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CFD Analysis of the Steady Flow across a Tapered Trapezoidal Cylinder

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Abstract— In this paper, two-dimensional incompressible laminar flow past a long tapered trapezoidal cylinder is examined in the steady flow regime. The governing Navier-Stokes equations along with appropriate boundary conditions are solved by using a finite volume method based CFD solver FLUENT (6.3). The computational grid is created in a commercial grid generator GAMBIT. The flow patterns are presented by streamline and velocity magnitude profiles. The wake length and drag are calculated for the range of conditions studied here. The wake length increases with increasing value of the Reynolds number; however, the mean drag coefficient decreases with increasing value of the Reynolds number for the range of conditions investigated here.

Index Terms— **Drag, steady flow, trapezoidal cylinder, wake length**

I. INTRODUCTION

In recent years, flow over an obstacle of non-circular cross-section (e.g., square and triangular cylinders) has been the topic of research of many researchers. In spite of this, the flow around bluff body of trapezoidal cross-section has not been investigated in the same extent. It has many engineering applications, such as electronic cooling, heat exchange systems, offshore structures, suspension bridges, chimneys, flow metering devices, probes and sensors, etc. The relevant literature for the problem under consideration is given below.

Lee [1] studied the characteristics of the developing recirculation region behind the tapered trapezoidal cylinder and its interaction with the separating shear layer from the leading edges of the cylinder, numerically for an impulsively started laminar flow. In his study, the range of Reynolds number considered is 25 to 1000. He identified main flow and sub flow regimes by an analysis of the evolution of the flow characteristics. He found that, for a given trapezoidal cylinder, flow starts with no separation. With the advancement of time, the symmetrical standing zone of recirculation develops aft of the trapezoidal cylinder. He found that the rate of growth in width, length and structure of the aft end eddies depends on the Reynolds number. In time, separated flow from the leading edges of the trapezoidal cylinder develops and forms growing separation bubbles on the upper and lower inclined surfaces of the cylinder grow towards the downstream regions and merge with the symmetrical eddies aft of the trapezoidal cylinder. Chung and Kang [2] reported that the Strouhal number from trapezoidal cylinders depends not only the Reynolds number, but the height ratio as well. The distribution of the Strouhal number shows that it has minimum values for Reynolds numbers of 100 and 150 at height ratios of 0.7 and 0.85, respectively. It is noticed that at the Reynolds number of 200, the variation of Strouhal number increases as the height ratio decreases. Kahawita and Wang [3] carried out 2-D numerical simulations of the Benard-von Karman hydrodynamic instability behind trapezoidal bluff bodies using the spline method of fractional steps. They found that the maximum amplitude of the velocity component oscillating with the fundamental frequency follows fairly well the scaling law. The influence of the trapezoidal shape on the value of the critical Reynolds number and on the vortex shedding is briefly discussed. It appears that the influence of the height of the trapezoidal cylinder has the dominant influence on the value of Strouhal number, when compared with the effect of the smaller trapezoidal base width. Chen et al. [4] investigated the 2-D flow around a porous expanded trapezoidal cylinder using finite volume method, based on the body-fitted, non-orthogonal grids and multi-block technique. The flow range considered was varied from steady state to unsteady Reynolds number and different porosities, Darcy numbers and stress jump parameters were considered. With a large Darcy number, the Reynolds number has to be higher before the vortex shedding phenomena occurs. The effects of the stress jump parameters are given for Reynolds number = 20 to 200. At large Darcy number, the fluctuation-amplitude of drag coefficient decreases. Also, a large porosity cylinder results in a smaller drag coefficient and larger lift amplitude.

Thus, based on the above discussion, it can be concluded here that very limited work is available on the flow over a long tapered trapezoidal cylinder, especially in the steady flow regime. Therefore, the objective of this study is to investigate the fluid flow over a long obstacle of tapered trapezoidal cross-section in the steady flow regime.

II. GOVERNING EQUATIONS AND BOUNDARY CONDITIONS

For an incompressible, 2-D and steady flow, non-dimensional forms of the continuity, x and y components of Navier-Stokes equations can be written as

Continuity equation:

$$\frac{\partial V_x}{\partial x}+\frac{\partial V_y}{\partial y}=0 \qquad (1)$$

x-Momentum equation:

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$$\frac{\partial V_x}{\partial t}+\frac{\partial V_xV_x}{\partial x}+\frac{\partial V_yV_x}{\partial y}=-\frac{\partial p}{\partial x}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 V_x}{\partial x^2}+\frac{\partial^2 V_x}{\partial y^2}\right) \tag{2}$$

y-Momentum equation:

$$\frac{\partial V_y}{\partial t}+\frac{\partial V_xV_y}{\partial x}+\frac{\partial V_yV_y}{\partial y}=-\frac{\partial p}{\partial y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 V_y}{\partial x^2}+\frac{\partial^2 V_y}{\partial y^2}\right) \tag{3}$$

In this study, the Reynolds number is defined as $bU_\infty \rho / \mu$. where b is the upstream side of the trapezoidal cylinder, U_∞ is the uniform velocity at the inlet, ρ is the density of the fluid and μ is the viscosity of the fluid. Also, V_x, V_y, p and t are the dimensionless velocities in x and y directions, dimensionless pressure and dimensionless time, respectively.

The dimensionless boundary conditions may be written as follows (Fig. 1).

- **At the inlet boundary,** $V_x = 1$; $V_y = 0$
- **On upper and lower boundaries,** $\partial V_x / \partial y = 0$; $V_y = 0$
- **On the surface of the trapezoidal cylinder,** No-slip boundary condition, i.e.,$V_x = 0$; $V_y = 0$
- **At the exit boundary,** The default outflow boundary condition in FLUENT, which assumes a zero diffusion flux for all flow variables, is used. This is similar to Neumann boundary condition, i.e.,$\partial \mathrm{V_x} / \partial \mathrm{x} = 0$, $\partial \mathrm{V_y} / \partial \mathrm{x} = 0$

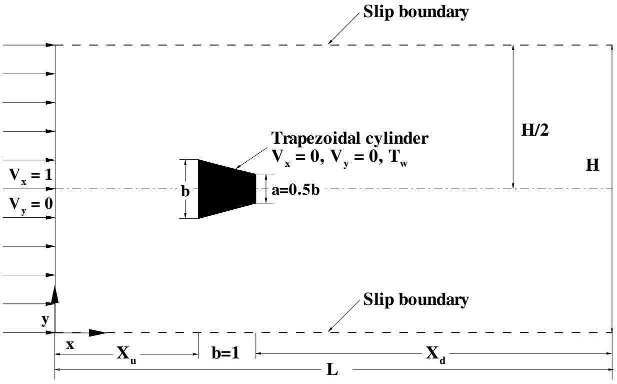

Figure 1: Schematics of the flow around a tapered trapezoidal cylinder

III. NUMERICAL DETAILS

The problem under consideration is solved by using a finite volume method based CFD solver FLUENT. The grid is generated by using GAMBIT, a pre-processor to Fluent. A very fine grid of cell size of 0.002 units is used near the cylinder to capture the wake-wall interactions and the larger size grids are used away from the trapezoidal cylinder. The second order upwind scheme is used to discretise convective terms of momentum equations, whereas the diffusive term is discretised by central difference scheme. The resulting algebraic equations are solved by Gauss-Siedel iterative scheme.

IV. RESULTS AND DISCUSSION

The present study concentrates on the 2-D incompressible laminar flow across a long (horizontal) tapered trapezoidal cylinder in an unconfined domain for Reynolds numbers of 10 and 20 in the steady flow regime with an upstream length (X_u) of 12b and downstream length (X_d) of 20b. The length (L) and height (H) of the computational domain are taken as 33b and 30b, respectively.

The benchmarking of the present results is done with the results of Lee [1] for the values of the Reynolds number of 25 and 50. An excellent agreement is found between the present results and that of Lee [1]. The maximum deviations in the values of the drag coefficients are found to be less than 4 %.submitting author is responsible for obtaining

FLOW PATTERNS

In this study, flow patterns close to the tapered trapezoidal obstacle are presented for the Reynolds number of 10 and 20. The flow patterns are presented by streamline and velocity magnitude profiles in Figs. 2 and 3, respectively. Similar to the long square obstacle case, the symmetry in the flow field about the mid plane (i.e., at y = 15b) can be seen here from these figures. In the steady flow regime, two symmetric vortices are formed behind the tapered trapezoidal cylinder and the size of these vortices increases with increasing value of the Reynolds number (Figs. 2 and 3).

WAKE LENGTH AND DRAG COEFFICIENT

The variation of the recirculation length and the total drag coefficient for the values of the Reynolds numbers of 10 and 20 is presented in Table 1. The wake length increases with increasing value of the Reynolds number. However, the mean drag coefficient decreases with increasing value of the Reynolds number in the steady regime.

Table 1: Variation of the mean drag coefficient and the recirculation length with Reynolds number

Reynolds number	Mean drag coefficient	Recirculation length
10	2.9838	0.125
20	2.1837	0.613

V. CONCLUSION

In the present study, flow across a long tapered trapezoidal cylinder is investigated for the Reynolds number of 10 and 20 in the steady flow regime. The flow patterns are represented here by streamline and velocity magnitude profiles. Two symmetric vortices are formed behind the long trapezoidal bluff body and the size of these vortices increases with increasing value of the Reynolds number in the steady flow regime. The wake length increases as the Reynolds number increases; however, the mean drag coefficient decreases with increasing value of the Reynolds number for the range of conditions covered here.

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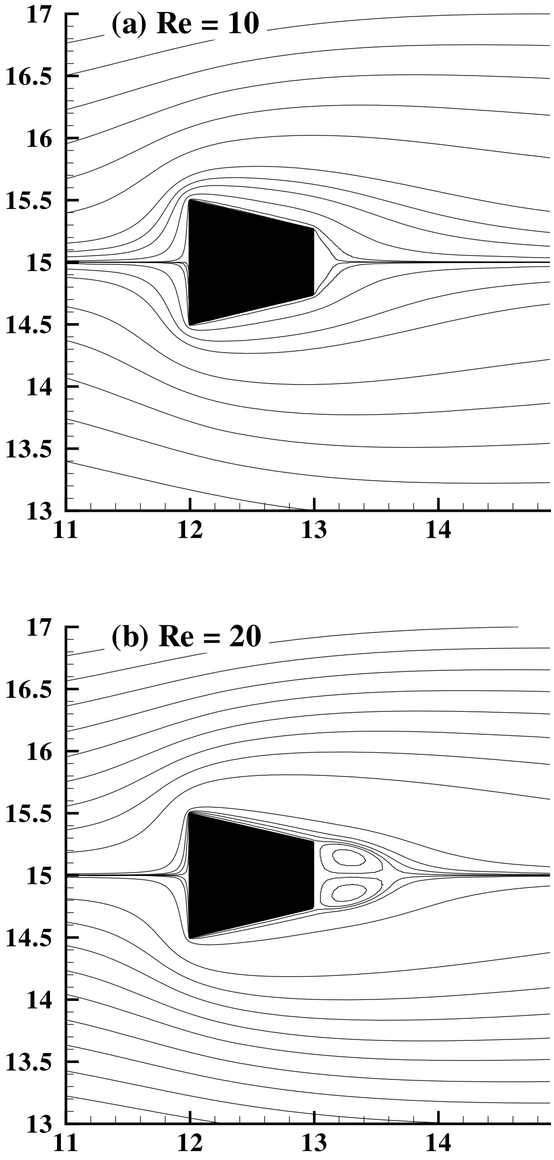

Figure 2: Streamline profiles for (a) Re = 10 and (b) Re = 20

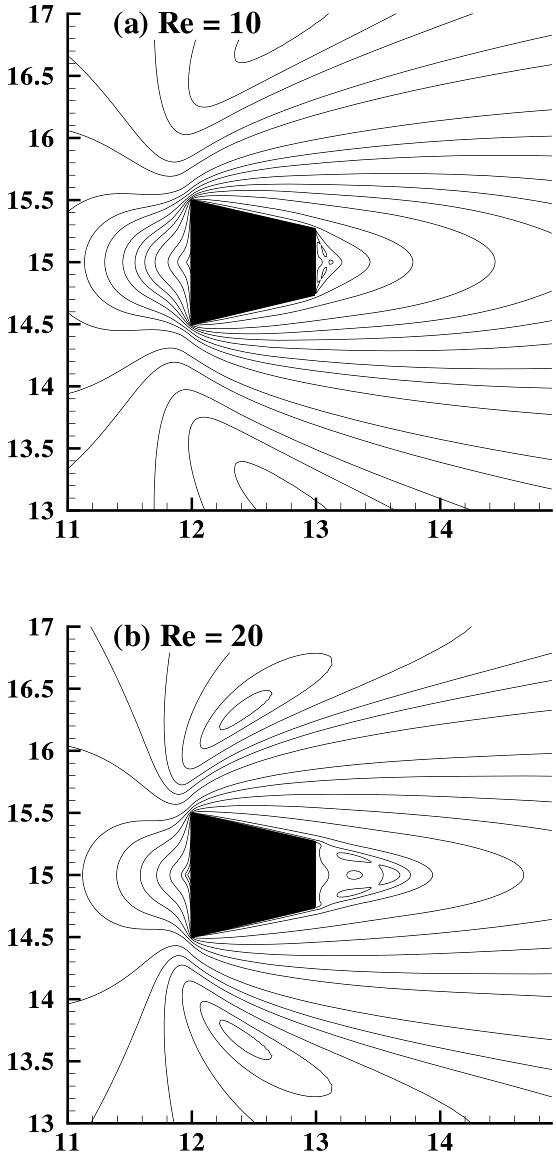

Figure 3: Velocity magnitude profiles for (a) Re = 10 and (b) Re = 20