The Simultaneous Influence of the Skin Effect, Environmental Conditions and Variable Resistivity on Current and Temperature Distribution in Overhead Conductors

Oscar Chávez and Federico Méndez

Abstract— **In the present work, we have theoretically analyzed the heat conduction of alternating current that flows in a circular electric conductor, taking into account variations of electric resistivity with temperature, which leads to a coupled thermo-electric model. In this approach, we have considered the presence of the skin effect that yields important radial differences of temperature by solving numerically the corresponding Maxwell's equations for the pass of the electrical current, together with the heat conduction equation to predict the radial variations of temperature. The results are presented in dimensionless form in order to reduce the number of physical variables. It is well-known that these high-voltage transmission lines are constructed with aluminum conductors that are composed by strands of the same material. For this reason, trapped-air between these strands complicates the analysis of these phenomena. In this direction, the mathematical model for the heat conduction equation, therefore, takes into account that the air-aluminum system is represented by a porous media. Finally, the imposed thermal boundary conditions reveal that the environmental conditions are a fundamental factor to control the above effects.**

Index Terms— **Ampacity, Skin effect, Thermal behavior.**

I. Introduction

Currently it is necessary to get a better model that determines more accurate temperature profiles in order to prompt the fuller utilization of overhead transmission lines; therefore, we have to consider all possible physical laws that describe the phenomenon.

Analytical and experimental studies of the thermal behavior in cables were initially carried out measuring conductor temperature and meteorological conditions in real-time systems, which did not take into account temperature variations within conductor, for this reason it does not allow using the full function of the conductor [1].

For this reason, the estimation of the temperature for a conductor is an important issue. In early models, a uniform temperature approximation has been considered. Normally, it is determined by an energy balance between the generated heat due to the Joule effect and the removed heat by environmental conditions. Reference [2] shows a thermal model that does not consider the existence of temperature gradients and was solved by the transient and steady-state cases.

On the other hand, in stranded cables the gradients of temperature are larger than the corresponding gradients in solid conductors, due to air trapped between the wires forming the cable. Reference [3] shows in detail the temperature profile for the steady –state case of monometallic cables, considering both cases: the cable as solid and as well as a stranded solid, under the assumption of uniform current density. He also developed a novel analysis to calculate the equivalent thermal conductivity for stranded cables. Reference [4] developed a comparison between the temperature gradients in a steady regime with the results obtained with the aid of a lumped model, showing that the temperature of the center of the conductors is always higher than the temperature calculated on the assumption of constant temperature.

At present, due to computational progress is possible to obtain easily the solution of the equation of heat transfer in transient state and thus to predict the thermal behavior of the electrical conductor.

In the past, many mathematical models have been developed to show the thermal behavior of electrical conductors in order to make a better design of overhead-line conductors. Early models were made under the assumption that electrical current is uniform in the cross section of the conductor. However, In order to have a better representation of the thermal behavior is necessary to take into account the electromagnetic effects, which affect seriously the electrical performance. The above is particularly valid when alternating current flows through a conductor and a redistribution of current density is developed [5].

Therefore, in the present work we develop a mathematical model for which the presence of simultaneous electromagnetic and thermal effects has been taken into account. The corresponding governing equations are solved by finite difference method. In this manner, the temperature profiles influenced by the electromagnetic effects are obtained for realistic cases and the influence of the environmental conditions is clarified.

II. Description Of The Phenomenon

The case of study is here represented by a sudden flow of electric current in a cylindrical metallic conductor that is initially found at ambient temperature. The alternating current

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that is flowing in this electrical medium leads to a redistribution of current density, causing the well-known skin effect. Under this condition, the current density tends to flow over the surface of the conductor, depending strongly on the frequency of the electrical signal. Recognizing the existence of a specific electrical resistance associated with the conductor, this component suffers an increment of its temperature, generating a finite quantity of heat due to the Joule effect, which originates an increase of temperature of the conductor, and therefore an increase of the electric resistivity, which causes a redistribution of current density.

Normally, the heat is generated in regions near to the surface of the conductor, because of skin effect. Therefore, we expect that during the transient state the heat is transferred to the core of the cable by heat conduction and considering that the cable is stranded, we use the theory of porous media to know the equivalent thermal properties. We assume that the heat transferred to the environment is characterized by an appropriate Biot numbers.

A. *Electromagnetic Model*

From the well-known Maxwell's equations, we can readily derive a wave equation to analyze the electromagnetic propagation. Therefore, the current density is governed by the following equation [6]:

$$\nabla^2 \lambda \bar{J} = \mu\left(\frac{\partial \bar{J}}{\partial t} + \gamma \frac{\partial^2 \lambda \bar{J}}{\partial t^2}\right), \tag{1}$$

In the above equation, λ is the electric resistivity, $\bar{J}$ is the current density, μ is the magnetic permeability, γ is the electric permittivity and t is the physical time.

We consider that exist only variations of the current density in the radial direction and the alternating current behaves like a sinusoidal wave. Therefore, the current density can write as $\bar{J} = J_s(r)e^{i\omega t}$. On the other hand, the electrical resistivity has a linear variation with temperature which can be written as $\lambda = \lambda_\infty\left[1+\phi(T-T_\infty)\right]$, and introducing it into (1) we obtain that,

$$\begin{aligned}&\frac{d^2 J_s}{dr^2} + \left(\frac{2\phi}{1+\phi(T-T_\infty)}\frac{\partial T}{\partial r} + \frac{1}{r}\right)\frac{dJ_s}{dr} + \\ &+\frac{\phi}{1+\phi(T-T_\infty)}\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right]J_s = \\ &= \left\{\frac{\mu\omega}{\lambda}\left[i + \gamma\omega\lambda_\infty\left(\frac{\phi}{\omega^2}\frac{\partial^2 T}{\partial t^2} + \frac{2i\cdot\phi}{\omega}\frac{\partial T}{\partial t} - \left(1+\phi(T-T_\infty)\right)\right)\right]\right\}J_s,\end{aligned} \tag{2}$$

where ω is the frequency of the electrical signal, r is the radial coordinate, T is the temperature, T_∞ is the environment temperature, J_s is the current density function depending only on radial coordinate, ϕ is the temperature coefficient for resistivity and i is the imaginary number $\sqrt{-1}$.

In practical cases, the term $\gamma\omega^2\mu$ is smaller than $i\omega\mu/\lambda$ and can be neglected in a first approximation. In addition, we can introduce the well-known conductor skin depth parameter, δ, defined by $\delta = (2\lambda/\omega\mu)^{1/2}$, which can be found elsewhere [7]; in this form then (2) can be written as

$$\begin{aligned}&\frac{d^2 J_s}{dr^2} + \left(\frac{2\phi}{1+\phi(T-T_\infty)}\frac{\partial T}{\partial r} + \frac{1}{r}\right)\frac{dJ_s}{dr} + \\ &+\frac{\phi}{1+\phi(T-T_\infty)}\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right]J_s = \\ &= \frac{2i}{\delta^2\left(1+\phi(T-T_\infty)\right)}J_s.\end{aligned} \tag{3}$$

The above equation must be solved with the following boundary conditions:

$$\text{at} \quad r = 0: \quad \frac{dJ_s}{dr} = 0, \tag{4}$$

$$\text{at} \quad r = R: \quad J = J_R. \tag{5}$$

Here, R represents the radius of the metallic conductor and J_R is the current density at the surface of the conductor and should be determinate with the following restriction:

$$I = \int_S J_s \cdot dA,$$

where A is the cross section area, and I is the total circulating electrical current. In our case, we assume known values for this variable given lines below.

B. *Thermal Model*

In order to determine the gradients of temperature generated by the Joule effect, we must solve the heat conduction equation in transient state, regarding only temperature variations in the radial coordinate [8]:

$$k_{ef}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \lambda_\infty\left(1+\phi(T-T_\infty)\right)\left|J_s\right|^2 = (\rho c)_{ef}\frac{\partial T}{\partial t}, \tag{6}$$

and subject to the following boundary and initial conditions:

$$r = 0: \quad \frac{\partial T}{\partial r} = 0, \tag{7}$$

$$r = R: \quad -k_{ef}\frac{\partial T}{\partial r} = h(T-T_\infty), \tag{8}$$

$$t = 0: \quad T = T_\infty. \tag{9}$$

In the above equations, k is the thermal conductivity of the metallic conductor, ρ is the density, c is specific heat, h is the convective heat transfer coefficient and subscript ef denotes effective properties.

C. *Order of magnitude analysis and dimensionless variables*

In order to reduce the number of physical parameters and variables, we can develop an order of magnitude analysis.

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First, we identify various scales: the characteristic convective time scale $t_C \sim \rho c R/h$. On the other hand, the suitable spatial scale corresponds directly to the radius of the conductor $r \sim R$. Furthermore, the characteristic temperature drop ΔT_c can be obtained through an energy balance between the heat generation term and the transient term, i.e.:

$$\frac{\Delta T_C h}{R} \sim \lambda_\infty J_R^2, \tag{10}$$

thus,

$$\Delta T_c \sim \frac{\lambda_\infty R^2 J_R^2}{k_{ef} \cdot Bi}, \tag{11}$$

and the dimensionless parameter Bi in the above relationship represents the Biot number defined as

$$Bi = \frac{hR}{k_{ef}}. \tag{12}$$

With the above set of characteristic geometrical and physical scales, the electromagnetic and thermal models can be considerably simplified by introducing the following dimensionless variables

$$\theta = \frac{T - T_\infty}{\Delta T_c}, \qquad \eta = \frac{r}{R}, \qquad \tau = \frac{t \cdot h}{\rho c R},$$

$$\kappa = \frac{\phi \lambda_\infty R^2 J_R^2}{k_{ef} \cdot Bi}, \quad \text{and} \quad \varphi = \frac{J_s}{J_R}.$$

D. Dimensionless Electromagnetic Model

Therefore, (3)-(5) with the aid of the above considerations can be written as,

$$\frac{d^2\varphi}{d\eta^2} + \left(2\frac{\kappa}{(1+\kappa\theta)}\frac{\partial\theta}{\partial\eta} + \frac{1}{\eta}\right)\frac{d\varphi}{d\eta} + \\ + \frac{\kappa}{(1+\kappa\theta)}\left(\frac{\partial^2\theta}{\partial\eta^2} + \frac{1}{\eta}\frac{\partial\theta}{\partial\eta}\right)\varphi = \frac{2i}{(1+\kappa\theta)\varepsilon^2}\varphi, \tag{13}$$

Together with,

$$\eta = 0: \qquad \frac{\partial\varphi}{\partial\eta} = 0, \tag{14}$$

$$\eta = 1: \qquad \varphi = 1. \tag{15}$$

In the above system of equations, ε is a dimensionless parameter related directly with the intensity of the skin effect and given by $\varepsilon = (\delta / R)$.

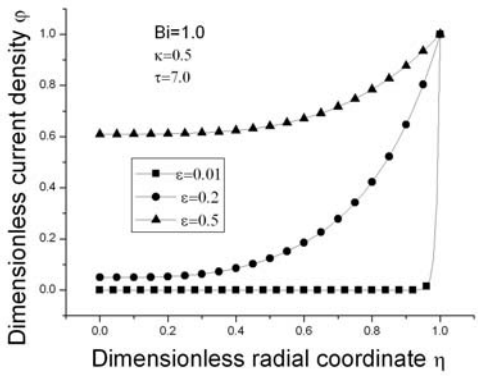

Figure 1. Variation of dimensionless current density distribution, φ, for different values of ε

E. Dimensionless Thermal Model

In the same manner, we can use the dimensionless variables for obtaining the following dimensionless thermal model,

$$\frac{1}{\eta}\frac{\partial}{\partial\eta}\left(\eta\frac{\partial\theta}{\partial\eta}\right) + Bi(1+\kappa\theta)|\varphi|^2 = Bi\frac{\partial\theta}{\partial\tau}; \tag{16}$$

together with the associated boundary and initial conditions given above.

$$\eta = 0: \qquad \frac{\partial\theta}{\partial\eta} = 0, \tag{17}$$

$$\eta = 1: \qquad \frac{\partial\theta}{\partial\eta} = -Bi \cdot \theta, \tag{18}$$

$$\tau = 0: \qquad \theta = 0. \tag{19}$$

The above dimensionless heat conduction equation together with boundary and initial conditions, here represented by the system (13)-(19) were solved using the conventional finite-differences method [9].

III. RESULTS

In this paper we have numerically solved the current density equation together with the unsteady heat conduction equation with a non-uniform heat generation and here represented by the alternating current. Both equations were solved using the well known finite-differences method.

In particular, we show the current densities and the temperature profiles as functions of the radial coordinate, considering the influence of the skin effect and the variation of electrical resistivity with temperature. In this direction, we prove the existence of two dimensionless parameters -here denoted by the symbols ε and κ-, which measures the electrical thickness of the current density and the level of coupling between both models –electric and thermal models- respectively.

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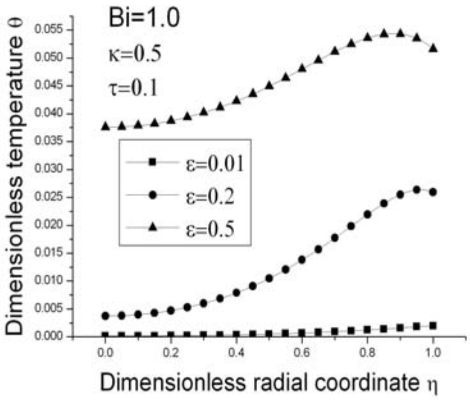

Figure 2. Dimensionless temperature profile, θ , for different values of the dimensionless parameter ε and τ =0.1.

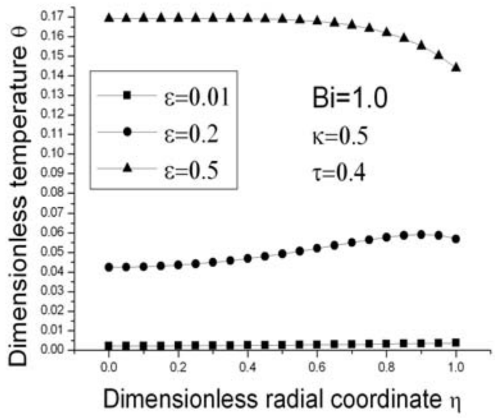

Figure 3. Dimensionless temperature profile, θ , for different values of the dimensionless parameter ε and τ =0.4.

A. *Influence of* ε

In Fig. 1 we show the current density as a function of the dimensionless radial coordinate for three different values of the skin parameter ε . Clearly, for small values of this parameter, the stronger skin effect is enhanced because the current density distribution tends to flow in regions closer to the surface

For the following Figs. 2-5, we show the results for the heat transfer process following this sequence: in order to see the influence of skin effect on the dimensionless temperature profiles, we choose a fixed values of the Biot number $Bi = 1$, and $\kappa = 0.5$, different values of the dimensionless time τ (=0.1, 0.4, 1.0, 7.0) and three different values of the skin parameter ε (=0.01, 0.2, 0.5). Therefore, Figs. 2-5 show the thermal behavior of dimensionless temperature as a function of the dimensionless radial coordinate. These figures reveal clearly the influence of the skin effect: for increasing values of ε the domain of the spatial variation of the temperature is going to be more relevant. Together with the above considerations, on the other hand, these figures reveal that for increasing values of the dimensionless time the temperature profiles tend to reach a steady-state condition.

B. *Influence of* Bi

In Fig. 6 we show the current density as a function of the dimensionless radial coordinate for three different Biot numbers Bi. Here, it is possible to appreciate redistribution in dimensionless current density behavior, due to environment conditions. A small value of Bi means that the heat transfer process to the environment is weaker and therefore the above figure suggest increasing values of the dimensionless current density. It implies smaller values of the surface current density J_R in order to guarantee the relationship

$$I = \int_S J_s \cdot dA.$$

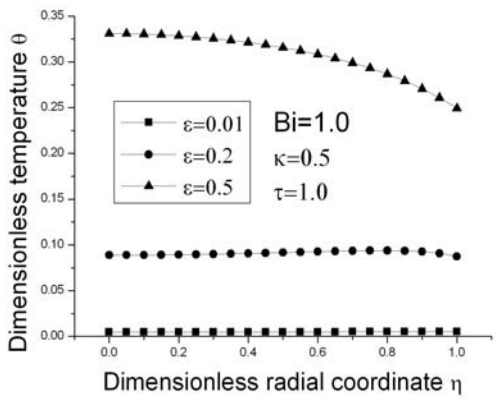

Figure 4. Dimensionless temperature profile, θ , for different values of the dimensionless parameter ε and τ =1.0.

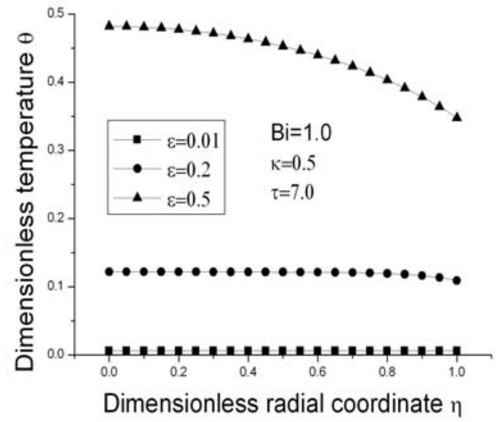

Figure 5. Dimensionless temperature profile, θ , for different values of the dimensionless parameter ε and τ =7.0.

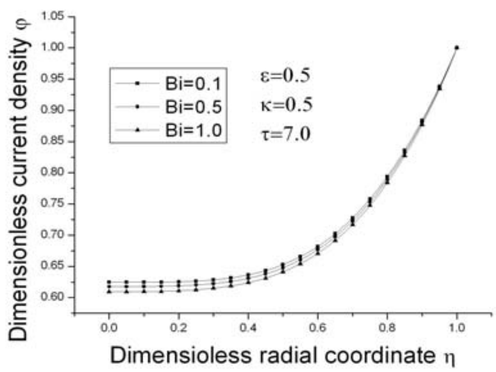

Figure 6. Variation of dimensionless current density distribution, φ , for different values of Biot number Bi

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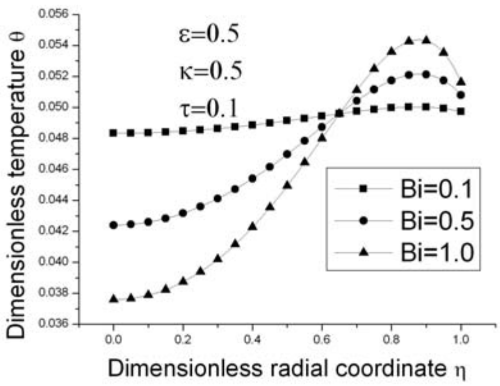

Figure 7. Dimensionless temperature profile, θ, for different values of Biot number Bi and τ =0.1.

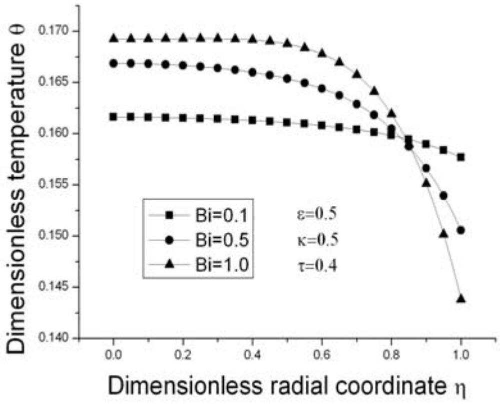

Figure 8. Dimensionless temperature profile, θ, for different values of Biot number Bi and τ =0.4.

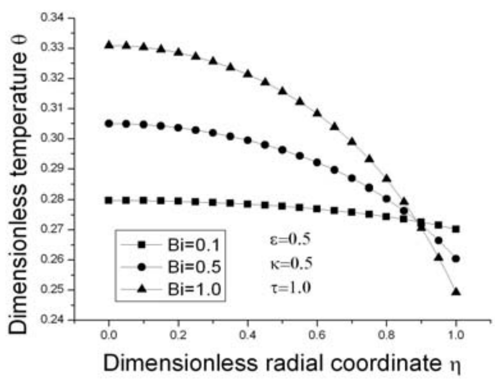

Figure 9. Dimensionless temperature profile, θ, for different values of Biot number Bi and τ =1.0.

Therefore, we can appreciate the influence of environmental conditions showing in Figs. 7-10 the temperature profiles under the influence of the Biot number, Bi (=0.1, 0.5, 1); keeping uniform values for the parameters $\varepsilon = 0.5$, and $\kappa = 0.5$ and varying the dimensionless time τ (=0.1, 0.4, 1.0, 7.0). Thus, for increasing values of Bi the spatial variation of the temperature is going to be more relevant; otherwise, we obtain practically uniform values for the temperature and in this case, the classical lumped method can be used.

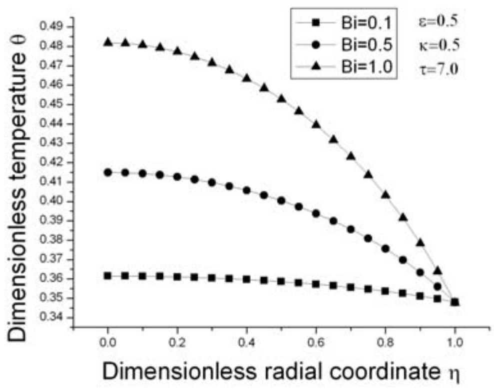

Figure 10. Dimensionless temperature profile, θ, for different values of Biot number Bi and τ =7.0.

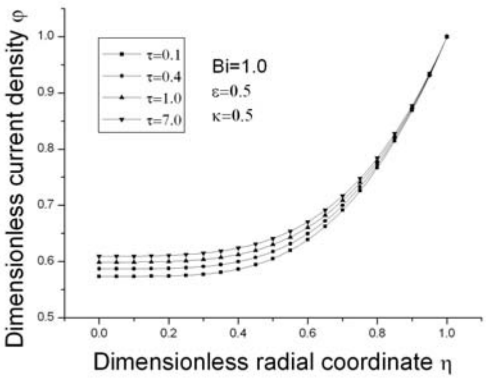

Figure 11. Transient state of dimensionless current density distribution, φ

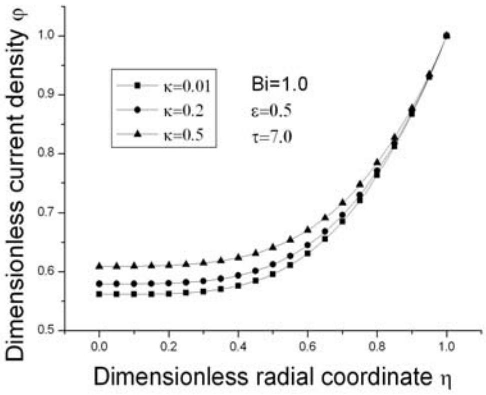

Figure 12. Variation of dimensionless current density distribution, φ, for different values of κ

C. *Influence of* κ

In Fig. 11 we show the current density as a function of the dimensionless radial coordinate in transient state for four different characteristic times and keeping fixed the values of the parameters $Bi = 1.0$, $\varepsilon = 0.5$ and $\kappa = 0.5$. In order to observe the influence of κ on the current density distribution, in Fig. 12 we keep invariable the parameters $Bi = 1.0$ and $\varepsilon = 0.5$ for three different values of the parameter κ (=0.01, 0.2, 0.5).

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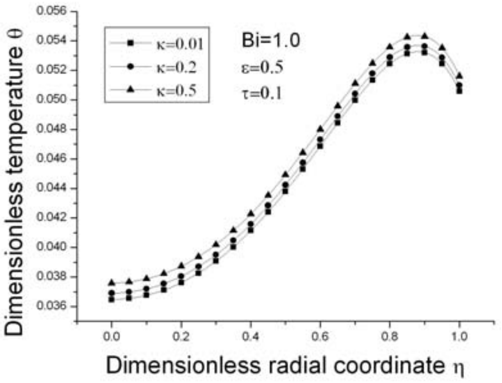

Figure 13. Dimensionless temperature profile, θ , for different values of the dimensionless parameter κ and τ =0.1.

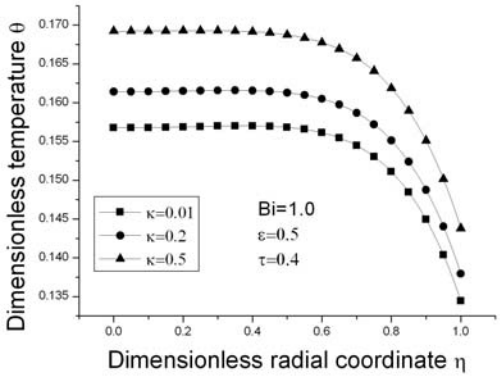

Figure 14. Dimensionless temperature profile, θ , for different values of the dimensionless parameter κ and τ =0.4.

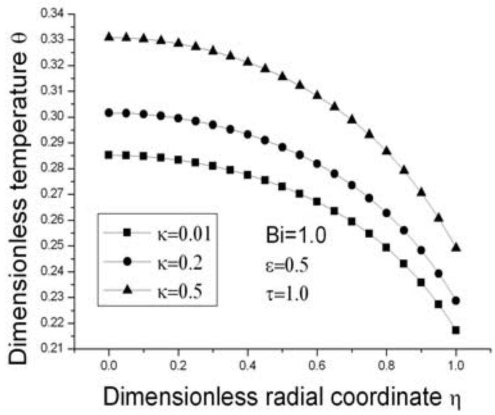

Figure 15. Dimensionless temperature profile, θ , for different values of the dimensionless parameter κ and τ =1.0.

For the following Figs. 13-16, we show the transient state of temperature profile. In order to show the effect of the parameter κ we kept uniform values of the parameters $Bi = 1.0$ and $\varepsilon = 0.5$ and varying the dimensionless time τ (=0.1, 0.4, 1.0, 7.0). Thus, for increasing values of κ the spatial variation of the temperature is going to be more relevant; otherwise, we can considerate both models separately.

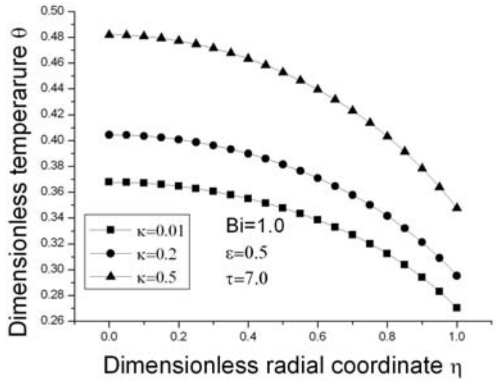

Figure 16. Dimensionless temperature profile, θ , for different values of the dimensionless parameter κ and τ =7.0.

IV. Conclusion

In summary, the temperature profiles are strongly influenced by the following parameters: Bi , κ ,and ε . For small values of these parameters, the physical consequences are direct: small values of the Biot number means an inefficient heat transfer process to the environment; therefore, the cable is overheated; while small values of skin parameter ε has a undesirable effect, because a great amount of electric current flows through a small area, leading it also to an overheated of the cable, and finally a small value of κ means that electric and thermal model are weakly coupled and can be solved separately. The above results are very interesting from a practical point of view, because the global electrical performance and design of high-tension cables is seriously altered by the influence of these factors.

References

[1] M. W. Davis, "A new thermal rating approach: the real time thermal rating system for strategic overhead conductor transmission lines. Part I. General description and justification of the real thermal rating system," *IEEE Trans. on Power Apparatus and Systems*, vol. 96, No. 3, pp. 803-809, 1977.

[2] W. Z. Black and W, R, Byrd, "Real-time ampacity model for overhead lines," *IEEE Trans. on Power Apparatus and Systems*, vol. 102, No. 7, pp. 2289-2293, 1983.

[3] V. T. Morgan, "The radial temperature distribution and efective radial thermal conductivity in bare solid and stranded conductors," *IEEE Trans. on Power Delivery*, vol. 5, pp. 1443-1452, 1990.

[4] W. Z. Black, S. S. Collins, and J. F. Hall, "Theoretical model for temperature gradients within bare overhead conductors," *IEEE Trans. on Power Delivery*, vol. 3, No. 2, pp. 707-715, 1988.

[5] A. Jordan, A. Barka, and M. Benmouna, "Transient state temperature distribution in a cylindrical conductor with skin effect," *Int. Journal of Heat and Mass Transfer*, vol. 30, No. 11, pp. 2446-2447, 1987.

[6] J. D. Jackson, *Classical Electrodynamics*, Ed. John Wiley & Sons, Inc., USA, 1975, pp. 209-268.

[7] A. A. Ghandakly, and R. L. Curran, "A model to predict current distributions in bundled cables for electric glass melters," *IEEE Trans. on Industry Applications,* vol. 26, No. 6, pp. 1043-1048, 1990.

[8] M. Özisik, *Heat Conduction*, John Wiley & Sons, Inc., USA, 1993.

[9] M. Özisik, *Finite Difference Methods in Heat Transfer*, CRC, USA, 1994.

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