Residual Modes on Non-linear Resonant Decay Method

Dr Mehdi Sarmast, Professor Jan R. Wright

ABSTRACT

Non-linear Resonant Decay method (NL-RDM) addresses the identification of multi-degree of freedom non-linear systems. This method offers a practical approach to the identification of lumped parameter and continuous systems by producing a non-linear extension of the classical linear modal model. The method is introduced and its potential as a practical identification approach explained.

This paper is concerned with the inclusion of residual modes, above and below any region of interest.

Any structure that is not supported to earth will have rigid body modes, being modes that have a natural frequency at zero Hertz; a good example is an aircraft. These modes are sometimes ignored but for a complete mathematical model obtained from NL-RDM the effects of rigid body modes must be analysed. NL-RDM relies on a method of curve fitting to generate modal characteristics, in terms of force, displacement, velocity and acceleration in modal space.

The lower residual region was observed to contain rigid body modes, and these were observed to affect significantly the results in the region of interest. Monitoring this effect was shown to be too difficult currently for the NL-RDM, given technological restrictions. A Mass Substitution method was generated to model the system response more accurately. Its accuracy was demonstrated through case studies.

Most systems also contain an indefinite number of residual modes, these being modes that occur at frequencies greater than the frequency range of interest. The section will attempt to determine the effects of excluding residual modes on the accuracy of NL-RDM.

The effect of the upper residual area on the modelling of the system within the region of interest was observed through several case studies. Simple validation methods have been considered to preserve the accuracy of the model, taking into account both harmonic and coupled residual modes and their effects.

Keywords: NL-RDM, Residual Modes, Rigid Body Mode, Out of Range modes.

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I. METHODOLOGY FOR NL-RDM

The method is a development of the Resonant Decay Method, the Restoring Force Surface method based in modal space and the Force Appropriation methodology [1, 2]. It makes use of an appropriated excitation vector applied in a burst to reduce the number of modes required in the curve fit.

In this section, the mathematical form of the extended modal model for non-linear systems will be described. Consider a lumped parameter system or a continuous one that has been discredited as N degrees of freedom (DOF). The equation of motion in physical space for a dynamic system including stiffness non-linearities is

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K_L]\{x\}+[K_{NL}\{x\}]\{x\}=\{F(t)\} \qquad (1)$$

where [M] is the mass matrix, [C] is the damping matrix, $[K_L]$ is the linear stiffness matrix, $[K_{NL}]$ is the non-linear stiffness matrix, {F(t)} is the vector of applied nodal forces, and {x(t)} is the vector of physical displacements. The transformation between physical and modal space is defined by

$$\{x(t)\}=\sum_{r=1}^{N}\{\phi\}_r\, p_r(t)=[\phi]\{p(t)\} \qquad (2)$$

where {p(t)} is a vector of modal amplitudes, and $[\phi]$ is the modal matrix of the N modes $\{\phi\}_r$, r = 1,2…*N* of the underlying linear system, which may be obtained by solving the classical eigenvalue problem for un-damped free vibration. The corresponding un-damped natural frequencies are ω_r (r = 1, 2 … *N*). In practice, the number of degrees of freedom required to model the system with reasonable accuracy can be reduced by considering only those modes *NR* << *N* having natural frequencies in the frequency range of interest, or that are considered important in the response. In this case, the modal matrix $[\phi]$ is reduced to dimension N by NR and the vector of modal amplitudes {p(t)} to dimension NR by 1. Substituting the modal expansion into the system equations of motion and pre-multiplying by $[\phi]^T$ yields:

$$\begin{aligned}&[\phi]^T[M][\phi]\{\ddot{p}(t)\}+[\phi]^T[C][\phi]\{\dot{p}(t)\}+\\&\left([\phi]^T[K_L][\phi]+[\phi]^T[K_{NL}][\phi]\right)\{p(t)\}=[\phi]^T\{F(t)\}\end{aligned} \qquad (3)$$

Using the orthogonality of the modes, this equation of motion in modal space becomes

$$[m]\{\ddot{p}(t)\}+[c]\{\dot{p}(t)\}+[k_L]\{p(t)\}+\{f_{NL}(t)\}=\{f(t)\} \quad (4)$$

where the modal mass matrix [m] and linear modal stiffness matrix $[k_L]$ are diagonal and the modal damping matrix [c] is diagonal for proportionally damped systems. {f} is the applied modal force vector and $\{f_{NL}\}$ is the vector of non-linear restoring forces in modal space. Clearly, in order to be able to perform this transformation, the modal matrix $[\Phi]$ is assumed to be known from low force level results. For a particular mode of a proportionally damped non-linear system, equation (4) is in the form of a single degree of freedom system, namely

$$m_r\ddot{p}_r + c_r\dot{p}_r + k_r p_r + f_{rNL} = f_r(t) \quad r = 1,2...NR \quad (5)$$

where p_r is the r^{th} modal displacement, m_r, c_r and k_r are the r^{th} mode modal mass, damping and stiffness, and f_r is the corresponding applied modal force. The modal mass and stiffness for the linear system are related by the un-damped natural frequency from the equation $k_r = \omega_r^2 m_r$ [3]. Non-proportional damping would lead to the presence of modal damping coupling terms [4]. The term f_{rNL} refers to the r^{th} mode non-linear modal restoring force and in general includes other modal co-ordinates to allow for non-linear cross coupling

$$f_r(p_1, p_2,.., p_{NR})_{NL} = \sum_{s=1}^{NR}\sum_{i=2}^{3} A_r^{(s^i,s^0)} p_s^i + \sum_{s=1}^{NR-1}\sum_{t=s+1}^{NR}\sum_{i=1}^{2}\sum_{j=1}^{3-i} A_r^{(s^i,t^j)} p_s^i p_t^j \quad (6)$$

where the A_r term is the amplitude of the non-linear term in the r^{th} mode equation that is associated with the respective modal displacements p_s and p_t (for modes identified by the indices s and t), raised to the powers defined by i and j. For example, the direct cubic non-linear stiffness term associated with the r^{th} mode would be $A_r p_r^3$ (s=r, t=r, i=3, j=0) and a typical cross-coupling term between modes r and q might be $A_r p_r^2 p_q$ (s=r, t=q, i=2, j=1). Note, in this case, that the set of possible basis functions is restricted to terms of 3^{rd} order involving no more than two modes. Also, the basis functions in equation (6) are polynomials whereas non-polynomial terms may be used to cater for different types of non-linearity (e.g. discontinuous) [5]. There is no reason in principle why non-linear modal velocity terms cannot also be included in the extended modal model to cope with damping non-linearity.

A. SIMPLE EXAMPLE

Non-linearity is not easy to define in a general manner and so the NL-RDM cannot be applied to a general case. Thus, it is recommended that the consideration of the method start with a simple example such as a two degree of freedom system with some kind of non-linearity, which could be the common cubic stiffness, to take advantage of a relatively simple case. Some other type of non-linearity such as quadratic damping, friction and backlash will not be investigated in this paper.

II. RIGID BODY AND OUT OF RANGE MODES

The previous paper [6] attempted to determine the effects the use of imperfect data has on NL-RDM, assuming use of the correct model. This paper will assume the use of perfect data and test process, but will analyse some of the effects of using an incorrect model. The analysis will include the effects of rigid body modes, of problems associated with harmonics, and of the residual part of the data. The effects will be analysed in terms of derived system parameters and the FRF.

Any structure that is not supported to earth will have rigid body modes, being modes that have a natural frequency at zero Hertz; a good example is an aircraft. These modes are sometimes ignored but for a complete mathematical model obtained from NL-RDM the effects of rigid body modes must be analysed. NL-RDM relies on a method of curve fitting to generate modal characteristics, in terms of force, displacement, velocity and acceleration in modal space.

Most systems also contain an indefinite number of residual modes, these being modes that occur at frequencies greater than the frequency range of interest. This chapter will attempt to determine the effects of excluding residual modes on the accuracy of NL-RDM. Finally, harmonic effects will be analysed. Specifically a system with a cubic stiffness non-linearity will be observed, when one mode natural frequency occurs at precisely 3 times the natural frequency of the other mode.

A. IDENTIFYING RIGID BODY MODES

Rigid body motion occurs in many systems and affects the dynamic behaviour, so should be investigated. A rigid body mode is a mode with a natural frequency of zero Hertz and mode shapes which normally consist of pure translation or rotation; rigid body modes occur in a perfect free-free structure. Experimentally, rigid body motion is observed to cause no strain in any elastic element in the system. Rigid body behaviour can include six types of motion, which are translation and rotation in 3 dimensions.

The way in which the NL-RDM would approach the analysis of rigid body modes needs to be discussed, as a complete model is required in order for the dynamic behaviour of the system to be investigated. The problems encountered by this approach give rise to three practical methods of analysing structures with rigid body modes. Firstly, the mode is identified using the NL-RDM in the same way as for any other mode. Secondly, the rigid body mode can simply be ignored. The effects of this on accurate system representation are analysed by comparison between practical results for models with and without rigid body modes. Thirdly, the rigid body mode is simulated by a substituted modal mass element without the need for any specific identification.

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A.1. NON-LINEAR MDOF MODELS WITH RIGID MODY MODES

In this section, two systems are introduced for the evaluation of the practical method, i.e. to determine the effectiveness of approximating a free-free system by a lightly sprung fixed-fixed system. The three degree of freedom lumped parameter system with stiffness non-linearity between the second and third masses (free-free structure) in Figure 1, is compared to another structure, identical except for being supported at both ends on soft springs (practical structure) in Figure 2.

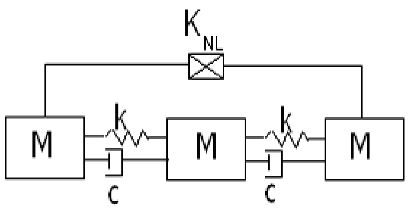

Figure 1: Schematic Diagram of 3 Degree of Freedom Free-free System with Stiffness Non-linearity (perfect free-free structure)

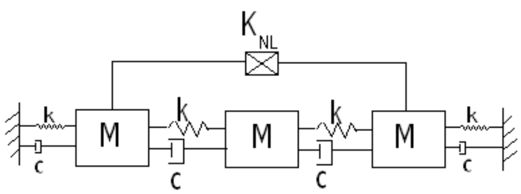

Figure 2: Schematic Diagram of 3 Degree of Freedom Free-Free System with Stiffness Non-linearity but supported on Soft Springs (practical structure)

The FRF results for the perfect free-free and practical systems are shown in Figures 3 and 4 respectively. Comparison between the first and second modes, i.e. the rigid body mode and the first flexible mode, gives an indication of the suitability of the practical system for approximating the free-free system. At a ratio of 1/10 the first mode is considered close enough to zero Hertz to not have any significant effect on the flexible mode; in this example, the rigid body mode natural frequency is 0.4 Hz and the first flexible mode is 5.06 Hz. For better comparison between the perfect and practical system, the mode shapes or modal matrix for the two cases are as follows

$$ModalMatrix_{Perfect} = \begin{pmatrix} 1 & 1 & -0.5 \\ 1 & 0 & 1 \\ 1 & -1 & -0.5 \end{pmatrix} \qquad (7)$$

and

$$ModalMatrix_{Practical} = \begin{pmatrix} 0.9967 & 1 & -0.5017 \\ 1 & 0 & 1 \\ 0.9967 & -1 & -0.5017 \end{pmatrix} \qquad (8)$$

Clearly, the support has little effect on the modal characteristics. The FRFs of the perfect and practical structures introduced in the previous section, are compared in Figures 3 and FRFs phases in Figure 4. The FRFs show strong agreement, indicating the accuracy of the rigid body mode approximation and justifying the analysis of the supported structure for rigid body mode identification.

Figure 5 shows the MMIF [7] for the 3 degree of freedom practical structure, where the rigid body mode on soft supports is included. Clearly, appropriated force patterns could be obtained from such an analysis.

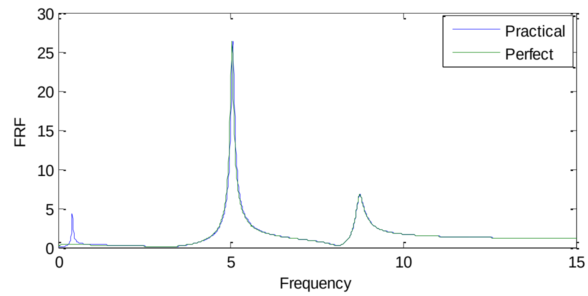

Figure 3: H_{11} FRFs for the Perfect free-free Structure and the Practical System

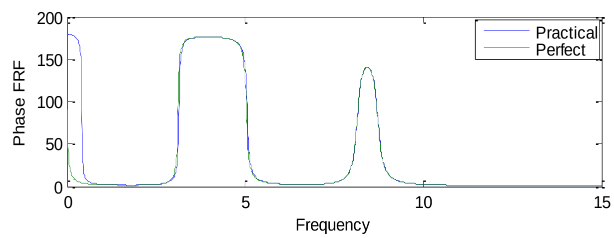

Figure 4: H_{11} FRFs Phases for the Perfect free-free Structure and the Practical System

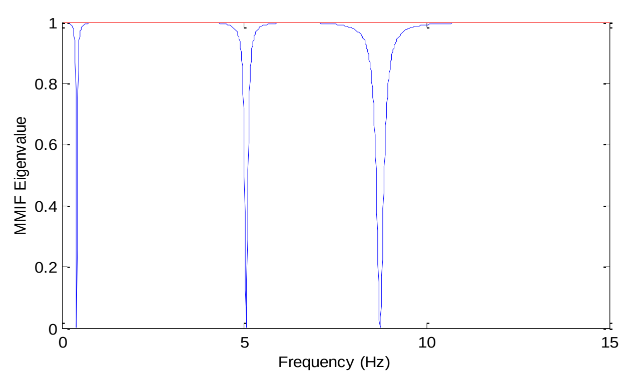

Figure 5: MMIF for the 3DOF Practical Free-free System

A.2 IDENTIFICATION OF A RIGID BODY MODE USING NL-RDM

Having confirmed that it is acceptable to consider a free-free structure to be supported on soft springs, one possibility for identifying the rigid body behaviour is to simply apply the NL-RDM [8] approach just as for a flexible mode. The NL-RDM is perfectly suited to identify a free-free structure by simulation, but problems arise when seeking to identify a structure in practice, i.e. experimentally.

Several problems hinder the analysis of rigid body modes practically as the rigid body mode may need to be investigated with very low frequencies. This may create a significant problem with noise, as the level of noise is closer to the low acceleration levels experienced at low frequencies. The very low frequencies create further problems experimentally when generating force and monitoring response. Piezo-electric transducers are not accurate at low frequency levels as the characteristics roll-off at low frequencies. However, it would be possible to use accelerometers that operate down to DC (e.g. piezo-resistive accelerometers) or even displacement transducers.

Commonly, shakers would be used to generate force and accelerometers used to monitor the motion, so the ability to generate acceptably low levels of force and monitor the low levels of motion, and hence obtain accurate results, is limited by the capability of these pieces of equipment. It is well known that it is difficult to use an electrodynamic shaker to generate a sinusoidal force at low frequency.

Further problems are encountered when using the NL-RDM to identify the rigid body mode. Problems may arise when preparing for force appropriation. A poor MMIF, specifically a poorly identified rigid body mode causes an inaccurate force vector and poor tuning of the system.

These problems are critical with a lower flexible mode frequency. For example, with a first flexible mode around 5 Hz, the rigid mode frequency would be around 0.5 Hz and the procedure of detecting and identifying the mode becomes much more difficult than when a structure's first flexible mode frequency appears around 100 Hz and the rigid body frequency could be more like 10 Hz. For such a stiffer structure, there would be no particular problem in using the NL-RDM to identify the rigid body mode linear characteristics; no non-linear behaviour would be expected. Typically, the instrumentation problems mentioned above become worse below about 4 Hz, corresponding to a flexible mode at 40 Hz if the rigid / flexible mode frequency ratio is 10%.

Theoretically, using improved force generation / recording, with better shakers and accelerometers could produce better, more accurate results at very low frequencies. Technologies are already available or being developed that could improve the accuracy of these devices (e.g. laser based measurement systems). Clearly, improved low-level accuracy of components would have a significant effect on the difficulties associated with rigid body mode experimentation. With accurate enough components, the rigid body mode could theoretically be analysed by the NL-RDM in the same way as any other mode.

However, if direct identification of very low frequency rigid body modes is too difficult to perform accurately, maybe there are other approaches? Perhaps the rigid body mode could simply be ignored or a theoretically based estimate included?

A.3 NEGLECTING THE RIGID BODY MODE IN THE Non-IDEALIZED MODEL

Another method that could be employed to analyse the system response in a non-linear system with a rigid body mode is to entirely ignore the rigid body mode itself. For example, in the flutter analysis of an aircraft, often only the flexible modes are included. The non-linear system is then analysed like any other system, in terms of its flexible modes. The ease of this method is clearly its greatest advantage, but what may be a significant mode is missed out. Case studies are carried out in this section to demonstrate the accuracy of this approach as compared to identifying the rigid body mode from measured experimental results.

The same system as introduced in Figure 2 is chosen for analysis. Figures 6 and 7 show the calculated FRFs for the system when the rigid body mode is ignored (Green colour), and compares them to results from a complete analysis, with the rigid body mode on soft supports included in the practical system (Blue colour). While the general shape of the FRFs match around the two flexible mode peaks, it is clear that response levels are all somewhat inaccurate when the rigid body mode is ignored. The effect occurs not only where the rigid body peak is not modelled, but at other frequencies too, because the influence of the rigid body mode extends across the entire frequency range of the system. Thus, while this method can be used as an approximation to the true system, its accuracy for generating system response and thus its application may be seriously limited. Figure 8 implements a flowchart, which indicates the way the comparison between 2 systems is done.

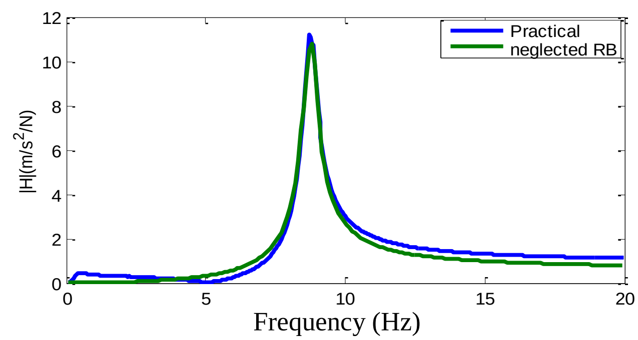

Figure 6: Comparisons of FRF (H_{22})

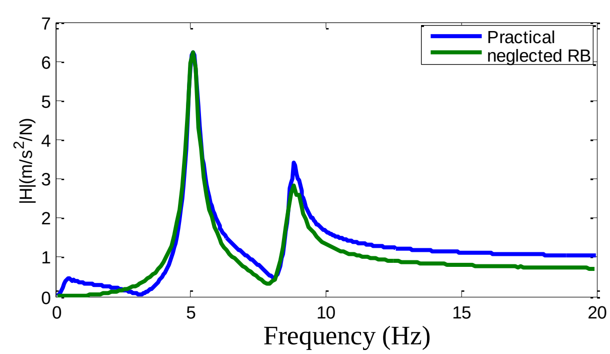

Figure 7: Comparisons of FRF (H_{11})

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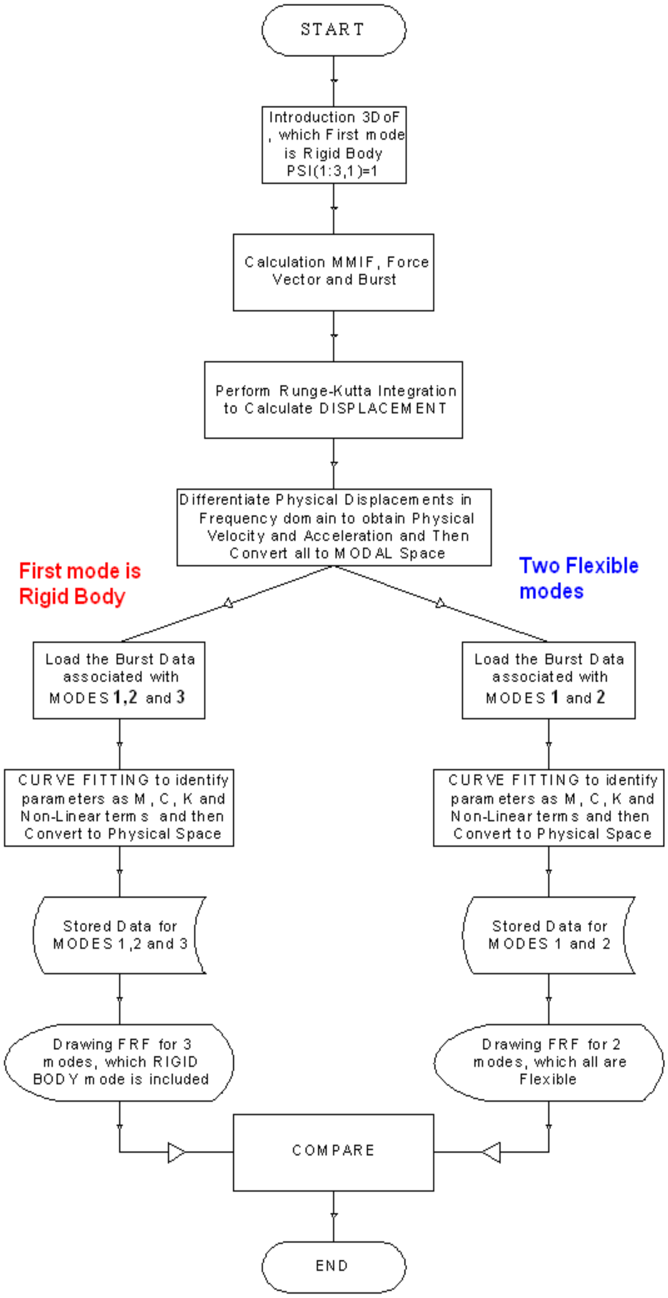

Figure 8: Comparison between 2 Systems – Rigid Body Ignored or Included

A.4 REPRESENTING THE RIGID BODY MODE WITHOUT SPECIFIC IDENTIFICATION

Due to the inaccuracy of measurement of low frequency rigid body modes, a complete identification using NL-RDM may be considered too inaccurate to be workable; certainly this is the case for an aircraft where the flexible modes occur at low frequencies (typically 2-5 Hz). Therefore a method is required that will readily allow for the rigid body mode effects in the identified model; for example, in curve fitting an FRF, it is common to include a lower residual term in the model to approximate the rigid body effect [9]. In this section, a method is proposed that will simply add in a modal equation to allow for the rigid body mode contribution.

The mass representation method involves analysing the rigid body mode in terms of an added modal mass, that mass being derived from a mass summation over the structure, assuming a rigid body mode shape. This method has the advantage of a more complete system analysis, without making any major changes or additions to the original method.

The approach will be illustrated for the simple case of the 3 DOF system considered earlier. The extension to more complex problems will be outlined later. The rigid body modal mass M_{RB}, which is also in a sense a mass summation, could be calculated as

$$M_{RB} = \Phi_{RB}^T * M_{Physical} * \Phi_{RB} \tag{9}$$

where the rigid body mode shape for the 3 DOF system is

$$\Phi_{RB} = \{1 \quad 1 \quad 1\}^T \tag{10}$$

and $M_{Physical}$ is the physical mass matrix of the system. Thus the rigid body modal mass is

$$M_{RB} = (1 \quad 1 \quad 1) * \begin{bmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{bmatrix} * \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{11}$$

or

$$M_{RB} = M_1 + M_2 + M_3 \tag{12}$$

or more generally

$$M_{RB} = \sum M_i = M_{Total} \tag{13}$$

Thus, if the total mass of the structure is known by direct measurement or calculation, and if the rigid body translation mode shape is assumed to have unit values, the rigid body modal mass is simply equal to the total mass of the structure.

In order to include the rigid body mode behaviour as an additional equation to accompany the identified flexible mode equations, then the appropriate modal force and possibly modal mass and damping values should be estimated.

By analogy to the analysis for mass performed above, the rigid body modal stiffness K_{RB} will equal the summation of the support stiffnesses (e.g. bungees) and the modal damping C_{RB} will be the summation of the support damping coefficients, both transformed into modal space.

The rigid body modal force may be determined through the physical forces to be applied to the system, which means that:

$$F_{RB} = \Phi_{RB}^T * F_{Physical} \tag{14}$$

$$F_{RB} = (1 \quad 1 \quad 1) * \begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix} \tag{15}$$

$$F_{RB} = F_1 + F_2 + F_3 \tag{16}$$

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or more generally

$$F_{RB} = \sum F_i = F_{Total} \tag{17}$$

Thus the rigid body modal force is simply equal to the summation of the physical forces applied to the structure.

The final modal equation for the rigid body mode (including supports if desired) will then be

$$M_{RB}\ddot{q}_{RB} + C_{RB}\dot{q}_{RB} + K_{RB}q_{RB} = F_{RB} \tag{18}$$

where q_{RB} is the rigid body modal coordinate. This modal equation is simply added to those identified for the non-linear flexible modes.

It should be noted that no non-linear modal couplings will exist between rigid body and flexible modes, provided the support stiffnesses are linear, because no non-linear forces will be generated by rigid body motion and forces generated in motion of a flexible mode will not do any work in the rigid body mode.

For implementation in the simulation programme, a system with 3 degree of freedom with cubic stiffness non-linearity and one rigid body mode is considered. Figure 2 shows the schematic system of the practical structure. Firstly, only the rigid body modal mass is included (i.e. effect of support stiffness is ignored) and then the support effects are included through the modal stiffness K_{RB}.

The accuracy is clearly improved in comparison with the 'Neglected…' method, with the FRFs being mostly coincident. The only variation is seen in the representation of the rigid body mode itself, around which point the methodically calculated FRF levels off, giving a lower response level around the rigid body mode.

Thus this approach is a simple and practical one to including rigid body mode effects in the identified non-linear model without needing to carry out difficult low frequency measurements. The approach represents the rigid body effects accurately, provided the estimated mass is reasonably accurate; indeed, the representation of the rigid body modes is considered to be more accurate than using the lower residual term in a curve fit. If support stiffnesses were only included to make the test possible, and a model of the true free-free system required, then only the rigid body modal mass and force need be included in the final model, as the support stiffness and damping effects can be ignored.

A.5 PRACTICALITIES FOR MORE COMPLEX STRUCTURES

In reality a structure could have as many as six rigid body modes. This will affect the mass representation method.

For the 3 translation modes, the modal masses will all equal the total mass of the structure and the mode shapes will be equal translation of all points in the same direction, i.e. simple extension of the above approach. Thus, for a heave mode,

$$M_{RB}^{Heave} = \sum M_i = M_{Total} \tag{19}$$

and so on.

The other rigid body modes will involve rigid body rotation about the centre of mass (e.g. roll, pitch and yaw). In this case, the substitution would be an inertia representation, as opposed to being mass based for a translation type mode.

Thus for a pitch mode, it may be shown that

$$M_{RB}^{Pitch} = \sum I_i = I_{Total} \tag{20}$$

where I_i is the pitch moment of inertia of the i^{th} component of the structure about the centre of mass and I_{Total} is the total pitch moment of inertia of the structure about the centre of mass. The pitch mode modal force will be the sum of applied moments of each of the physical forces about the centre of mass. The corresponding rotation mode shape must be a 1 radian rotation if the modal mass is the moment of inertia.

In practice, if the support stiffness effects are to be included in the rigid body modal equations, the values may be found by direct measurement of the spring stiffnesses or by determining the rigid body natural frequencies by exciting or disturbing the structure into a rigid body motion, even though the data may be noisy, and then calculating the modal stiffness from the modal mass. Flexible modes would be identified as normal using NL-RDM.

B. UPPER RESIDUAL EFFECTS

It is possible to divide the range of frequencies into three regions. Of central concern is the particular region of interest, which will often include several important flexible and maybe non-linear modes. If this area is considered in isolation, it may be modelled inaccurately, as the energy from modes is less than expected in that region. The preceding section has considered the low frequency region below the region of interest, containing one or more rigid body modes, and observed the effect of its presence on the region of interest; approaches for allowing for the rigid body effects were proposed. Additionally, the region of frequencies higher than the region of interest should be considered and its effects on the region of interest observed. These higher frequency modes occur at lower levels of displacement, and so are more likely to be linear, and therefore arguably may be less of a concern.

The problem with such out of range high frequency modes is that their effects will appear in the measured data, up to around the Nyquist frequency [10], but the model constructed by the NL-RDM approach will only cover a limited number of modes, likely to be smaller than the number of modes in the measured range. In curve fitting of an FRF, it is common to include an upper residual term to allow crudely for the modes above the range analysed. However, there appears to be no obvious way of allowing for out of range modes above the range of interest; adding a representative mode would seem to be rather artificial. It seems the only way is to ensure that the frequency range

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analysed is somewhat wider than that for which the model is really required; this is more important for a non-linear structure than for a linear one as will be shown later.

B.1 EFFECTS OF MISSED MODES

In a continuous system, there are a great many modes present. It would be impractical to model all of them, and so the effect of missed modes should be observed. These missed modes may be coupled to included modes and so the effect of those missed modes should not be neglected.

To observe the effect of missed modes on a simple example, a three degree of freedom non-linear lumped parameter system is modelled, and force, displacement, velocity and acceleration data derived. Then, in this study case, the third and highest mode is omitted from the fitted model (but not its effects from the data) and the curve fit would be run for both systems (3 and 2 DOF). The FRF result is shown in Figure 9. Results have shown some loss of energy and inaccuracy on the FRF. However, the estimated parameters for the two identified modes were actually the same (i.e. not in error); this is because there was no non-linear coupling and 3 shakers were used so the appropriation was perfect. Errors could be present if the appropriation was imperfect and if non-linear couplings were present; any error would then depend upon how large the missed mode contribution was and how close in frequency.

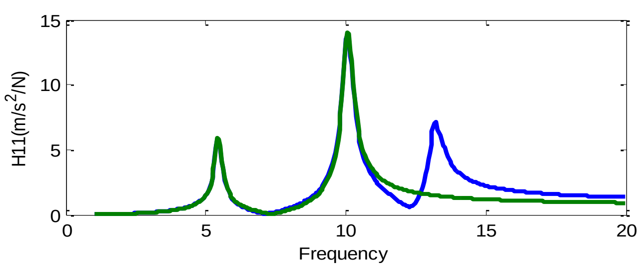

Figure 9: FRF 11, when Third mode is missed

There is no way to eliminate these errors; the only approach is to ensure that any modes with a large response are included and that the frequency range chosen is adequately large so that the effect of higher frequency modes is small.

B.2 HARMONIC EFFECTS

One particular case of out of range modes that is of interest is when an out of range mode occurs at a harmonic of a mode within the range of interest. Harmonic effects need to be considered for a non-linear system. The harmonic behaviour depends on the order of non-linearity, which means that if a system has a quadratic or cubic non-linearity, harmonic effects may be observed at two or three times any natural frequency respectively. The so called ‘harmonic modes’ outside the region of interest may store a significant amount of energy, and so shouldn’t be neglected.

The effects of these harmonic modes are considered through the modelling of a system with two natural frequencies, at 7.12 and 21.36 Hz, the second mode occurring at the third harmonic of the first mode. The simulation programme is run for cases around and at exactly three times the ratio between natural frequencies to indicate any effects on the results. When the system was identified with both modes present, the identified parameters were not affected because all the relevant terms were included in the analysis; however, consideration of this harmonic issue is required when the 3 times natural frequency is outside the region of interest.

The results showed the response of the higher frequency (harmonic) mode is clearly much larger in this case, especially for the acceleration. Another comparison based on examining time domain signals showed that the modal acceleration for mode 2 when seeking to excite mode 1 was doubled when the frequencies differed by a factor of 3.

What is of more interest is that when mode 1 was analysed alone using NL-RDM, with the presence of mode 2 completely ignored, the modal parameters identified were more in error when the two modes differed by exactly a factor of 3.

B.3 PRACTICALITIES

Practically, it may be important to validate the response model generated within the region of interest. This would be best achieved through modelling of the system with one additional mode outside this region, and observing the effect of the inclusion of this mode on the originally identified parameters and fit to a validation excitation. If there is no significant change, then the range chosen is arguably adequate. If there is a significant difference, then the range needs to be extended.

In recent sections has been concerned with the inclusion of residual modes, above and below any region of interest. The lower residual region was observed to contain rigid body modes, and these were observed to possibly affect significantly the results in the region of interest. Obtaining rigid body parameters experimentally was shown to be too difficult currently for the NL-RDM, given technological restrictions. A mass representation method was generated to model the system response more accurately by adding in a modal equation for each rigid body mode. Its accuracy was demonstrated through case studies.

The effect of the upper residual area on the modelling of the system within the region of interest was observed through several case studies. No obvious way of allowing for higher frequency modes seems available and so it is important that an adequate range of data is considered. Modes whose frequency is at a harmonic factor of modes of interest need to be considered carefully as if they are omitted, significant errors may occur. Simple validation methods have been considered to preserve the accuracy of the model, taking into account both harmonic and coupled residual modes and their effects.

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III. CONCLUSIONS

In this paper, problems with residual effects have been investigated, including rigid body mode effects in the lower residual area, and missed / coupled modes in the upper residual area. Harmonic effects, where one mode natural frequency was an integer multiple of another mode were also investigated. Measuring any rigid body mode is difficult because, even though the structure is supported on low stiffness mounts, the frequency will usually be very low and excitation and measurement are inaccurate. So, some methods were introduced in the simulation programme for 2 and 3 degree of freedom systems to determine or maybe neglect a rigid body mode. The first option could be to neglect the rigid body mode. The results show some inaccuracy not just around the rigid mode that neglected, but also at other frequencies even when the rigid body mode was at a significantly lower frequency than the flexible mode(s). Another option could be considered when, instead of identifying the rigid body mode, the modal mass may be represented theoretically by a summation of physical mass and inertia parameters and a theoretical modal equation added to the overall model; a modal stiffness term may be estimated if the rigid body frequencies on the supports can be estimated. The results show good agreement when compared with neglecting the rigid body mode. In a real structure, estimating accurate mass and inertia parameters could become a problem although the FE model could be used.

In real structures, because a significant number of modes are present in the frequency domain, only regions of interest are considered but then how could upper residual effects be allowed for? Effects of this region have been investigated. Missing or including a mode in a region of interest is compared for a 3 degree of freedom system. Unlike linear identification, there seems no way of including an upper residual term in the non-linear model to allow for the effect of the out of range modes. To avoid this problem, it is recommended that extra modes be added to the region of interest and, the effect of the increased size model on the validation checked to see if the difference is significant; so maybe more modes need to be included than for a linear identification. Alternatively, the level of energy in the out of range modes could be assessed to see if they need to be included.

One more issue, is about the so-called harmonic effect where an out of range mode frequency is at an integer multiple of a mode in the region of interest. As explained, the harmonic effect should be considered carefully in a non-linear system and depends on the non-linear order (i.e. is non-linearity of second or third order). For example for cubic stiffness non-linearity, 3 times frequency should be examined and, depending on the level of energy in that mode, could be considered or left out of the region of interest.

The overall study showed that the NL-RDM could allow a non-linear modal model to be identified. The levels of inaccuracy present for expected levels of error was considered to be acceptable; even though high levels of inaccuracy were sometimes encountered, the errors were higher than might be expected from modern instrumentation and methodologies. This is the nature of non-linearity for which there is no perfect solution to non-linear system identification up to now. The method needs to be modified, but at the end of the day, NL-RDM is still in relatively early stages of exploration and development should continue.

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