in variables ξ_1 and ξ_3.

According to principle of generalized superposition [2], by means of corresponding transformations, replacements u_2 can be performed by infinite series for the unknown coefficients A_n and cylindrical Hankel and Bessel functions:

$$(u_2)_{B_l} = \sum_{(n)} (A_{nl} H_n^{(1)}(\omega_0 r_l) + (S_{nl}^{(1)} + S_{nl}^{(2)}) J_n(\omega_0 r_l)) e^{in\kappa_l}, \quad (7)$$

$$n = \overline{-\infty, +\infty},\; l = \overline{1,M},\; r_l < R_{lj}^{(1)},\; r_l < R_{lj}^{(2)},\; j \neq l,\; j = \overline{1,M}$$

where $S_{nl}^{(1)} = \sum\limits_{(q=1, q\neq l)}^{M} \sum\limits_{(p)} A_{pq} H_{p-n}^{(1)}(\omega_0 R_{lq}^{(1)}) e^{i(p-n)\kappa_{lq}^{(1)}}$,

$$S_{nl}^{(2)} = \sum_{(q=1, q\neq l)}^{M} \sum_{(p)} A_{pq} (-1)^{p+n} H_{p+n}^{(1)}(\omega_0 R_{lq}^{(2)}) e^{-i(p+n)\kappa_{lq}^{(2)}},$$

$(R_{lq}^{(s)}, \kappa_{lq}^{(s)})$ – Coordinates of O_l pole in $(r_q^{(s)}, \kappa_q^{(s)})$ system of coordinates, $q = \overline{1,M}$, $s = 1,2$.

Going to the same procedure as for holes deep located in massif, with using expressions for contour points we can write complex-valued potentials for l-s contour points as [11]:

$$H_n^{(1)}(\omega_0 r_l) e^{in\kappa_l} = \sum_{(p)} Q_{npl}^{(1)} e^{ip\theta_l},\; J_n(\omega_0 r_l) e^{in\kappa_l} = \sum_{(p)} Q_{npl}^{(2)} e^{ip\theta_l}. \quad (8)$$

These expressions allow us to present $(u_2)_{B_l}$ contour replacement as

$$(u_2)_{B_l} = \sum_{(n,p)} (A_{nl} Q_{npl}^{(1)} + (S_{nl}^{(1)} + S_{nl}^{(2)}) Q_{npl}^{(2)}) e^{ip\theta_l}, \quad l = \overline{1,M} \quad (9)$$

Here $Q_{npl}^{(1)}$, $Q_{npl}^{(2)}$ - complex-valued expressions, see [11].

Expressions for stresses σ_{n2}, $\sigma_{\tilde{\gamma}2}$ give us possibility to transform the boundary conditions into the next form:

$$(|\widetilde{\omega}'|\sigma_{n2} + |\widetilde{\omega}'|\sigma_{n2}^*)_{\Gamma l} = 0 \text{ or } (|\widetilde{\omega}'|\sigma_{\tilde{\gamma}2} + |\widetilde{\omega}'|\sigma_{\tilde{\gamma}2}^*)_{\Gamma_l} = 0$$

For obtaining a system of linear algebraic equations for the unknown coefficients it will be necessary to define expressions for stresses such as:

$$(|\widetilde{\omega}'|\sigma_{n2})_{B_l} = \sum_{(n,p)} (A_{nl}\lambda_{np}^{(1)} + (S_{nl}^{(1)} + S_{nl}^{(2)})\lambda_{np}^{(2)}) e^{ip\theta_l},$$

$$(|\widetilde{\omega}'|\sigma_{\gamma 2})_{B_l} = \sum_{(n,p)} (A_{nl}\widetilde{\lambda}_{np}^{(1)} + (S_{nl}^{(1)} + S_{nl}^{(2)})\widetilde{\lambda}_{np}^{(2)}) e^{ip\theta_l}$$

Here $\lambda_{np}^{(1)}$, $\lambda_{np}^{(2)}$, $\widetilde{\lambda}_{np}^{(1)}$, $\widetilde{\lambda}_{np}^{(2)}$ - complex-valued expressions, see [11].

Further we can obtain the expressions for contour replacements $(u_2^*)_{B_l}$ and stresses $(|\widetilde{\omega}'|\sigma_{n2}^*)_{B_l}$, $(|\widetilde{\omega}'|\sigma_{\gamma 2}^*)_{B_l}$ in a falling shift wave by means the formula of exponential functions expansion in the harmonic series:

$$(u_2^*)_{B_l} = V_a^* \sum_{(p)} \widetilde{Q}_{pl} e^{ip\theta_l} \cdot \quad (|\widetilde{\omega}'|\sigma_{n2}^*)_{B_l} = (1/2) V_a^* R \sum_{(p)} \widetilde{\alpha}_{pl}^{(1)} e^{ip\theta_l},$$

$$(|\widetilde{\omega}'|\sigma_{\tilde{\gamma}2}^*)_{B_l} = (1/2) V_a^* R \sum_{(p)} \widetilde{\alpha}_{pl}^{(2)} e^{ip\theta_l}, \qquad (10)$$

where $\widetilde{Q}_{pl}$, $\widetilde{\alpha}_{pl}^{(1)}$, $\widetilde{\alpha}_{pl}^{(2)}$ - complex-valued expressions derived by taking into account the wave reflection from the flat border in case of holes absence, which are similar to expressions for a falling wave in case of deep located holes. See [11].

To satisfy the boundary conditions we can use the equating of left and right sides at the $e^{ip\theta_l}$ member in boundary conditions. Then after appropriate substitutions these transformations may lead to the infinite system of linear algebraic equations for the unknown coefficients A_n, S_n in next form:

$$\sum_{(n)} A_{nl}\lambda_{npl}^{(1)} + (S_{nl}^{(1)} + S_{nl}^{(2)})\lambda_{npl}^{(2)} = -(i/2)V_a^* \widetilde{\alpha}_{pl}^{(j)}, \quad (11)$$

$$(p = -\infty, +\infty), \ (j = 1,2), (l = \overline{1, M})$$

which can be solved by reduction method.

V. Analysis of Results and Conclusion

For the dynamic analysis we give the solutions for single circular hole shallow located in transtropic massif (siltstone) with the next elastic properties: E_1=1,074*10^5 kg/cm^2, E_2=0,523*10^5 kg/cm^2, G_2=0,12*10^5 kg/cm^2, ν_1 =0,413, ν_2 =0,198; wave propagates at an angle α to the horizontal axis Ox_1 ; change of wave frequencies is taken in the range from 1 to 100 Hz, changing the angle of inclined plane of isotropy φ is taken from $0°$ to $180°$.

According to theoretical solution for quality and quantity analysis of strain-stresses conditions of circular single hole traversed at a depth δ in transtropic massif the algorithm and programming package in Matlab7.0 program environment were developed.

In order to test the program we have carried out the analysis of elastic stresses and replacements distribution close to circular hole in half plane from impact of SH shift wave in case when the depth $\delta \geq 10R$. Here we compare the results with the same solutions for strain-stresses condition of deep located hole. We have observed the identical results as for deep located hole with using the similar initial parameters.

The results of calculation of circular contour shear stresses $|\sigma_{\theta 2}|$ are obtained based on the solution of system of equations (11), which has held by no more than 25 equations. This will satisfy the boundary conditions with an error less than 1% for stresses $|\sigma_{\theta 2}|$ in relation of amplitude values of stresses in a falling wave.

As it turned out, changing the angle of inclined plane of isotropy φ significantly affects the qualitative and quantitative parameters of the contour stresses and replacements distribution.

It may be noted that the values of shear stresses $|\sigma_{\theta 2}|$ for angles $0° \leq \varphi \leq 90°$ are equal to values of shear stresses $|\sigma_{\pi-\theta,2}|$ for $90° \leq \varphi \leq 180°$.

Significantly that the maximal values of replacements are observed at inclination of plane of isotropy $\varphi = 90°$ for angles of falling waves in range $0° < \alpha \leq 90°$ and at $\varphi = 0°$ for range of falling wave angles $90° < \alpha \leq 180°$.

After analyzing the dependence of replacements on a size of dike between the hole and plane border it was found that at the process of relocation the hole close to the plane border replacement values increase sharply, but after increasing the distance between hole and plane border these replacements have a tendency to decrease. And when the depth $\delta \geq 10R$ contour replacement values remain virtually unchanged and equal to contour replacement values for deep located circular hole (see fig.2).

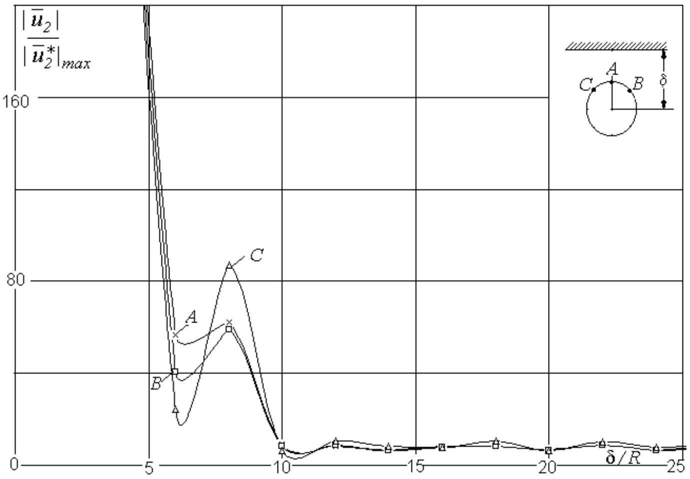

Figure 2.

References

[1] Walter Wittke, Berndt Pierau, Claus Erichsen "*New Austrian Tunneling Method (NATM) Stability Analysis and Design*" (Statik und Konstruktion der Spritzbetonbauweise. Geotechnik in Forschung und Praxis), WBI-PRINT 5, VGE-Verlag Glückauf GmbH, Essen 2002, ISBN 3-7739-1305-2

[2] A.N.Guz, V.D.Kubenko, M.A.Cherevko, "*Diffraction of elastic waves*", Kiev, Publishing House "Naukova dumka", 1978, Russian.

[3] A.N.Guz, V.T.Golovchan, "*Diffraction of elastic waves in multicoherent bodies*", Kiev, Publishing House "Naukova dumka", 1972, Russian

[4] N.A.Shulga, V.I. Colodii. *"Diffraction of SH-waves on noncircular cylindrical cavity"*, Applied mechanics, 1984, Vol.17, No3, Russian

[5] A.S.Kosmodamianski, V.I.Storozhev, *"Dynamical tasks of elasticity theory for anisotropic environments"*, Kiev, Publishing House Naukova dumka", 1985, Russian

[6] A.S.Kosmodamianski, L.B.Obodovski,V.I.Storozhev *"Diffraction of elastic waves on anisotropic cylindrical inclusion inside the multicoherent areas"*, Proceedings of Science Academy of USSR, Vol. A, 1980, No10, pp.38-41,Russian House "Gosstroiizdat", 1959, Russian.

[7] J.Pao *"Dynamical concentration of stresses in elastic plane, Applied mechanics"* (Proceedings of American Society of Civil Engineers (ASCE)): collection of translations, 1962, No2, pp. 147-154, Russian.

[8] J.Mau, L.Mente *"Dynamical stresses and moving near of cylindrical surface of break from a flat harmonious shift wave"*, Applied mechanics (Proceedings of American Society of Civil Engineers (ASCE)): collection of translations, 1963, No4, Russian.

[9] Zh.S.Erzhanov, Sh.M.Aitaliyev, L.A.Alekseeva, *"Dynamic of tunnels and underground pipelines"*. – Alma-Ata, Publishing House "Nauka", 1989, Russian

[10] Zh.S.Erzhanov, Sh.M.Aitaliyev, Zh.K.Masanov, *"Seismically stressed state of underground constructions in anisotropic layered massif"*, Alma-Ata, Publishing House "Nauka", 1980, Russian

[11] Lyazzat Atymtayeva, Zhailau Masanov, Bagdat Yagaliyeva , Gulzada Myrzakhmetova *"Stationary diffraction of elastic SH-waves onto mutually influencing cavities of arbitrary structure and deep lying in transtropic massif"*, Proceedings of the 3rd IASME / WSEAS International Conference on CONTINUUM MECHANICS (CM'08), Cambridge, UK, February 23-25, 2008, ISBN: 978-960-6766-38-1, ISSN: 1790-2769, p. 5-9

ISBN: 978-988-18210-7-2
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)