Singularity Analysis of the New Parallel Manipulator with 6 Degree-of-Freedom

Zhumadil Zh. Baigunchekov, and Myrzabay B. Izmambetov

Abstract **— In this paper we are presenting some results of the singularity analysis of the new parallel manipulator with six degrees of freedom. This manipulator comprises two platforms connected by six legs, each of which consists of two links, one revolute and two spherical joints. Such structure gives the manipulator six degrees of freedom, and all revolute joints placed on the fixed platform are actuated. Thus, each leg has one actuator. We have derived the differential kinematic relations between two vectors: mobile-platform velocity and the active-joint rates. These relations comprise two matrices, the forward - and the inverse - kinematics Jacobians. The analytical approach to the research of these relations, based only on linear algebra, has yielded interesting results on identification of the singular configurations of the parallel manipulator.**

Index Terms **— Jacobian matrix, mobile platform, parallel manipulator, singularity analysis.**

I. Introduction

A parallel manipulator, also called the platform manipulator, is the closed-loop mechanism in which the moving platform with an operation point is connected to the base by at least two serial kinematic chains. It is well known that such manipulators possess inherent advantages of higher stiffness, higher payload capacity, and lower inertia to the manipulation problem than comparable serial manipulators. However, the closed-loop nature of parallel manipulators limits the motion of the platform and creates complex kinematic singularities. An important limitation of a parallel manipulator is that singular configurations may exist within its workspace where the manipulator gains one or more degrees of freedom and completely loses its stiffness [1], [2].

One of the first works devoted configuration singularities of general closed-loop mechanisms belongs to Gosselin and Angeles [3]. In this work, configuration singularities were classified into three main types, based on the properties of the Jacobian matrices of the mechanism. These matrices define the differential kinematic relations between the vectors mobile-platform velocity $\dot{\mathbf{x}}$ (vector of output velocities) and the active-joint rates $\dot{\mathbf{q}}$ (vector of input generalized velocities) as follows:

$$\mathbf{A}\dot{\mathbf{x}} = \mathbf{B}\dot{\mathbf{q}} \qquad (1)$$

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where **A** and **B** are the Jacobian matrices. Depending on which of the matrix is singular, the singularities of a parallel manipulator can be classified into the three types. *The first type* of singularity occurs when matrix **B** is singular (det **B**=0), and in a corresponding configuration a manipulator loses 1 or more degrees of freedom. *The second type* of singularity occurs when matrix **A** is singular (det **A**=0), and a manipulator gains 1 or more degrees of freedom. And at last, *the third type* of singularity is possible when matrices **A** and **B** are simultaneously singular. Tsai [2] has noted that matrix **A** is associated with the direct kinematics and matrix **B** is associated with the inverse kinematics. He determined the above mentioned types of singularities as inverse, forward, and combined singularities, respectively.

Thus, the singularity analysis of parallel manipulators requires the Jacobian analysis that is a much more difficult problem than the same analysis of serial manipulators. It is connected to the fact that these manipulators consist of many links, which form a number of closed loops. Different approaches to the singularity analysis of the manipulators based on Grassmann geometry, the screws theory and a machinery of Clifford algebra, etc. have been proposed [4]-[10]. In works [11], [12] the concept of screw reciprocity for the Jacobian analysis of parallel manipulators is presented.

The above mentioned advantages of the parallel manipulators became motivation for development of the new six-legged parallel manipulator (PM) with six degree of freedom (DOF), based on an RSS structure [13], [14]. This paper is devoted to the singularity analysis of this PM. The procedure of the formation of the differential kinematic relations (1) is described. The analytical approach of the singularity analysis based only on linear algebra is proposed and some results for identification of the first and second types of singularities of this PM with 6 DOF are presented.

II. Geometry of the PM with 6 DOF

The six-legged PM with 6 DOF contains a moving platform *3*, connected with the fixed base *0* by six legs of RSS kind, where R - a revolute joint, S - a spherical joint (Fig. 1). The first joint in each leg is an active one, i.e. all revolute joints at points O_i (i=1, 2, … 6) are active, and all spherical joints are passive. This PM is intended for reproduction of movement of a moving platform or the local coordinates system (the frame) $Px_Py_Pz_P$ attached to it, with respect to the base frame *OXYZ*

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$$\left.\begin{array}{l} X_P = X_P(\mathbf{q}(t)),\ Y_P = Y_P(\mathbf{q}(t)),\ Z_P = Z_P(\mathbf{q}(t)) \\ \gamma_P = \gamma_P(\mathbf{q}(t)),\ \alpha_P = \alpha_P(\mathbf{q}(t)),\ \beta_P = \beta_P(\mathbf{q}(t)) \end{array}\right\} \quad (2)$$

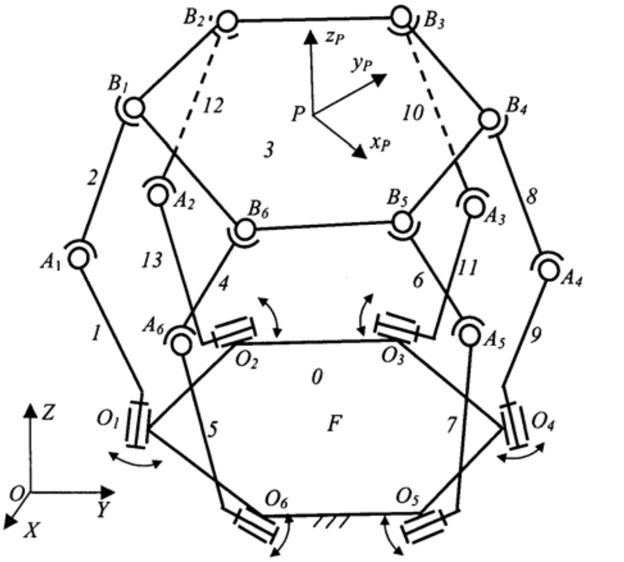

Fig. 1. Six-legged PM with 6 DOF

where $\mathbf{q}(t)=[\theta_1(t),\ \theta_2(t),\ldots,\ \theta_6(t)]^T$ - a vector of the input generalized coordinates; $\mathbf{x}(t)=[X_P,\ Y_P,\ Z_P,\ \gamma_P,\ \alpha_P,\ \beta_P]^T$ - a vector of the output coordinates (the position of the mobile platform); γ_P, α_P and β_P - the components of relative orientation of coordinates systems $Px_Py_Pz_P$ and $OXYZ$.

In the description of this manipulator in work [14] the rules of the choice of six parameters proposed by Sheth and Uicker [15] for the definition of a positional relationship of two frames are used. Three of these parameters a_{jk}, b_{jk}, c_{jk} correspond to linear shifts, and other three parameters $\alpha_{jk}, \beta_{jk}, \gamma_{jk}$, correspond to angular deviations of k-th frame with respect to j-th frame. Corresponding 4x4 transformation matrix $\mathbf{T}_{jk}$ which has been used to describe the geometry of separate links and to derive the symbolical formula of the parallel manipulator as a whole is obtained. Thus orientation of *k-th* frame with respect to *j-th* frame is determined by 3×3 orthogonal rotation submatrix $\mathbf{R}$ of matrix $\mathbf{T}_{jk}$ as follows:

$$^{j}\mathbf{R}_k = \mathbf{R}(\alpha_{jk}, \beta_{jk}, \gamma_{jk}) = \\ = \begin{bmatrix} c\gamma_{jk}c\beta_{jk} - s\gamma_{jk}c\alpha_{jk}s\beta_{jk} & -c\gamma_{jk}s\beta_{jk} - s\gamma_{jk}c\alpha_{jk}c\beta_{jk} & s\gamma_{jk}s\alpha_{jk} \\ s\gamma_{jk}c\beta_{jk} + c\gamma_{jk}c\alpha_{jk}s\beta_{jk} & c\gamma_{jk}c\alpha_{jk}c\beta_{jk} - s\gamma_{jk}s\beta_{jk} & -c\gamma_{jk}s\alpha_{jk} \\ s\alpha_{jk}s\beta_{jk} & s\alpha_{jk}c\beta_{jk} & c\alpha_{jk} \end{bmatrix}, \quad (3)$$

where $c\alpha_{jk} = \cos\alpha_{jk}$, $s\alpha_{jk} = \sin\alpha_{jk}$, and so on. Relative linear shift between these two frames can be determined by following 3×1 submatrix $\boldsymbol{\tau}$

$$^{j}\boldsymbol{\tau}_k = \boldsymbol{\tau}(a_{jk}, b_{jk}, c_{jk}, \alpha_{jk}, \beta_{jk}, \gamma_{jk}) = \\ = \begin{bmatrix} a_{jk}c\gamma_{jk} + b_{jk}s\gamma_{jk}s\alpha_{jk} \\ a_{jk}s\gamma_{jk} - b_{jk}c\gamma_{jk}s\alpha_{jk} \\ c_{jk} + b_{jk}c\alpha_{jk} \end{bmatrix}. \quad (4)$$

The identical architecture of each leg of the PM allows us to show all its geometrical parameters with respect to arbitrary i-th leg, i.e. with respect to kinematic chain $OO_iA_iB_iP$ (Fig. 2). Other five legs are not illustrated in Fig.2 to make the manipulator sketch simpler.

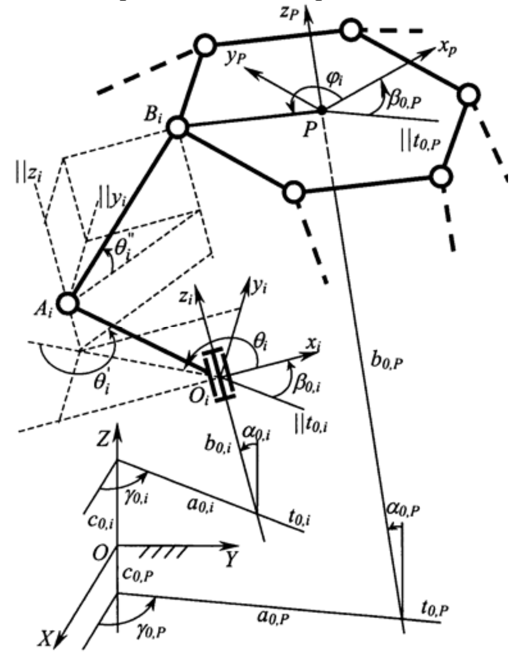

Fig. 2. Geometry of the i-th leg of the PM

The following parameters are given: $l_{O_iA_i} = f_i$ $l_{A_iB_i} = g_i$, $i = 1, 2, \ldots, 6$ - lengths of mobile links; $l_{PB_i} = h_i$, φ_i, $i = 1,2,\ldots,6$ - polar coordinates of spherical kinematic pairs B_i with respect to local frame $Px_Py_Pz_P$; $a_{0,i}, b_{0,i}, c_{0,i}, \alpha_{0,i}, \beta_{0,i}, \gamma_{0,i}$, $i = 1, 2, \ldots, 6$ - the parameters defining a local frame $O_ix_iy_iz_i$ attached to active joints O_i, with respect to base frame $OXYZ$; values of constant angles ψ_i defining deviations of links O_iA_i from a direction of an rotation axis of i-th active joint.

Let the positional relationship of local frame $Px_Py_Pz_P$ with respect to base frame $OXYZ$ be defined by a set of parameters $a_{0,P}, b_{0,P}, c_{0,P}, \alpha_{0,P}, \beta_{0,P}, \gamma_{0,P}$, and the vector $\mathbf{g}_i = \overline{A_iB_i}$ be defined by the spatial polar coordinates $\{g_i,\ \theta_i^{'},\ \theta_i^{''}\}$.

III. Generation of Jacobian Matrices

The loop-closure equation for each of legs of the PM, i.e. for kinematic chain $OO_iA_iB_iPO$, can be written as

$$\mathbf{r}_P = \mathbf{r}_{O_i} + \mathbf{f}_i + \mathbf{g}_i - \mathbf{h}_i, \quad i = 1, 2, \ldots, 6, \quad (5)$$

where

$$\mathbf{r}_P = [X_P, Y_P, Z_P]^T = {}^{0}\boldsymbol{\tau}_P = \\ = \boldsymbol{\tau}(a_{0,P}, b_{0,P}, c_{0,P}, \alpha_{0,P}, \beta_{0,P}, \gamma_{0,P}), \quad (6)$$

$$\mathbf{r}_{O_i} = [X_{O_i}, Y_{O_i}, Z_{O_i}]^T = {}^{0}\boldsymbol{\tau}_i = \\ = \boldsymbol{\tau}(a_{0,i}, b_{0,i}, c_{0,i}, \alpha_{0,i}, \beta_{0,i}, \gamma_{0,i}), \quad (7)$$

$$\mathbf{f}_i = \overline{O_iA_i} = {}^{0}\mathbf{R}_i\, {}^{i}\mathbf{f}_i,\quad \mathbf{g}_i = {}^{0}\mathbf{R}_i\, {}^{i}\mathbf{g}_i,\quad \mathbf{h}_i = \overline{PB_i} = {}^{0}\mathbf{R}_P\, {}^{P}\mathbf{h}_i, \quad (8)$$

$$^{0}\mathbf{R}_i = \mathbf{R}(\alpha_{0,i}, \beta_{0,i}, \gamma_{0,i}),\quad {}^{0}\mathbf{R}_P = \mathbf{R}(\alpha_{0,P}, \beta_{0,P}, \gamma_{0,P}), \quad (9)$$

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$$ {}^{i}\mathbf{f}_i = f_i\begin{bmatrix} s\psi_i c\theta_i \\ s\psi_i s\theta_i \\ c\psi_i \end{bmatrix},\ {}^{i}\mathbf{g}_i = g_i\begin{bmatrix} c\theta_i^{''} c(\theta_i+\theta_i^{'}) \\ c\theta_i^{''} s(\theta_i+\theta_i^{'}) \\ s\theta_i^{''} \end{bmatrix},\ {}^{P}\mathbf{h}_i = h_i\begin{bmatrix} c\varphi_i \\ s\varphi_i \\ 0 \end{bmatrix}. \quad (10) $$

Differentiating (5) with respect to time, we obtain

$$ \dot{\mathbf{r}}_P = \dot{\boldsymbol{\theta}}_i \times \mathbf{f}_i + \boldsymbol{\omega}_i \times \mathbf{g}_i - \boldsymbol{\omega}_P \times \mathbf{h}_i, \quad i = 1, 2, \dots, 6 \quad (11) $$

where $\dot{\mathbf{r}}_P = [\dot{X}_P, \dot{Y}_P, \dot{Z}_P]^T = [\vartheta_{P_X}, \vartheta_{P_Y}, \vartheta_{P_Z}]^T$ - a vector of a velocity of point *P*; $\dot{\boldsymbol{\theta}}_i = \dot{\theta}_i \cdot \mathbf{e}_i$ or ${}^{i}\dot{\boldsymbol{\theta}}_i = \dot{\theta}_i \cdot {}^{i}\mathbf{e}_i$ - vectors of angular velocity of the active joints with respect to base or local frame $O_i x_i y_i z_i$, respectively, and $\mathbf{e}_i$ or ${}^{i}\mathbf{e}_i = [0,0,1]^T$ - corresponding unit vectors indicating the direction of the rotation axis $O_i z_i$ of the *i*-th active joint; $\boldsymbol{\omega}_i$ - an angular velocity of *i*-th passive link A_iB_i; $\boldsymbol{\omega}_P = [\omega_{P_X}, \omega_{P_Y}, \omega_{P_Z}]^T$ - a vector of an angular velocity of a mobile platform.

Dot-multiplying both sides of (11) by $\mathbf{g}_i$, leads to

$$ \mathbf{g}_i \cdot \dot{\mathbf{r}}_P = \dot{\boldsymbol{\theta}}_i \cdot (\mathbf{f}_i \times \mathbf{g}_i) - \boldsymbol{\omega}_P \cdot (\mathbf{h}_i \times \mathbf{g}_i),\ i = 1, 2, \dots, 6. \quad (12) $$

Equation (12) can be presented in the matrix form like (1) by using the following denotation: $\dot{\mathbf{x}} = [\dot{\mathbf{r}}_P^T, \boldsymbol{\omega}_P^T]^T$ - a vector mobile-platform velocity, $\dot{\mathbf{q}} = [\dot{\theta}_1, \dot{\theta}_2, \dots, \dot{\theta}_6]^T$ - a vector of the active-joint velocities. Thus both Jacobian matrices **A** and **B** have dimensions 6×6, and are defined as follows

$$ \mathbf{A} = \mathbf{A}(\mathbf{x},\mathbf{q}) = \begin{bmatrix} \mathbf{g}_1^T & (\mathbf{h}_1 \times \mathbf{g}_1)^T \\ \mathbf{g}_2^T & (\mathbf{h}_2 \times \mathbf{g}_2)^T \\ \vdots & \\ \mathbf{g}_6^T & (\mathbf{h}_6 \times \mathbf{g}_6)^T \end{bmatrix}, \quad (13) $$

$$ \mathbf{B} = \mathbf{B}(\mathbf{x},\mathbf{q}) = $$
$$ = diag\left[\mathbf{e}_1^T \cdot (\mathbf{f}_1 \times \mathbf{g}_1), \mathbf{e}_2^T \cdot (\mathbf{f}_2 \times \mathbf{g}_2), \dots, \mathbf{e}_6^T \cdot (\mathbf{f}_6 \times \mathbf{g}_6)\right]. \quad (14) $$

Elements a_{ij} and b_{ij} of matrices **A** and **B** are defined by means of constant geometrical and variable kinematics parameters of PM by the following equations, respectively

$$ \begin{aligned} a_{i1} &= \mathbf{g}_i \cdot \mathbf{e}_X = g_{iX} = \\ &= g_i\{c\theta_i^{''} c(\theta_i+\theta_i^{'})(c\gamma_{0i}c\beta_{0i} - s\gamma_{0i}c\alpha_{0i}s\beta_{0i}) - \\ &- c\theta_i^{''} s(\theta_i+\theta_i^{'})(c\gamma_{0i}s\beta_{0i} + s\gamma_{0i}c\alpha_{0i}c\beta_{0i}) + s\theta_i^{''} s\gamma_{0i}s\alpha_{0i}\}, \\ a_{i2} &= \mathbf{g}_i \cdot \mathbf{e}_Y = g_{iY} = \\ &= g_i\{c\theta_i^{''} c(\theta_i+\theta_i^{'})(s\gamma_{0i}c\beta_{0i} + c\gamma_{0i}c\alpha_{0i}s\beta_{0i}) + \\ &+ c\theta_i^{''} s(\theta_i+\theta_i^{'})(c\gamma_{0i}c\alpha_{0i}c\beta_{0i} - s\gamma_{0i}s\beta_{0i}) - s\theta_i^{''} c\gamma_{0i}s\alpha_{0i}\}, \\ a_{i3} &= \mathbf{g}_i \cdot \mathbf{e}_Z = g_{iZ} = g_i\{c\theta_i^{''} c(\theta_i+\theta_i^{'})s\alpha_{0i}s\beta_{0i}) - \\ &- c\theta_i^{''} s(\theta_i+\theta_i^{'})s\alpha_{0i}c\beta_{0i} - s\theta_i^{''} c\alpha_{0i}\}, \end{aligned} \quad (15) $$

$$ a_{i4} = (\mathbf{h}_i \times \mathbf{g}_i) \cdot \mathbf{e}_X = n_{iX} = h_{iY}g_{iZ} - h_{iZ}g_{iY}, $$
$$ a_{i5} = (\mathbf{h}_i \times \mathbf{g}_i) \cdot \mathbf{e}_Y = n_{iY} = h_{iZ}g_{iX} - h_{iX}g_{iZ}, $$
$$ a_{i6} = (\mathbf{h}_i \times \mathbf{g}_i) \cdot \mathbf{e}_Z = n_{iZ} = h_{iX}g_{iY} - h_{iY}g_{iX}, \quad (16) $$

$$ b_{ii} = \mathbf{e}_i^T \cdot (\mathbf{f}_i \times \mathbf{g}_i) = f_i \cdot g_i \cdot \sin\psi_i \cos\theta_i^{''} \sin\theta_i^{'}, $$
$$ b_{ij} = 0,\ \text{for}\ i \neq j,\quad i = 1,2,\dots,6;\ j = 1,2,\dots,6, \quad (17) $$

where $\mathbf{e}_X$, $\mathbf{e}_Y$, $\mathbf{e}_Z$ - unit vectors indicating the direction of axes *OX*, *OY*, and *OZ*, respectively, and components of a vector $\mathbf{h}_i$ are defined by the equations

$$ \begin{aligned} h_{iX} &= h_i\{(c\gamma_{0P}c\beta_{0P} - s\gamma_{0P}c\alpha_{0P}s\beta_{0P})\cos\varphi_i - \\ &\quad - (c\gamma_{0P}s\alpha_{0P} + s\gamma_{0P}c\alpha_{0,P}c\beta_{0P})\sin\varphi_i\}, \\ h_{iY} &= h_i\{(s\gamma_{0P}c\beta_{0P} + c\gamma_{0P}c\alpha_{0P}s\beta_{0P})\cos\varphi_i + \\ &\quad + (c\gamma_{0P}c\alpha_{0P}c\beta_{0P} - s\gamma_{0P}s\beta_{0P})\sin\varphi_i\}, \\ h_{iZ} &= h_i\{s\alpha_{0P}s\beta_{0P}\cos\varphi_i + s\alpha_{0P}c\beta_{0P}\sin\varphi_i\}. \end{aligned} \quad (16) $$

IV. Singularity Analysis of the PM With 6 DOF

A. The first type of singularity

This type of singularities occurs, when the determinant of matrix **B** vanishes. According to (17), in such configurations at least one of diagonal elements of this matrix is equal to zero, i.e. we get that $\det \mathbf{B} = 0$, if:

1^0. $\theta_i^{''} = \pm\pi/2$, i.e. $\mathbf{g}_i \parallel O_i z_i$ (18)

2^0. $\theta_i^{'} = 0$ or π, i.e. $(A_iB_i) \cap O_i z_i \neq \varnothing$ (19)

Thus, the first type of singularities of a configuration can take place whenever the passive link A_iB_i, the actuated link O_iA_i, and the axis of rotation $O_i z_i$ of the active joint are located in one plane σ_i, as shown in Fig. 3. Moreover, since the solution of the inverse kinematics problem leads to two branches of solutions per leg (a spherical surface of radius g_i and centered in point B_i and a circumference of radius $f_i \cdot \sin\psi_i$ and with the center in a point $O_i^{'}$ on axis $O_i z_i$, in general can be intersected in two points) when the prescribed platform coordinates are located inside the workspace, or to no real solution (the indicated sphere and the circumference have no cross points) when platform coordinates are outside the workspace. Hence, the boundary of workspace is determined

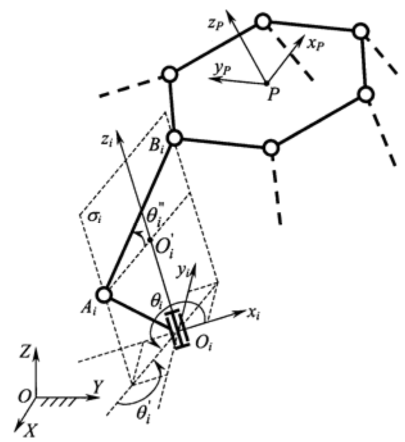

Fig. 3. The first type of singularity of the PM

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by the set of points for which the inverse kinematics problem gives only one solution (a sphere and a circumference adjoin to each other in one point and have a common tangent), that is equivalent to the conditions (18), (19). In other words, this kind of singularity consists of the set of points where there are different branches of the inverse kinematics solutions, where the inverse kinematics problem is understood here as the computation of the values of the input coordinates at a given values of the output coordinates of the PM.

Since in this type of a configuration i-th leg and the axis of i-th actuated revolute joint are located in one plane σ_i, the set of mobile-platform velocities that correspond to a velocity of an attachment point of i-th leg to this platform along passive link A_iB_i, cannot be reproduced. This set of the mobile-platform velocities is determined by the set of rotations of the platform about an arbitrary line of a plane containing the i-th attachment point of a platform and orthogonal to the link A_iB_i. Moreover, any force applied to the mobile-platform along plane σ_i, and also a couple of forces applied at this platform parallel to the same plane σ_i, will not affect the actuator. This is so because the moments of these loadings (force and a couple of forces) about a rotation axis of i-th active-joint are equal to zero.

B. The second type of singularity

The second type of singularity is associated with degeneration of matrix **A**, i.e. such singularity occurs when the determinant of **A** vanishes. This type of singularity lies within the workspace of the PM and corresponds to a point or a set of points where different branches of the direct kinematics problem meet. In the direct kinematics problem, the values of the output variables from given values of the input variables should be obtained. For this type of configuration there exist nonzero mobile-platform velocities $\dot{\mathbf{x}}$, which are mapped into the zero vector by matrix **A.** These velocities of the platform are possible even when the actuated joints are locked. We shall write matrix **A** in the following form

$$\mathbf{A} = \begin{bmatrix} g_{1X} & g_{1Y} & g_{1Z} & n_{1X} & n_{1Y} & n_{1Z} \\ g_{2X} & g_{2Y} & g_{2Z} & n_{2X} & n_{2Y} & n_{2Z} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ g_{6X} & g_{6Y} & g_{6Z} & n_{6X} & n_{6Y} & n_{6Z} \end{bmatrix}, \quad (20)$$

where elements of first three columns are components of vectors $\mathbf{g}_i$, and elements of last three columns are components of vectors $\mathbf{n}_i = \mathbf{h}_i \times \mathbf{g}_i$, that is

$$\mathbf{g}_i = [g_{iX}, g_{iY}, g_{iZ}]^T,$$
$$\mathbf{n}_i = \mathbf{h}_i \times \mathbf{g}_i = [n_{iX}, n_{iY}, n_{iZ}]^T, \ i = 1, 2, \dots, 6.$$

The analysis of the structure of matrix **A** leads to reviewing the question whether linear dependences between lines or columns of matrix **A** generating the second type of singularities are possible? Both parts of this question are interdependent and it would be more reasonable to review a linear dependence of columns of matrix **A** that leads to required results faster. However, in the greater degree we are wondered under what conditions the linear dependence of line vectors of matrix **A** is possible, since components of each i-th line vector of matrix **A** consists of coordinates of two different and orthogonal vectors $\mathbf{g}_i$, $\mathbf{n}_i$.

Let's assume that matrix **A** degenerates because of a linear dependence of its two arbitrary lines with numbers j and k. It means that numbers λ_1 and λ_2 exist, which simultaneously are not equal to zero and for which the following formula is fair

$$\lambda_1[\mathbf{g}_j^T, \mathbf{n}_j^T]^T + \lambda_2[\mathbf{g}_k^T, \mathbf{n}_k^T]^T = \mathbf{0}, \ j \neq k \quad (21)$$

whence

$$[(\lambda_1\mathbf{g}_j + \lambda_2\mathbf{g}_k)^T, (\lambda_1\mathbf{n}_j + \lambda_2\mathbf{n}_k)^T]^T = \mathbf{0}, \ j \neq k. \quad (22)$$

These conditions should satisfy the corresponding configuration

$$\lambda_1\mathbf{g}_j + \lambda_2\mathbf{g}_k = \mathbf{0}, \Rightarrow \mathbf{g}_j = -\frac{\lambda_2}{\lambda_1}\mathbf{g}_k, \Rightarrow \frac{g_j}{g_k} = \left|\frac{\lambda_2}{\lambda_1}\right| = \mu, \quad (23)$$

$$\lambda_1\mathbf{n}_j + \lambda_2\mathbf{n}_k = \mathbf{0}, \Rightarrow \mathbf{n}_j = -\frac{\lambda_2}{\lambda_1}\mathbf{n}_k, \Rightarrow \frac{n_j}{n_k} = \left|\frac{\lambda_2}{\lambda_1}\right| = \mu. \quad (24)$$

The analysis of conditions (23) and (24) shows that equality to a zero vector of one of vectors $\mathbf{n}_j$ and $\mathbf{n}_k$ excludes a linear dependence of corresponding lines of matrix **A**. Indeed, let us assume that $\mathbf{n}_j = \mathbf{0}$, $\mathbf{n}_k \neq \mathbf{0}$. Then we receive that $\lambda_2 = 0$ from (24). But since $\mathbf{g}_j$ cannot be zero, we also obtain $\lambda_1 = 0$ from (23). I.e. numbers λ_1 and λ_2 are equal to zero simultaneously, hence, the considered lines cannot be linearly dependent.

Further we shall assume that $\mathbf{n}_j \neq \mathbf{0}$, $\mathbf{n}_k \neq \mathbf{0}$. Then, according to conditions (23) and (24) it should be $(\mathbf{g}_j \parallel \mathbf{g}_k)$ and $(\mathbf{n}_j \parallel \mathbf{n}_k)$, and with identical coefficient of proportionality μ. The specified requirements also mean the fulfillment of the following conditions

$$\mathbf{g}_j \uparrow\downarrow \mathbf{g}_k \ \Rightarrow \ \mathbf{n}_j \uparrow\downarrow \mathbf{n}_k \quad (25)$$

$$\mathbf{g}_j \uparrow\uparrow \mathbf{g}_k \ \Rightarrow \ \mathbf{n}_j \uparrow\uparrow \mathbf{n}_k. \quad (26)$$

Achieving a parallelism of vectors $\mathbf{g}_j$ and $\mathbf{g}_k$ is possible, therefore we admit that $(\mathbf{g}_j \parallel \mathbf{g}_k)$ and the condition (23) is satisfied. We shall check up whether a parallelism of the vectors $\mathbf{n}_j$ and $\mathbf{n}_k$ is possible under the condition of (24).

As $(\mathbf{n}_j \parallel \mathbf{n}_k) \Rightarrow (\mathbf{h}_j \times \mathbf{g}_j) \parallel (\mathbf{h}_k \times \mathbf{g}_k)$, the following two cases are possible:

Case 1⁰. Let's assume that $(\mathbf{h}_j \parallel \mathbf{h}_k)$ is possible. Obviously, it is one of necessary conditions of linear dependence of considered lines and it means that vectors $\mathbf{n}_j$ and $\mathbf{n}_k$ lay in planes Q_j and Q_k, which are perpendicular to vectors

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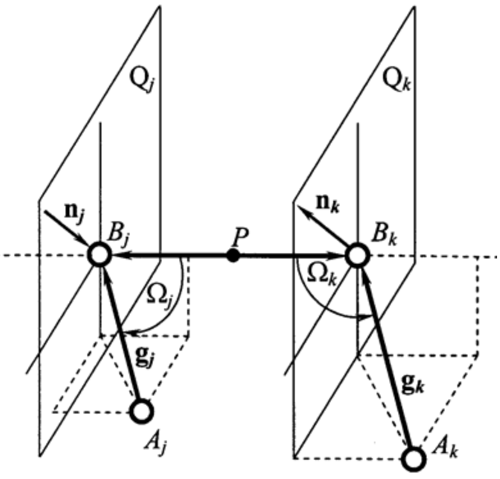

Fig. 3. Spatial layouts of vectors of *j*-th and *k*-th lines of a Jacobian **A** in a case when $\mathbf{n}_j \neq \mathbf{0}$, $\mathbf{n}_k \neq \mathbf{0}$, and $\mathbf{h}_j \parallel \mathbf{h}_k$

$\mathbf{h}_j$ and $\mathbf{h}_k$, respectively (Fig. 3). It is evident that $Q_j \parallel Q_k$. On the other hand, according to (24)

$$\frac{n_j}{n_k} = \frac{h_j \cdot g_j \cdot \sin\Omega_j}{h_k \cdot g_k \cdot \sin\Omega_k} = \mu ,$$

should be carried out, whence on the base of (23) and $\sin\Omega_j = \sin\Omega_k$, we obtain

$$h_j = h_k , \qquad (27)$$

i.e. pole *P* should be in the middle of $B_j B_k$. But at such layout of the pole *P,* conditions (25) and (26) are not satisfied. Indeed, from Fig. 3 we can see that $\mathbf{n}_j \uparrow\downarrow \mathbf{n}_k$ follows from $\mathbf{g}_j \uparrow\uparrow \mathbf{g}_k$. And also it is simple to be convinced that $\mathbf{g}_j \uparrow\downarrow \mathbf{g}_k$ corresponds to $\mathbf{n}_j \uparrow\uparrow \mathbf{n}_k$. Thus, the considered case does not lead to linear dependence of vectors lines of matrix **A** and singularity of a configuration of the PM.

Case 2^0. We shall consider a case when the vectors $\mathbf{h}_j$ and $\mathbf{h}_k$ are not parallel. Using an invariance of the (1) with respect to the choice of coordinates systems, we admit that vectors with which we operate are determined with respect to the base coordinates system instantly combined with local coordinates system $Px_Py_Pz_P$ by the following components

$$\mathbf{h}_j = [h_{jx},\ h_{jy},\ 0]^T,\ \mathbf{h}_k = [h_{kx},\ h_{ky},\ 0]^T,$$
$$\mathbf{g}_j = [g_{jx}, g_{jy}, g_{jz}]^T,\ \mathbf{g}_k = [g_{kx}, g_{ky}, g_{kz}]^T,$$

$$\mathbf{n}_j = (\mathbf{h}_j \times \mathbf{g}_j) = [h_{jy}g_{jz},\ -h_{jx}g_{jz},\ h_{jx}g_{jy} - h_{jy}g_{jx}]^T \qquad (28)$$

$$\mathbf{n}_k = (\mathbf{h}_k \times \mathbf{g}_k) = [h_{ky}g_{kz},\ -h_{kx}g_{kz},\ h_{kx}g_{ky} - h_{ky}g_{kx}]^T . \qquad (29)$$

Further, from $(\mathbf{n}_j \parallel \mathbf{n}_k) \Rightarrow (\mathbf{n}_j \times \mathbf{n}_k) = \mathbf{0}$, by using (28) and (29), we obtain

$$\begin{cases} h_{jx}g_{jz}(h_{kx}g_{kx} - h_{ky}g_{kx}) + h_{kx}g_{kz}(h_{jx}g_{jy} - h_{jy}g_{jx}) = 0, \\ h_{jy}g_{jz}(h_{kx}g_{kx} - h_{ky}g_{kx}) + h_{ky}g_{kz}(h_{jx}g_{jy} - h_{jy}g_{jx}) = 0, \\ g_{jz}g_{kz}(h_{jx}h_{ky} - h_{jy}h_{kx}) = 0, \end{cases} \qquad (30)$$

where, in relation to expressions in parentheses, it is possible to establish the following

$$(h_{kx}g_{ku} - h_{ky}g_{kx}) = \begin{vmatrix} h_{kx} & h_{ky} \\ g_{kx} & g_{ky} \end{vmatrix} \neq 0, \qquad (31)$$

as vectors $\mathbf{h}_k$ and $\mathbf{g}_k$ are not parallel;

$$(h_{jx}g_{jy} - h_{jy}g_{jx}) = \begin{vmatrix} h_{jx} & h_{jy} \\ g_{jx} & g_{jy} \end{vmatrix} \neq 0, \qquad (32)$$

as vectors $\mathbf{h}_j$ and $\mathbf{g}_j$ are not parallel;

$$(h_{jx}h_{ky} - h_{jy}h_{kx}) = \begin{vmatrix} h_{jx} & h_{jy} \\ h_{kx} & h_{ky} \end{vmatrix} \neq 0, \qquad (33)$$

as vectors $\mathbf{h}_j$ and $\mathbf{h}_k$ are not parallel.

Taking into consideration (31) - (33), we obtain from (30) the following

$$g_{jz} = 0, \quad g_{kz} = 0 , \qquad (34)$$

i.e. the vectors $\mathbf{g}_j$ and $\mathbf{g}_k$ or the passive links corresponding to them should be located in a plane of mobile platform Px_Py_P (Fig.4). Substituting (34) in (28) and (29), we obtain

$$\mathbf{n}_j = [0,\ \ 0,\ \ h_{jx}g_{jy} - h_{jy}g_{jx}]^T , \qquad (35)$$

$$\mathbf{n}_k = [0,\ \ 0,\ \ h_{kx}g_{ky} - h_{ky}g_{kx}]^T , \qquad (36)$$

i.e. the vectors $\mathbf{n}_j$ and $\mathbf{n}_k$ should be parallel to axis Pz_P.

Thus, (34) or (35) and (36) serve as a primary hint of possible occurrence of the second type of singularity configuration. I.e. they are taken as necessary conditions of linear dependence of considered line vectors of matrix **A** but they are not enough for degeneration of matrix **A**, and plus to this it is also necessary to require satisfaction of the condition (24)

$$\frac{|\,\mathbf{n}_j\,|}{|\,\mathbf{n}_k\,|} = \frac{n_j}{n_k} = \frac{h_j \cdot g_j \cdot \sin\Omega_j}{h_k \cdot g_k \cdot \sin\Omega_k} = \mu , \qquad (37)$$

where $\Omega_j = (\widehat{\mathbf{h}_j, \mathbf{g}_j})$ $\Omega_k = (\widehat{\mathbf{h}_k, \mathbf{g}_k})$. Whence, given (23), we obtain the following condition

$$h_j \cdot \sin\Omega_j = h_k \cdot \sin\Omega_k , \qquad (38)$$

i.e. projections of line segments PB_j and PB_k (modules of

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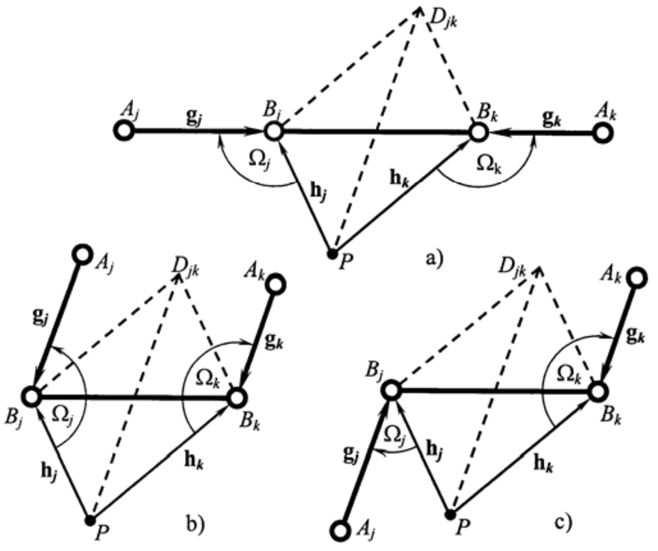

Fig. 4. Layouts of *j*-th and *k*-th passive links of PM in a plane of a mobile platform

orthogonal projections of the vectors $\mathbf{h}_j$ and $\mathbf{h}_k$) on a direction which perpendicular to vectors $\mathbf{g}_j$ and $\mathbf{g}_k$, should be equal. However, we do not know, how vectors $\mathbf{g}_j$ and $\mathbf{g}_k$ are directed. We only know that they are parallel and lay in one plane with pole *P* of a platform according to (34). That's why at first we determine those directions where the considered line segments have equal projections. It is possible to show that such directions are perpendicular to diagonals of the parallelogram constructed on vectors $\mathbf{h}_j$ and $\mathbf{h}_k$. It is necessary consideration of different variants of a placement of vectors $\mathbf{g}_j$ and $\mathbf{g}_k$ determined by directions of these diagonals (Fig. 4): along a straight line (B_jB_k) and in parallel to a straight line (PD_{jk}). However, the analysis shows that only four variants of a layout of passive links along a straight line (B_jB_k) satisfy conditions (25), (26), one of which is shown in Fig. 4a. Examples of a layout of the same links which do not generate singularity configuration are shown on Fig. 4b, c.

Let's note that consideration of a case when both vectors $\mathbf{n}_j$ and $\mathbf{n}_k$ are simultaneously equal to zero brings to the same results.

V. Conclusion

As parallel manipulators are constructed on the basis of the closed kinematic chains, they become more complex and exhibit a much broader range of kinematic behavior than serial manipulators on open chains. Therefore the understanding of parallel manipulator singularities is needed in order to avoid possible undesirable consequences. In this paper the singularity analysis of the new six-legged parallel manipulator with six DOF has been considered. It was shown that the first type of singularities occur whenever the passive link, the actuated link, and the rotation axis of the active joint have a layout in one plane for any of legs. Physical interpretation of this singularity configuration of the manipulator is given. Also one of the conditions of occurrence of the second type of singularity configuration of this parallel manipulator is established on the basis of consideration of degeneracy of direct-kinematics Jacobian matrix in consequence of linear dependence of its two arbitrary lines. According to this condition the passive parallel manipulator links corresponding to two linearly dependent lines of a Jacobian matrix have a layout along a direct line which passes through attachment points of these links with a mobile platform. The obtained conditions of configuration singularities can be expressed through constants and variable parameters of the parallel manipulator that is important for the control of such configurations. The further publications of research results of the second type singularity of the considered parallel manipulator because of a linear dependence of three and more line vectors, and also of the columns vectors of a direct-kinematics Jacobian matrix are being planned.

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